Abstract—In this paper, power system modes are monitored for the purpose of stability prediction using an innovated recursive Subspace System Identification (SSI) method. SSI methods can process large package of sampled data using powerful mathematical tools. However, their application for monitoring requires some changes in SSI algorithms in order to provide their recursive versions. SSI algorithms have some troublesome steps which take enormous processing load. For instance, Singular Value Decomposition (SVD) which we avoid it using some innovations from propagator and projector methods, in order to propose a Recursive SSI (RSSI) algorithm. There are some difficulties in application of SSI for monitoring purposes which we process them in different sections of the paper. Finally, we provide a recursive SSI algorithm and apply it to different large scale power systems using computer simulations. Simulation result expresses good performance of the algorithms for power system mode identification and tracking.

Keywords—Recursive, subspace, identification, monitoring, power system, mode tracking.

I. INTRODUCTION

In the last decay, recursive subspace system identification (RSSI) has extended its application to many engineering fields [1-3]. Applications usually are proposed for fault identification and adaptive control. SSI methods have some advantages such as application of mathematical and geometrical techniques for processing of input-output data. Thus, linear algebra has a prominent role in SSI algorithms. However, most of linear algebra tools applied to SSI is not suitable for recursive SSI. Among them, Singular Value Decomposition (SVD) is not suitable for online implementation and recursive applications. In this case, avoiding such a troublesome processing tool is one of the difficulties in developing RSSI.

In [3], it is expressed that RSSI has not been applied to power system mode identification. Therefore in this paper, we present an optimized RSSI algorithm for power system mode tracking. This algorithm uses the similarities between signal processing techniques for estimation of direction of arrival (DOA) and Yung measure. In this way, we illustrate that if we use instrumental variable version of Projection Approximation and Subspace Tracking (PAST) method, an unbiased algorithm is achieved.

II. RECURRENCE SSI OF SYSTEM STOCHASTIC MODEL

Stochastic Model of a $n^{th}$ linear time invariant system which has $l$ outputs and $m$ inputs is:

$$
\begin{align*}
X(t+1) &= AX(t) + Bu(t) + w(t) \\
Y(t) &= CX(t) + Du(t) + v(t)
\end{align*}
$$

where $w$ is $n$ dimensional process noise vector and $v$ is $l$ dimensional output noise.

The main problem in stochastic SSI is to estimate $i^{th}$ order column space of extended observability matrix $\Gamma_i$ using input-output samples.

$$
\Gamma_i = [C^T (CA)^T (CA^2)^T \ldots (CA^{i-1})^T]
$$

We can estimate system matrices using estimation of above observability matrix and a similarity transformation. Identification of $\Gamma_i$ starts from Hankels of data matrices:

$$
Y_{t,i,j} = \Gamma_i X_{t,i,j} + H_t U_{t,i,j} + G_t W_{t,i,j} + V_{t,i,j}
$$

$$
X_{t,j} = [x(t) \ x(t+1) \ \ldots \ \ x(t+j-1)]
$$

$$
Y_{t,j} = \begin{bmatrix}
y(t) & \ldots & y(t+i-1) \\
y(t+1) & \ldots & y(t+j) \\
\vdots & \ddots & \vdots \\
y(t+i-1) & \ldots & y(t+i+j-2)
\end{bmatrix}
$$

$$
G_i = \begin{bmatrix}
0 & 0 & \ldots & 0 \\
C & 0 & \ldots & 0 \\
CA & C & \ldots & 0 \\
\vdots & \ddots & \ddots & \ddots \\
CA^{i-2} & CA^{i-3} & \ldots & 0
\end{bmatrix}
$$

$$
U_{t,i,j} = \begin{bmatrix}
u(t) & \ldots & u(t+j-1) \\
u(t+1) & \ldots & u(t+j) \\
\vdots & \ddots & \vdots \\
u(t+i-1) & \ldots & u(t+i+j-2)
\end{bmatrix}
$$

There are several non-recursive SSI algorithms based on above equation. What’s more, we propose a recursive algorithm in the following lines.

