Another Look at Fourier Coefficients: Forecasting Non-Stationary Time-Series using Time Dependent Fourier Coefficients

Uchenwa Linus O'kafor and Oladejo, M. O

Abstract— A new method for forecasting non- stationary time series by Harmonic Analysis is developed in this paper. The process uses an adaptive method that assigns weights to each Fourier coefficients on a proportionate basis. The proposed method gives a better result than that obtained by the traditional Fourier series method. The new method shows how to tackle unstable systems in electrical appliances and other devices whose temperature rises with time.

Keywords— Adaptive method, Fourier Coefficients, forecasting, non-stationary, Time dependent weights.

I. STATEMENT OF THE PROBLEM

In the Fourier analysis of time series, it is assumed that the amplitudes of the waves hover over a mean value, DASS

[4], DEAN [5]. This is only true for a stationary time series but when the time series is non-stationary Fourier coefficients obtained will no longer have amplitudes meet the mean value condition and there is therefore the need to make amplitudes of the waves to match the movement of the trend in the nonstationary data. Bloomfield [1] said that no sinusoid can match oscillations that grow in amplitude. Most of observed time series generated by the real life world have a trend and nonstationary. In a monotonic time series the trend is modelled as a function of time and filtering is used to obtain variance stabilization. Most of the work done in non-stationary time series data are by non-parametric methods, Box and Jenkins^[2], Parezen ^[15], Kendall^[12], But Nagpaul ^[13], DeLurago [6], Cowpertwait [3], Stoica et al, Harold[8] and a host of authors who have used the parametric method. Yang [17] called for an extension of models that allow for time varying amplitudes and phases. Harmonic Analysis is concerned with the discovering of periodicities in a given time series data and is used when the data is either in tabular or graphical form, Harold [12] reports that it started with a paper published by Lagrange [13] but it was known Leonard Euler [14] that an analytic function could be represented by means of a series of sine's, and cosines, namely, by the series

$$Y_t = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi t}{L} + b_n \sin \frac{n\pi t}{L} \right) \quad \text{,for } -a \le t \le a$$
(1)

Uchenwa Linus O'kafor¹ and Oladejo, M. O.², Mathematics Department, Nigerian Defence Academy, Kaduna Nigeria.

¹.linusokafor@gmail.com, +2348097524668

It was Foureir [15] who showed how the constants a_n and b_n could be evaluated

II. Aim

The aim of this paper is to derive Fourier coefficients that will match the nature (satisfy) stationary conditions) of the non-stationary data.

III. OBJECTIVE

The objective of this paper is therefore using an adaptive method to obtain coefficients of the Fourier series that will make the amplitudes of the waves to be in accordance with trend of the time series data so as obtain a minimum squares of errors for fitted values.

In order to achieve the stated aim, the following additional objectives will be followed through: Determine the Fourier coefficients a_k and b_k up to the sixth harmonics for the Traditional Method and the Adaptive method using monthly Air passengers data, determine the frequencies that minimize the Sum of Squares of Error (SSE), obtain amplitudes for Traditional method or stable amplitude and the Adaptive method or unstable system.

The work will not be concerned with complex Fourier series at this stage. The monthly Air Line Passenger 1948-196p,of Box and Jenkins constitutes.

IV. DATA SET AND MATERIALS

The monthly Air Line Passenger 1948-196p,of Box and Jenkins constitutes the data set. Two statistical packages NCSS (TRAIL VERSION) and EXCEL will be employed to obtain results.

V.METHODOLOGY

Since the pioneer work of [15] in 1822, when he stated that a function of the form:

$$Y = f(t) \tag{2}$$

Could be expressed between the limits t=0 and $t=2\pi$ by the infinite series that is given in the form in equation (2):

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\frac{n\pi t}{L} + b_n \sin\frac{n\pi t}{L}\right) \quad (3)$$

².mikeoladejo2003@yahoo.com, +2348033430043

This is for a single valued function which is continuous, has a finite number but without discontinuities, where it is given that coefficient

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(t) \cos(kt) dt \tag{4}$$

And

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(t) \sin(kt) dt$$
 (5)

if the data is continuous.

