Optimal DG Placement and Sizing For Voltage Stability Improvement Using Backtracking Search Algorithm

Ruhaizad Ishak, Azah Mohamed, Ahmed N. Abdalla, and Mohd Zamri C. Wanik

***Abstract*—**The increase in power demand and limitation of transmission capacities have led to strong concern among electrical power industrials. The large deployement of distributed generation (DG) sources in distribution network can be an efficient solution to overcome power system technical problems and economical challenges. Unlike centralized electrical generation, the DGs generate electrical energy near to the load centres with small generating capacity. Installing DG may influence power system stability and losses. To maximize such benefits, the optimal location and sizing of DGs are very important. This paper presents a method to identify optimal DG location based on maximum power stability index while minimization of real power loss and also DG cost is considered in optimizing the DG size. A new evolutionary algorithm known as backtracking search algorithm (BSA) is opted in solving the optimization problem. The applicability of proposed method is verified using the IEEE 30-bus transmission network.

***Keywords*—**Loadability, maximum power transfer, stability index, voltage collapse prediction.

I. Introduction

IN the modernization of power system industry, distributed generation sources are inevitably becomes important source of electricity supplies. To satisfy the increasing power demand, huge power plants have to be constructed. However, the intention of building new power plants face with many challenges such as finding a proper place for establishment, costs incurred due to the transfer of the electricity from distant power plants and technical impairments due to long travelling distance of electricity over the transmission lines. The continuous growth in power demand, lack in active power generation as well as limitations of traditional power system structure have led to increased interest in DG utilisation. Distributed generators can be strategically placed in power systems for improving system reliability and efficiency such as improving voltage profiles and reducing power losses [1]–[3]. The DG resources are normally deployed at distribution networks which are closer to consumption centres. The units are relatively small in size and also modular in structure. Two major aspects, namely the location and sizing of distributed generators strictly require careful attention in the units deployment. A common strategy to find the location of DG is to asses the power losses parameter in the system [4]-[6].

In many parts of the world, the number of DG units being installed in power systems shows an increasing trend. As the installed capacity of DG units increase, it becomes important to study the effects of these sources on electrical systems. Among the major effects include voltage violations and grid losses [7]-[9]. The voltage profile is directly a function of load consumption while network losses are a function of power flow. The presence of DG has changed the flow of power from unidirectional to bidirectional.

With reference to DG studies, it is observed that less attention is being given on voltage stability issues. The problem of voltage stability in distribution networks with DG units is considered new with few reported works. As such, this study focuses on assessing voltage stability as well as power losses which are considered as vital system parameters in measuring the effectiveness of DG installations. This paper utilizes the voltage index stability approach to identify the weakest buses in distribution networks. Section II presents the mathematical formulation of the proposed index. Section III describes the methodology for finding optimum DG placement and sizing. A new method for DG placement is introduced in this study based on the bus stability index. For determining the DG size, the proposed algorithm considers minimization of active power loss. Section IV presents the results of the optimization and voltage stability indices.

II. Formulation of New Voltage Stability Index

A new voltage stability index is developed to predict the network state of stability by assessing the voltage collapse

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point due to load active power increment. The derivation of the voltage stability index is given in this section.

A. Reduced Network Circuit

The assessment of stability in large networks can be an exceptionally demanding task. In this work, network model reduction is utilized to develop a fast and easier way to assess voltage stability of a complex system. The use of equivalent representation of a complex system is required to reduce the lengthy calculations. Accordingly, the formulation of MPSI begins with the transformation of a multi-node network into a 2-node model as shown by Figures 1a and 1b.

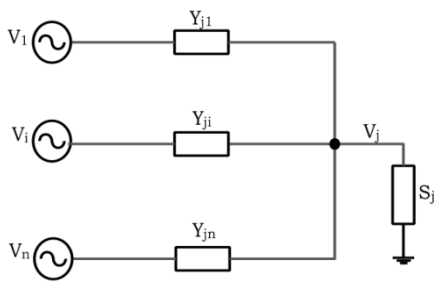

Fig. 1a Typical local network

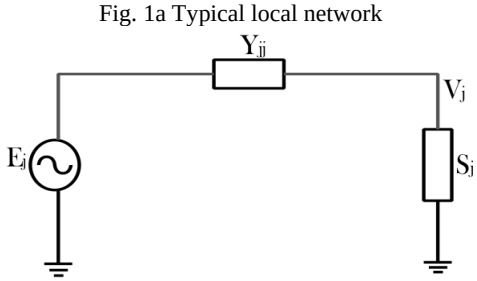

Fig. 1b Simplified network.