We have to provide a recursive method for estimation of $\Gamma_i$. This estimation needs SVD which we should avoid it in a recursive algorithm to achieve a fast algorithm. In this case, remember the Hankel matrices and suppose that in $t+1$, there is a new sample of $y(t+1)$. Therefore, the following column will be added to each of the above matrices:

$$
\gamma_i(t+1) = [y^T(t+j) \ \ldots \ y(t+i+j-1)]^T
$$

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Therefore, we can rewrite the system data equation as following:

\[ y_i(t + 1) = \Gamma_i x(t - i + 2) + H_i u_i(t + 1) + G_i w_i(t + 1) + v_i(t + 1) \]  

(9)

Now, we should find a way for estimating column space basis of \( \Gamma_i \) which has a minimized processing cost. The solution is in the field of array signal processing where there are several adaptive algorithms for replacing SVD [4, 5]. Those algorithms are based on the following data producer model:

\[ z(t + 1) = \Gamma(\theta) s(t + 1) + b(t + 1) \]  

(10)

where \( z \) is antenna array sensor output, \( \Gamma(\theta) \) is steering matrix of DOA of \( \theta \). \( S \) is the vector of attacking waves and \( b \) is measurement noise.

Relation of RSSI model and the above mentioned model becomes clearer when we rewrite (9) as below:

\[ z_i(t + 1) = y_i(t + 1) - H_i u_i(t + 1) \]

(11)

In the following table, the similarities have been distinguished:

<table>
<thead>
<tr>
<th>SSI Array Signal Processing</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_i(t + 1) \in \mathbb{R}^{l \times 1} )</td>
</tr>
<tr>
<td>( \Gamma' \in \mathbb{R}^{l \times n} )</td>
</tr>
<tr>
<td>( x(t - i + 2) \in \mathbb{R}^{m \times 1} )</td>
</tr>
<tr>
<td>( b_i(t + 1) \in \mathbb{R}^{l \times 1} )</td>
</tr>
</tbody>
</table>

Therefore, applying the above concepts, in a RSSI algorithm, first, we should update \( z_i \) using the new data \( z_i(t + 1) = y_i(t + 1) - H_i u_i(t + 1) \) and then estimating \( \Gamma' \) using the following observation:

\[ z_i(t + 1) = \Gamma_i x(t - i + 2) + b_i(t + 1) \]  

(13)

Obviously, both of above mentioned steps of a RSSI algorithm should recursively be done.

A. Updating Observation Vector

In every time sequence, a true estimate of observation vector \( z_i(t + 1) \) should be provided. There are two alternative methods:

1) First method

estimating \( \tilde{H}_i(t) \) in every time sequence and apply it to the following approximate equation [6]:

\[ z_i(t + 1) = y_i(t + 1) - \tilde{H}_i(t) u_i(t + 1) \]  

(14)

2) Second method

In order to avoid above approximation and calculation of a quantity containing the same information of \( z_i(t + 1) \), we can use the following RQ decomposition [1]:

\[
\begin{bmatrix}
U_{i,j+1} \\
V_{i,j+1}
\end{bmatrix}
= 
\begin{bmatrix}
R_{i1} & 0 \\
R_{i21} & R_{i22}
\end{bmatrix}
\begin{bmatrix}
Q_{i1}(t) \\
Q_{i2}(t)
\end{bmatrix}
\]

\[
\begin{bmatrix}
Q_{i1}(t) \\
Q_{i2}(t)
\end{bmatrix}
\begin{bmatrix}
Q_T(t) \\
Q_T^T(t)
\end{bmatrix} = I
\]

(15)

If a new data arrives, the above equation can be update as below:

\[
\begin{bmatrix}
U_{i,j+1} \\
V_{i,j+1}
\end{bmatrix}
= 
\begin{bmatrix}
R_{i1} & 0 \\
R_{i21} & R_{i22}(t)
\end{bmatrix}
\begin{bmatrix}
Q_{i1}(t) \\
Q_{i2}(t)
\end{bmatrix}
\]

\[
\begin{bmatrix}
Q_{i1}(t) \\
Q_{i2}(t)
\end{bmatrix}
\begin{bmatrix}
Q_T(t) \\
Q_T^T(t)
\end{bmatrix} = I
\]

(16)

There is always a sequence of matrix rotation named as \( G(t + 1) \) which returns \( R \) to its low-triangular structure.

\[
\begin{bmatrix}
R_{i1}(t) & 0 & u_i(t + 1) \\
R_{i21}(t) & R_{i22}(t) & y_i(t + 1)
\end{bmatrix}
\begin{bmatrix}
Q_{i1}(t) \\
Q_{i2}(t)
\end{bmatrix}
= G(t + 1)
\]

\[
\begin{bmatrix}
R_{i1}(t) \\
R_{i21}(t) \\
R_{i22}(t)
\end{bmatrix}
\begin{bmatrix}
z_i(t + 1)
\end{bmatrix}
\]

(17)

B. Updating Observability Subspace

In this section, we propose a technique for updating observability subspace using Projection Approximation and Subspace Tracking (PAST) method.