When the data is discrete the coefficients become

$$a_{n=\frac{2}{n}} \sum_{n=1}^{\infty} (f(t) * cos(kt))$$
(6)

$$b_n = 1/n \sum_{n=1}^{\infty} f(t)(\sin kt)$$
 (7)

In a situation where the data is trending or is non-stationary we will have the form of equation below:

Y=f(t)

$$a_{0}+a_{1}t+a_{2}t^{2}+a_{3}t^{3}+...+a_{m}t^{m}+\sum_{n=1}^{\infty}\left(a_{n}\cos\frac{n\pi t}{L}+b_{n}\sin\frac{n\pi t}{L}\right)$$
(8)

But using the 12th difference method proposed by DeLurigo [15] we shall have

$$Y = a_0 + b * t + \sum_{n=1}^{N} \left(a_n \cos \frac{n\pi t}{L} + b_n \sin \frac{n\pi t}{L} \right)$$
(9)

N=1,2,3,...,N. Where,

$$b = \sum_{13}^{N} (Yt - Yt - L)/L, \qquad (10)$$

Where L is the seasonal length

$$a_0 = N^{-1} (\sum_{t=1}^{N} Y_t) - [b(1+N)/2]$$
(11)

In our scheme the Fourier coefficients are obtained in this sequel:

$$a_{n} = \frac{1}{N(N+1)(N+2)} \sum_{n=1}^{N} (f(t)(t/N)\cos(kt))$$
(12)

$$a_{n} = \frac{6}{N(N-1)(N-2)} \sum_{n=1}^{N} (f(t)(t/N)\cos(kt))$$
(12a)

$$b_{n} = \frac{1}{N(N+1)(N+2)} \sum_{n=1}^{N} (f(t)(t/N) \sin(kt))$$
(13)

We are working on another scheme

$$b_{n} = \frac{1}{N(N-1)(N-2)} \sum_{n=1}^{N} (f(t)(t/N) \sin(kt))$$
(13a)

The model proposed equation thus becomes

$$Y = a_0 + \sum_{n=1}^{N} (\frac{2}{N}) t \left(a_n \left(\cos \frac{n\pi t}{L} \right) + (\frac{2}{N}) b_n t \sin \frac{n\pi t}{L} \right) + e_t$$
(14)

Note that a_{o} , represents the trend which can linear or of any degree.

The fitted values can now be obtained by the equation

$$\mathring{y} = a_0 + \sum_{n=1}^{N} \left(\frac{2t}{N} a_n \cos \frac{n\pi t}{L} + \frac{2t}{N} b_n \sin \frac{n\pi t}{L} \right)$$
(15)

We can replace equation (15) by the series

$$\mathring{\mathbf{y}} = a_0 + \sum_{n=1}^{N} \frac{2}{N} t \left(A_n \operatorname{Sin}(\frac{n\pi t}{L} + \tilde{\mathbf{\Omega}}n) \right)$$
(16)

Where,

$$A_{n} = (a_{n}^{2} + b_{n}^{2})^{1/2}$$
(17)

and

$$\tilde{\Omega}_{n} = \tan^{-1}(a_{n}/b_{n})$$
(18)

Similarly equation (16) may be expressed as

$$Y = a_0 + \sum_{n=1}^{N} \frac{2}{N} t \left(A_n \cos(\frac{n\pi t}{L} + \alpha n) \right)$$
(19)

Where

=

$$\tilde{\Omega}_n = \tan^{-1}(-b_n/a_n) \tag{20}$$

It has been [1]cautioned about one being careful in computing ${}_{\Omega n}$, as there are two possible values of ${}^{\widetilde{\Omega}}_{\Omega n}$ which satisfy either (16) or (19). Jakubauskas et al [15] stated:Jakubauskas et al [15] stated:

"Because the inverse tangent function only returns values in the interval $[-\pi,\pi/2]$, whenever $a_n < 0$ the modified $c_{\Omega n} = \tan^{-1}(a_n/b_n) + \pi$ must be used to obtain a true phase angel. It follows that $c_{\Omega n} \in [-\pi/2,3\pi/2]$ "

Shepherd et al [16], are of the view that using the equation (16),

$$Y = a_0 + \sum_{n=1}^{N} \frac{2}{n} t \left(A_n Sin(\frac{n\pi t}{L} + \tilde{\Omega}n) \right) \quad \text{gives a more}$$

accurate result than obtained by the superposition method According to Kendall [17] the golden rule of time series analysis is first to plot the data.