The simplified network of Figure 1b shows a load bus j being fed by a constant voltage source V_S through the line admittance Y_{jj}. The power equation at node j in the simplified network can be written as:

$$V_j[Y_j][V] = S_j \tag{1}$$

where
S_j is the apparent load power
V_j is the voltage magnitude at load bus j
[V] is the bus voltages matrix of local network
$[Y_j]$ is row j-th admittance matrix

Mathematically, the row j-th admittance matrix $[Y_j]$ in Eq. (1) can be written as:

$$[Y_j] = [-Y_{j1} - Y_{j2} - \ldots + Y_{jj} \ldots - Y_{jn}] \tag{2}$$

Where,
Y_{ji} is admittance between node j and node i
Y_{jj} is self admittance at node j

$$Y_{jj} = \sum_{i=1, i\neq j}^{n} Y_{ji}$$

The equivalent voltage of the network E_j obtained from node j can be written as:

$$E_j = \frac{\sum_{i=1, i\neq j}^{n} Y_{ji} V_i}{Y_{jj}} \tag{3}$$

Substituting (3) into (1) will give the load flow equation of the reduced network.

$$V_j^*(E_j - V_j)Y_{jj} = S_j \tag{4}$$

B. Critical Power

Referring to the simplified network of Figure 1b, the change in load characteristic affects the voltage, current and active power at the load bus. Figure 2 illustrates the characteristic of load variations on vital system parameters.

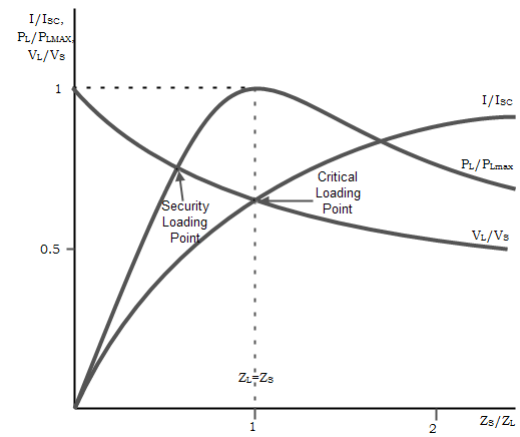

Fig.2 Variations of system parameters for simplified network

At a given operating point, if load increases but maintaining the power factor constant the current flowing through the feeding line increases. This causes an increase on voltage drop and a reduction in load voltage V_j. As shown in Fig. 2, the active power at load increases when load impedance Z_L is bigger than source impedance Z_S i.e. $Z_L > Z_S$. After reaching a maximum point at $Z_L = Z_S$, the transmitted active power to the load decreases when $Z_L < Z_S$, which makes the system become instable if the load consumed constant power.

It can be assumed that $Z_L = Z_S$ is the critical loading point where further increase beyond this stability point can easily cause the system voltage to collapse.

C. Stability Index

In the analysis of ac system, the theorem of maximum power transfer states the maximum power is transferred to a load when the load impedance Z_L is equal to the conjugate of source impedance Z_S. Again, referring to Fig. 1b, the active power at the load, P_L can be written as:

$$P_L = \tilde{V}_L \tilde{I}_L \cos\theta = Re(V_L I_L^*) = \frac{V_L^2}{Re\, Z_L} \tag{5}$$

Assuming that voltage and impedance of the source are priory known; therefore, the real power expression can be restated in terms of voltage and impedance of the source as:

$$P_L = Re(V_L I_L^*). \tag{6}$$

Where,

$$V_L = \frac{Z_L}{Z_{Th} + Z_L} V_{Th} \tag{7}$$

and

$$I_L^* = \left(\frac{V_{Th}}{Z_{Th}+Z_L}\right)^* \quad (8)$$

The complex power at the load S_L can be written as:

$$S_L = V_L I_L^* = \frac{V_{Th}^2}{|Z_{Th}+Z_L|^2} Z_L. \quad (9)$$

The active power can also be expressed as:

$$P_L = Re(V_L I_L^*) = Re\left(\frac{V_{Th}^2}{|Z_{Th}+Z_L|^2} Z_L\right) Re(Z_L) = \frac{V_{Th}^2 R_L}{(R_{Th}+R_L)^2+(X_{Th}+X_L)^2}. \quad (10)$$