Suppose a \( n_x \) dimensional stochastic vector \( z \) and the following unconditional measure:

\[ V(W) = E[\|z - WW^Tz\|^2] \]

\[ W \in \mathbb{R}^{n \times n}, n_x \geq n \]

(18)

Suppose that \( W \) is full-rank and \( R_x = E[zz^T] \) has the following eigenvalue decomposition which positive definite:

\[ R_x = Q \Lambda Q^H; \quad Q \in \mathbb{R}^{n \times n}, \Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{n_x}) \]

\[ \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{n_x} \]

Global minimum of \( V(W) \) is available only when \( W = Q_n T \) contains \( n \) dominant eigenvalue of \( R_x \), where \( T \) is an arbitrary Unitary matrix.

Now, we rewrite the cost function of above problem using limited number of data and a forgetting factor \( \lambda \).

\[ V(W(t)) = \sum_{k=1}^{t} \lambda^{t-k} \|z(k) - W(t)W^T(t)z(k)\|^2 \]

(20)

We can use the following recursive method to find a solution for above optimization problem. The key point is that we assume

\[ h(k) = W^T(k - 1)z(k) \]

(21)

in the above equations. This is called approximate projection which yields

\[ \hat{h}(t) = W(t)h(k) \]

(22)

\[ W(t) = R_{zh}(t)R_{n}^{-1}(t) \]

\[ \hat{R}_x(t) = \sum_{k=1}^{t} \lambda^{t-k} z(k)z^T(k) \]

(23)

\[ \hat{R}_{zh}(t) = \sum_{k=1}^{t} \lambda^{t-k} z(k)h^T(k) \]

If matrix inverse lemma is applied to above equation, recursive least square version of algorithm will be available [4].

An unbiased instrumental variable version of PAST is presented in [6] in order to cope with noisy observations. In this method, it is assumed that cross-correlation matrix \( R_{x,t} \) is associated with \( z \) and an instrumental variable vector \( \xi \) which is low rank.
\[ R_{z^2} = E[z\xi^T] = \Gamma \Phi \]  
(24)

where \( n_x \geq n, y \geq n, \Gamma \in \mathbb{R}^{n \times n}, \Phi \in \mathbb{R}^{n \times y}, \xi \in \mathbb{R}^{y \times 1} \)

and both of matrices are full rank. Now, we can rewrite the cost function of optimization problem

\[ V_N(W(t)) = \|R_{z^2} - W(t)W^T(t)R_{z^2}(t)\|^2_F \]  
(25)

The minimization problem with above cost function has \( W(t) = Q_n^T \) as its solution where \( Q_n \) contains \( n \) left eigenvector of \( R_{z^2} \) and \( T \) is an arbitrary unitary matrix. If we apply approximate projection on the above methods, recursive algorithm will be available [6].

Above method is based on approximation which yields insufficient solutions [7]. Therefore, it may encounter difficulties when using for some of tracking applications. In the following section, we propose a novel algorithm which defeats this problem using array signal processing.

III. RECURSIVE SSI USING PROPAGATOR METHOD

Propagator is a linear operator which is used for decomposition of signal and noise space in array signal processing [8]. To understand key point of this method, remember sensor array signal model (10). Suppose the steering matrix is full rank. By reconstruction of sensor outputs, we can categorize the steering matrix as below:

\[ I_i = \begin{bmatrix} I_{i1}^T \\ I_{i2}^T \end{bmatrix} \in \mathbb{R}^{n \times n}, \quad I_{i1} \in \mathbb{R}^{(l-n) \times n} \]  
(26)

Propagator is linear operator which (see [9])

\[ I_{i2} = P^T I_{i1} ; \quad P \in \mathbb{R}^{n \times (l-n)} \]  
(27)

where \( I_{i1} \) is a block of first \( n \) independent rows and \( I_{i2} \) is a matrix containing remained \( l-n \) rows. It is easy to write

\[ I_i = \begin{bmatrix} I_{i1} \\ I_{i2} \end{bmatrix} = \begin{bmatrix} I_{t} \\ P \end{bmatrix} I_{i1} = Q_n I_{i1} \]  
(28)