Fig 1. Is the plot of the Air line while Table 1. Shows the Actual time series data of Air Line travellers of Box and Jenkins as reported in [11]

TABLE I AIR LINE DATA

						0					
1949	1950	1951	1952	1953	1954	1955	1956	19578	1958	1959	1960
112	115	145	171	196	204	242	284	315	340	360	417
118	126	150	180	196	188	233	277	301	318	342	391
132	141	178	193	236	235	267	317	356	362	406	419
129	135	163	181	235	227	269	313	348	348	396	461
121	125	172	183	229	234	270	318	355	363	420	472
135	149	178	218	243	264	315	374	422	435	472	535
148	170	199	23 0	264	302	364	413	465	491	548	622
148	170	199	242	272	293	347	405	467	505	559	606
136	158	184	209	237	259	312	355	404	404	463	508
119	133	162	191	211	229	274	306	347	359	407	461
104	114	146	172	180	203	237	271	305	310	362	390
118	140	166	194	201	229	278	306	336	337	405	432



Fig. 1 Graph of the Air line passengers 1945-1960

VI. ESTIMATING THE COEFFICIENTS OF FOURIER FUNCTION

In the traditional Fourier series method, the constants an and bn, using (3), but the time dependent Fourier coefficients are obtained using (7).Table 2a and Table (2b) are displayed the coefficients from the traditional method and the time – dependent method respectively from the Air Line data. The periodograms are shown in Fig.2a and Fig.2b for the traditional method and the adaptive method in that order.

 TABLE 2(A)

 FOURIER COEFFICIENTS FROM THE TRADITIONAL METHOD

Harmonics(K)	Cosine Terms(a _n)	Sine Terms(b _n)	Amplitude (A _k)
1	-38.6610	-15.8145	41.7704
2	-4.8713	23.5088	23.8123
3	7.8807	-3.8880	8.7840
4	5.8122	7.3290	9.354
5	0.5933	6.0662	6095
6	0.7242	-	0.7242

 TABLE 2(B)

 FOURIER COEFFICIENTS FROM THE ADAPTIVE METHOD

 Harmonics(K)
 Cosine Tompo(c)
 Amplitude Sine Terms(b_n)
 Amplitude (A)

marinoines(ix)	Terms(a _n)	Sinc remis(0 _n)	(A_k)
1	12.14	-7.1867	14.109
2	-0.852	7.0789	7.123
3	-1.445	7.187	7.2936
4	1.642	0.4632	1.7061
5	1.955	0.6114	2.0481
6	0.687	-	0.687



Fig. 2a. Periodogram for Traditional Coefficients



VII. SEASONAL AMPLITUDES

Fig.3a shows the Amplitude for Traditional method, while Fig.3b is the Amplitude for the Adaptive method.



Fig.3a Amplitude for the Traditional Method



Fig.3b Amplitude for the Adaptive Method.

VIII. MODEL AND ESTIMATED MODEL EQUATION

Using the coefficients of Table2a.,to form the models, following model equations were obtained:

Estimated model for the Traditional coefficient for first harmonic

 $\dot{Y}_t = 92.00544 + 2.564t - 38.66*\cos(.5236t) - 15.8*\sin(0.5236t)$

Estimated model for the Adaptive coefficient for first harmonic

(21)

 $\dot{Y}_t = 92.00544 + 2.564t - 12.14 \cos(.5236t) - 7.187 \sin(0.5236t)$ (22)

IX. FITTED VALUES AND FITTED ERRORS

Table 3. Appendix 1 shows for the Traditional method, the Period(t) column 1,Actual values (Y_t) column 3,Fitted values(\dot{Y}_t) column 4, Fitted Errors(e_t) column 5, Amplitude column 6, squared Errors(e_t)² column 7,First difference error squared (e_t - e_{t-1})² column 7 and in Z-values column 8. Appendix 2 is for the Adaptive method.