According to [10], the maximum real power absorbed by the load P_{Lmax} is:

$$P_{Lmax} = \frac{V_S^2 R_L}{(R_L+R_L)^2+(-X_L+X_L)^2} = \frac{V_{Th}^2}{4R_L}. \quad (11)$$

As shown in Fig. 2, at the point where $Z_L = Z_{Th}$ the P_L to P_{Lmax} ratio is unity. Manipulating (5) and (11) with the real power ratio leads to the formulation of the proposed index:

$$MPSI = \frac{4V_L^2}{\left[\frac{\sum_{i=1, i\neq j}^{n} Y_{ji} V_i}{Y_{jj}}\right]^2} \quad (12)$$

The maximum power stability index (MPSI) shows value close to 0 for stable voltage condition meanwhile for critical operating condition the index value is close to 1.

III. Optimum Placement and Sizing of DG

A. Optimum placement of DG

The MPSI index is used to find the optimum placement of DG in the test system. The active power loading at each bus is randomly changed using the BSA algorithm and the index value is calculated at each bus. The bus with highest value of MPSI is assumed as the weakest bus in the system and DG will be placed at that respective bus. The mathematical formulation considered in the placement optimization is given as:

$$d = \text{Max}\ MPSI(i)\ for\ i = 1,2 \ldots, n_b \quad (13)$$

where n_b is the number of load buses

B. Optimum sizing of DG

The optimum DG sizing is identified by minimizing total active power losses. A set of random DG sizes is generated again through BSA algorithm and power flow is executed for each DG size. The fitness function is evaluated for total power losses in the system which provides the best solution for optimal DG size. The mathematical formulation considered in the loss calculation is given as:

$$f = \text{Min} \sum_{i=1}^{n} I_i^2 R_i \quad (14)$$

where n is the no. of lines.

C. System constraints

The search for optimum solution is subjected to the following constraints:

1. Power Balance Constraints: The power flow equations to be satisfied for each sizing and placement scheme are as follows:

$$\sum P_i^{DG} = P_i + P_i^D \quad (15)$$

$$\sum Q_i^{DG} = Q_i + Q_i^D \quad (16)$$

where

$$P_i = V_i \sum_{j=1}^{N} Y_{ij} V_j cos(\delta_i - \delta_j - \theta_{ij}) \quad (17)$$

$$Q_i = V_i \sum_{j=1}^{N} Y_{ij} V_j sin(\delta_i - \delta_j - \theta_{ij}) \quad (18)$$

where N the number of buses, P_i^{DG} generated real power delivered to bus i, Q_i^{DG} generated reactive power delivered to bus i, P_i^D real power demand at bus i and Q_i^D reactive power demand at bus i.

2. Voltage Constraints: The voltage magnitude of each bus should be kept within operating limits, as follows:

$$0.95\ p.u \leq V_i \leq 1.05\ p.u \quad (19)$$

3. Operating Limits of DG Units*:* DG units should be operated considering the limits of their primary resources, that is:

$$P_{DG}^{min} \leq P_{DG} \leq P_{DG}^{max} \quad (20)$$

Where, P_{DG} is the DG generation power; P_{DG}^{min} is zero and P_{DG}^{max} is 150% of total connected load.

The proposed optimization method is executed considering the following procedure as shown in Fig. 3.

IV. Results

The proposed method for DG placement and sizing is tested on the 69-bus distribution system having a total load of 3.80 MW [11] as shown in Fig. 4.

A. Sensitivity method

A number of simulations have been carried out for different combination of loadings. Loads are considered constant power loads and the generator reactive capability is taken into account. The loading index of all buses of the system is then evaluated through (13) using the bus voltage profile. Fig. 5 gives a summary of the index values. Table I gives the results of the five weakest buses in the system after loading all the load buses considering base case load and three cases of varying active load at different nodes. It shows that bus 65 is ranked first for base case load, however, bus 64 is ranked as the most critical bus under heavy loading conditions. As mentioned, the idea is to determine the weakest bus in the system. Obviously results in Table I also show the loading index at bus 64 has the highest average value and it constantly remains among the highest of all other buses. Thus, bus 64 can be considered as the weakest bus in the system from the voltage instability viewpoint.