It means that column space of observability matrix is a composition of independent columns. Thus, estimating \( P \) leads to an estimation of observability subspace.

\[ z_i(t) = \begin{bmatrix} I_{i} \\ P \end{bmatrix} I_{i1} x(t) + b(t) = \begin{bmatrix} z_{i1}^T \\ z_{i2}^T \end{bmatrix} \in \mathbb{R}^{n \times 1} \]  
(29)

If there was no noise (\( b_i \) was zero), it was easy to find propagator using

\[ z_{i2} = P^T z_{i1} \]  
(30)

But in the presence of noise, above equation doesn’t work and we have to use an optimization technique as below;

\[ \bar{V}(P) = E[z_{i2} - P^T z_{i1}]^2 \]  
(31)

We know that least square solution of above minimization problem, leads to a biased solution for \( P \) [10]. Therefore, we should use instrumental variables to achieve an unbiased solution [6]. Suppose that \( \xi \in \mathbb{R}^{y \times 1} \) is instrumental variables vector which is uncorrelated with noise but correlated sufficiently with state vector \( X(t) \). Now, we make a new cost function;

\[ \bar{V}(P(t)) = \sum_{k=1}^{t} \lambda^{t-k} \|z_{i2}(k)\xi^T(k) - P^T(t)z_{i1}(k)\xi^T(k)\|^2 \]  
(32)

The more number of instrumental variables, the more accurate estimation of \( P \) achieves [11, 12].

In attention to the concepts illustrated in this paper, we propose the following algorithm for finding observability matrix. It is called Extended Instrumentation Variable Propagator Method (EIVPM).

<table>
<thead>
<tr>
<th>EIVPM Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(t) = P(t-1) + (g(t) - P^T(t-1)\Psi(t))K(t) )</td>
</tr>
<tr>
<td>( g(t) = [\hat{z}<em>{i2}(t-1)\xi^T(t)z</em>{i1}(t)] )</td>
</tr>
<tr>
<td>( A(t) = -\xi^T(t)\xi(t) \lambda )</td>
</tr>
<tr>
<td>( q(t) = \hat{z}<em>{i1}(t) - \hat{z}</em>{i1}(t-1) )</td>
</tr>
<tr>
<td>( \Psi(t) = [q(t) \ z_{i1}(t)] )</td>
</tr>
<tr>
<td>( K(t) = (A(t) + \Psi^T(t)M(t-1)\Psi(t))^{-1}\Psi^T(t)M(t-1) )</td>
</tr>
<tr>
<td>( \hat{z}<em>{i2}(t) = \lambda\hat{z}</em>{i2}(t-1) + z_{i1}(t)\xi^T(t) )</td>
</tr>
<tr>
<td>( \hat{z}<em>{i1}(t) = \lambda\hat{z}</em>{i1}(t-1) + z_{i2}(t) )</td>
</tr>
<tr>
<td>( M(t) = (\hat{z}<em>{i1}(t)\hat{z}</em>{i2}(t))^{-1} )</td>
</tr>
</tbody>
</table>

IV. COMPUTER SIMULATION

A. Two-Area Power System

Reference [13] introduces the system which contains four machines in two different areas which are connected by a weak link. The system has four basic modes; an inter-area mode and two local modes. Damping factor of other modes are more than 60 percent. Therefore, their effects have been ignored. Increasing of network load destroys the stability of system and causes the first area local mode and inter-area mode move to the right half plane. However, Inter-area mode crosses imaginary axe sooner.

Now, we identify and track the three modes of system using proposed RSSI algorithm while increasing system loads gradually.

System signals are sampled in each duration of load increase and they are passed to the mentioned algorithm in order to identify system matrix \( A \). Then, eigenvalue decomposition provides system modes. Error! Reference source not found. illustrates good performance of algorithm for identification and tracking of two-area system modes.

B. Ten-Machine Power System

This system was introduced in [14]. Signal samples were measured through computer simulation of the power system while increasing system loads gradually. The algorithm was applied to sampled data in each step of load increase to identify the most dominant mode. Error! Reference source not found. illustrates good performance of algorithm for this case, too.

V. CONCLUSION

In comparison to methods presented in [15, 16], it is observed that the proposed algorithm can easily and simply be applied to large systems such as power systems. The performance of algorithm is also very good when estimate of dominant modes are under consider. The application of
algorithm is very straightforward and it can handle many signals and samples.

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Fig. 2: Mode Tracking of T-machine power system using EIVPM Algorithm.