The forecast error (e_t) is given as,

$$\mathbf{e}_{t} = \mathbf{Y}_{t} - \mathbf{\dot{y}}_{t} \tag{23}$$

X. THE SUM OF SQUARES ERRORS (SSE) OR ERROR LACK OF FIT

The sum of squares of errors is given by the equation

$$SSE = \sum_{t=1}^{n} (e_t^2)$$
 (24)

XI. ANALYSIS OF THE RESULTS

Results obtained using our coefficients and those of Fourier are shown in table 3. Figure 2a. Shows the seasonal amplitudes obtained from using the old method of Fourier coefficients showing a mean level dependence and a stable system. While the seasonal amplitudes from new method coefficients is as shown in Figure 2,B. indicating a trend level ,and hence an unstable system. While Table 3 a summary of the fit statistics obtained using the Traditional coefficients method and the Adaptive coefficients. The Adaptive method gave a lower Sum of Squares of Errors than the Traditional method.

XII. CONCLUSION

The adaptive or new method gives a better result in the statistics of both fitted and forecast.

TABLEIV

	IAI		
FIRST	New method,	Old method,	DIFFERENCE
,HARMONIC,N	SSE=94853	SSE=114525.7	=19673.7
=132			
FIRST	New method,	Old method,	DIFFERENCE
HARMONIC,	SSE =125063	SSE=150055	=24992
N=144			
SECOND	New method,	Old method,	DIFFERNCE=
HARMNIC,	SSE =184676	SSE=192260	7584
N= 132			
SECOND	New method,	Old method,	DIFFERENCE
HARMONIC,	SSE=250199.6	SSE=258429.8	=8230.2
N=144			

REFERENCES

- [1] Peter Bloomfield, Fourier Analysis of Time Series. An Introduction, Second Edition. Wiley Series In Probability and Statistics.1976
- [2] Box, G.E.P; G.M. Jenkins; and G.C. Reinsel. Time Series Analysis Forecasting and Control.3rd ed. Englewood Cliffs, NJ: Prentice Hall, 1994.
- [3] Paul S.P. Cowpertwait. Introductory Time Series with R.2006 Springer Science+ L Business Media, LLC.

- [4] H.K.DASS. Advanced Engineering Mathematics. S.CHAND & COMPANY LTD.RAM NAGAR, NEW DELHI.
- [5] Duffy. G. Dean. Advanced Engineering Mathematics.CRC Press, 2009.
- [6] Stephen A. DeLurigo. Forecasting Principles and Applications. Irwin McGraw-Hill.1998.
- [7] Leonhard Euler(1707-1783) in Davis T. Harold (1941,p28)
- [8] Davis T. Harold, The Analysis of Economic Time Series. The Principia Press Inc. Bloomington, Indiana. 1941 (download, 03 Feb 2013
- [9] Davis Thorold, The Analysis of Economic Time Series. The Principia Press Inc. Bloomington, Indiana. 1941 (download, 03 Feb 2013).
- [10] M.E. Jakubauskas, David R. and Jude H. Kastens, ''Harmonic Analysis of Time –Series AVHRR NDVI Data''. Photogrammetric , Engineering& Remote Sensing Vol67,No.4,Apri 12001,pp 461-470
- [11] J.L. Lagranrge. (1707-1783) in Davis T. Harold (1941, p28).
- [12] M.G. Kendall and A.B. Hill. 'The Analysis of Economic Time-Series-Part 1: Prices.'' Journal of Royal Statistical Society A(General), Volume 116,Issue 1(1953) 11-34.Stable URL: http://uk.jstor/org. Mon Dec 211:51:35 2002.

http://dx.doi.org/10.2307/2980947

- [13] P.S. Nagpaul. Time Series in WinDAMS .April 2005.downloadhep.
- [14] J. Shepherd, A.H. Morton &L.F. Spence. Higher Electrical Engineering. The English Language Book Society and PITMAN Publishing.1998.
- [15] Stephen ,A. DeLurigo. Forecasting Principles and Practice. Irwin McG N.Saraw-Hill.1998
- [16] P. Stoica and S Andgren, "Spectral analysis of irregularly –sampled data: Paralleling the regularly-sampled data approaches" Digit. Signal Process, Vol. 16. No.6, pp712-734, Nov. 2006. http://dx.doi.org/10.1016/j.dsp.2006.08.012
- [17] Emmanuel Parezen. Time Series Analysis. Technical Paper 1.19
- [18] Cheng Yang. Bayesian Time Series: Financial Models and Spectral Analysis. PhD Dissertation, 1997



Michael Olatunji Oladejo (PhD) Navy Captain (retired).Commissioned 1984.Retired Dec 2007. Instructor Mathematics Naval College, PortHacourt, 1984 – 1988.