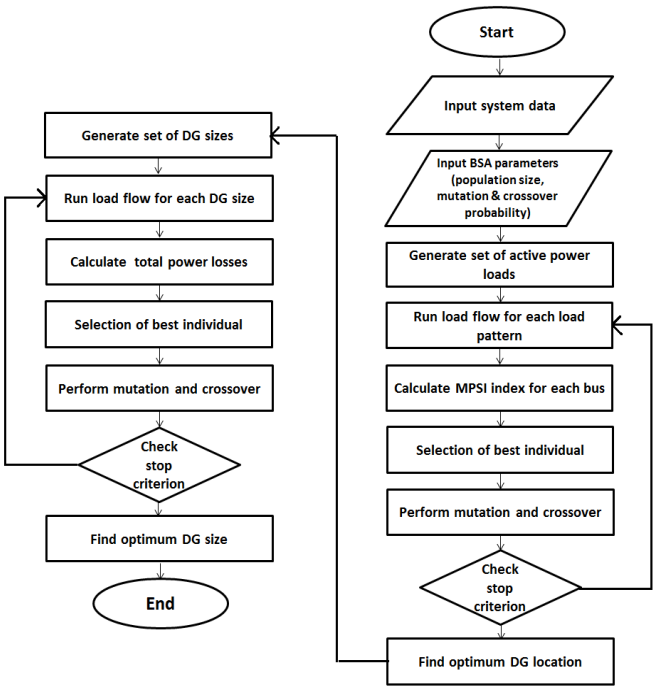

Fig. 3 Flowchart of the proposed optimizationmethod

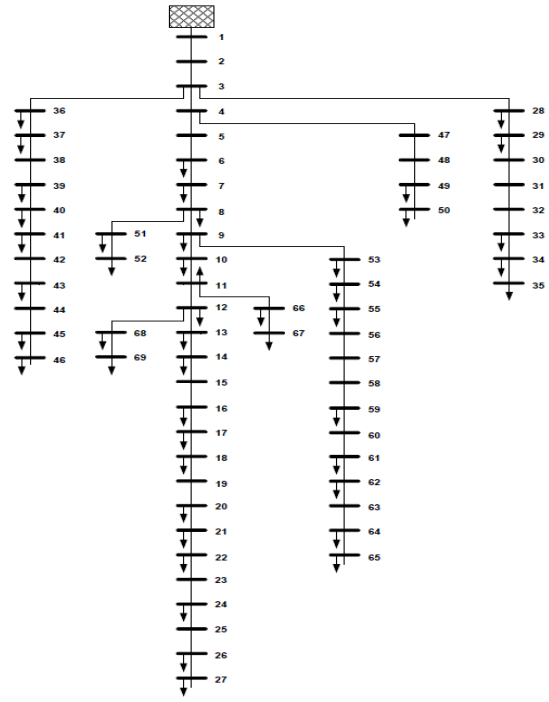

Fig. 4 The 69-bus radial distribution system

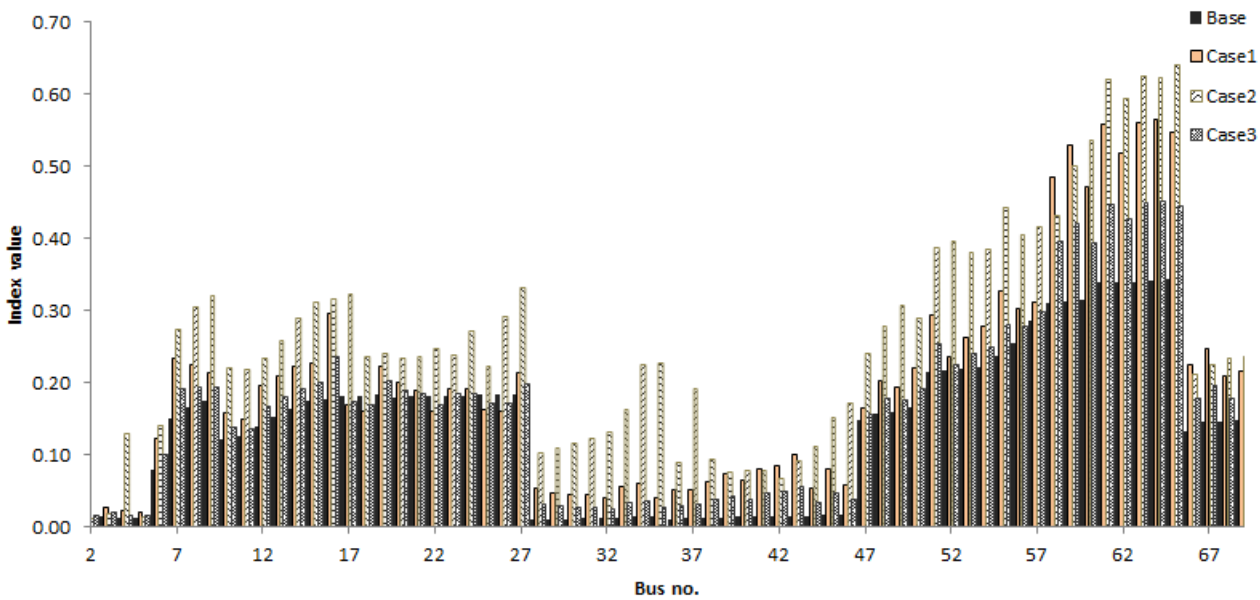

Fig. 5 MPSI index value for 69-bus system

TABLE I
5 HIGHEST INDEX CALCULATION

No.	Base case	Mean of 3 different loading cases
1	Bus 65	Bus 64
2	Bus 64	Bus 63
3	Bus 63	Bus 65
4	Bus 62	Bus 61
5	Bus 61	Bus 62

For finding the optimum sizes of DG at selected location, a loss sensitivity analysis is conducted to determine the minimum real power losses achievable in the system. Losses are investigated using DG of 500 kW capacity for its initial installation. The size of the DG is repeatedly increased in 500 kW step. Table II presents the results of the power loss solved by the load flow calculation.

TABLE II
POWER LOSS AT DIFFERENT DG CAPACITY

No.	DG size (MW)	Real power loss (MW)	Loss reduction (%)
Base	0	0.2259	0
Case 1	0.5	0.2171	3.89
Case 2	1.0	0.2073	8.23
Case 3	1.5	0.1965	13.01
Case 4	2.0	0.1814	19.70
Case 5	2.5	0.1727	23.55
Case 6	3.0	0.1682	25.54

The results obtained in Table II demonstrate that real power loss in the system is reduced when DG is installed at bus 64. It can be seen that as the capacity of the DG increases, the power loss savings also show better improvement. In the beginning, the increment in size of DG shows linear relation with the amount of loss reduction. However, the variance of loss reduction between case 5 and 6 decreases to the smallest value in contrary to increase in size of DG. As such, the optimum size of DG is 2.5 MW using the analytical method. The overall solution obtained above is assumed corresponding to local optimum and not global optimum solution.

B. Heuristic optimization method

To overcome the problem of local optimum, the backtracking search algorithm (BSA) is used to find the global optimum solution for DG location and sizing problem. In this simulation, two cases are considered. In case 1, only one DG unit is assumed while case 2 consists of two DG units. Table III shows the results obtained by using the BSA in terms of the the bus number where each DG unit is located, the active power injected and the loss reduction. For case 1, after a set of load pattern is randomly generated by the algorithm, the minimum value of voltage stability index is recorded at bus 61. Therefore, it indicates that bus 61 is very sensitive to voltage collapse and also has a minimum voltage. The optimum size of the DG at bus 61 is determined as 1.4 MW which gives a 33% of loss reduction. For case 2, the placement for second unit of DG is done after the placement of first unit is determined. Next, another set of random loading pattern is generated and the results indicate bus 12 is the suitable location for second DG with a capacity of 0.5 MW.

Fig. 6 shows the comparison of voltage profile before and after the installation of single DG unit. Note that the minimum voltage level for the base case is about 0.91 p.u. recorded at bus 65. However, after installing DG with active power generation at bus 61, the minimum voltage profile of the distribution network is increased to 0.96 p.u. It proves that DG has the ability to improve the voltage profile. Voltage improvement is obvious seen for buses located along the lateral where DG unit is installed where as for other laterals not much difference is observed.