Mathematics Lecturer Nigerian Defence Academy (NDA), Kaduna, 1988 – 1995.

Staff Officer 2, Research and Development, Naval Training Command Apapa, Lagos, 1997-1999.

Armed Forces Command And Staff College, Jaji, 2000 – 2001. Executive Officer Nigerian Navy Secondary School, Abeokuta, 2001-2003.

Mathematics Lecturer (NDA) 2003 – 2007.

Head of Department, Mathematics/Computer Science, Nigerian Defence Academy, Kaduna, Nigeria.

Qualification:

BSc University of Ibadan,1980.

MSc University of Nigeria, Nsukka 1982.

PhD University of Benin 1995 (Statistics, Operations Research, Industrial Engineering).

Postgraduate Diploma In Education National Teachers Institute, Kaduna 2009 Present Appointment: Lecturing 100, 200, 300, 400 levels and postgraduate, have supervised 15 PG Students (Masters and PhD).

Hobby: reading, travelling and watching nature.

Conferences:

International Conference on Mathematics Education (ICME 12), Seoul - Korea.

Publications:

i. Iyang S, Jumare MM, Oladejo MO, A geographic analysis of troop's concentration points and deployment route in Eastern Nigeria, Defence Studies 1, Vol. 6, Jul 1996.

ii. Oladejo MO, Ezigbo-esere MN, Ovuworie GC, Arrows Impossibility Theorem – Applied to Judgemental Assignments, NJEM, Vol. 7, No 4, Oct-Dec 2006.

iii. Oladejo MO, Ovuworie GC, Adequacy of C3I Models for Military Training, NJISS, Vol. 6, No. 4, 2007.

iv. Oladejo MO, Redundancy Consideration for Minimization of Manpower waste in NN GL, NDAJODS, Vol. 15 (July, 2008) pp 1-17.

v. Oladejo MO, Alhaji BB, Irhebhude M, Profile Analysis of Military Training Policy in the NDA: A Multivariate Approach, Blackwell Journal, 2010. vi. Okafor UL, Oladejo MO, ARIMA Model of Cadets sick's parade, Blackwell Journal, Vol. 2, No. 1, 2010.

vii. Oladejo MO, Udoh I, Application of Resource Allocation Queue Fairness Measure (RAQFM) to the Petroleum Product Pipelines Marketing Company (PPMC) to Suleja Fuel Depot, Niger State – Blackwell, IJMS, Vol. 3, No. 1, 2011.

viii. Oladejo MO, Spectral Analysis of Modelled N – Team Interacting DM with Bounded Rationality Constraints, IJRAS, 2013.

Plus 20 others

Books in Print:

i. Oladejo MO, Stochastic Processes, Queues and Games Models with Worked Examples.

ii. Oladejo MO, A First Course on Probability Principles and Applications with Worked Examples.



Wing Commander, Uchenwa Linus Okafor (retired) received B.Ed degree in Mathematics/Physics from the University of Ibadan in 1983, M.Ed. degree in Mathematics from the University of Jos in 1993, M.Sc in Applied Mathematics from the Post Graduate School of the Nigerian Defence Academy,

Kaduna, in 2008. Presently, he is doing his PhD in Applied Mathematics, his research interests are in the areas of time series analysis and prediction, signal processing and spectral analysis.

Okafor taught Mathematics and Physics at Air Force Military School, Jos, from 1984 to 1998 served as the Base Education Officer at Nigerian Air Force Base, Kaduna from Sep 1998-Nov 2000, was lecturer at the Department of Mathematics and Computer Science, Nigerian Defence Academy, Kaduna between Dec 2000-Sep 2003. He was the Commandant of Air Force Secondary School, Makurdi, from Sep 2003-Dec 2004, where he retired as a Wing Commander in 2004 and currently he is a lecturer at the Department of Mathematics at the Nigerian Defence Academy. He is a member of the International Commission on Mathematical Instruction (ICMI)