Also shown in Fig. 6, it can be noted that even after installation of DG, there are still buses which are near to the pre-specified voltage limit of 0.95 p.u. As such, a second DG unit is proposed to be installed in the system. In order to further see the effect on voltage stability, a comparison is made between the heuristic optimization method and analytical method. The analytical method results as given in Table I suggest the deployment of DG at bus 64. For the sizing results shown in Table II, case 3 is selected as the installed capacity. Note that, in (20) the DG operating size is specified not more than 150% of total connected load or equivalent to 1.9 MW.

The result of second DG unit installation is shown in Table III. The second DG is installed at bus 12. The resulting size is 0.5MW which corresponds to the limit given. Overall, with composition of 50% active power from DG; clearly the system restores voltage at all buses above operational voltage limit as shown in Fig. 7. In particular, a more convincing results is produced using the proposed BSA when DG is appropriately distributed in two different locations. The minimum voltage in the main lateral between bus 2 and bus 27 is increased above 0.96 p.u as compared to 0.95 p.u as from using the analytical method.

It is important to note that by determining the DG capacities with the optimum size at the selected buses, the line losses are significantly reduced to more than 40%, that is, double the amount of real power loss attained by having large capacity DG at one location. Thus, the proposed optimization method which combines the use of voltage stability index and power loss minimisation leads to more promising results.

V. CONCLUSION

In this work, a new voltage stability index is proposed as a fast assessment tool for identifying weak buses with small active power margins in a power system. The optimum distributed generator placement and size for improving voltage stability is determined by using the proposed voltage stability index and the backtracking search algorithm. The optimization problem has been successfully solved by considering minimization of real power loss and voltage stability improvement.

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TABLE III
RESULTS OF DG PENETRATION LEVEL FOR 69-BUS SYSTEM

Case	DG location	DG size (MW)	Total DG size (MW)	Total real power losses (MW)	Loss reduction (%)	DG Penetration (%)
Base	0	0	0	0.2259	0	0
Case 1	61	1.4	1.4	0.1518	33.02	26.32
Case 2	12 61	0.5 1.4	1.9	0.1272	43.69	50

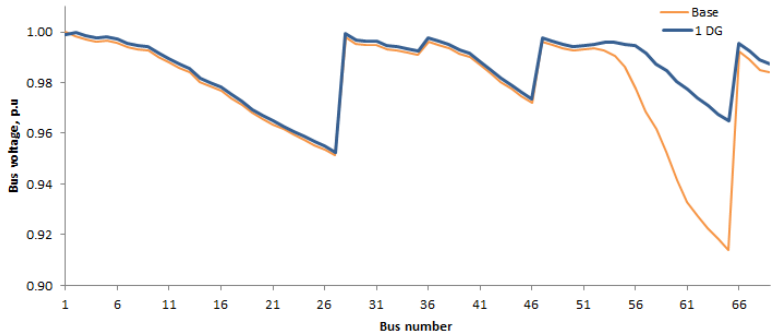

Fig. 6 Comparison of voltage profiles for the 69-bus system with DG

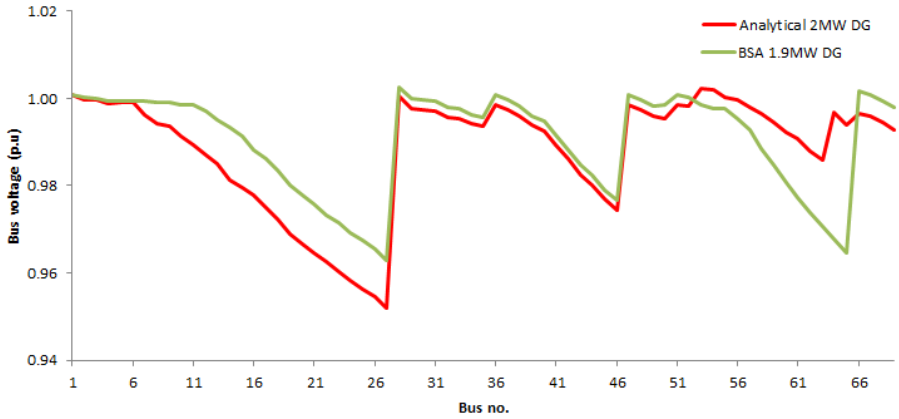

Fig. 7 Comparison of voltage profiles between BSA and analytical method