

# Multiple Model Adaptive Control in Uncertain Dynamic Model of Space Manipulator

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**Abstract**—In this paper, Multiple Model Adaptive Control approach is utilized in an uncertain dynamic model of free floating space manipulator. Uncertainty issue in the Space manipulator is a significant concern that must be resolved by an efficient control approach. The uncertainty challenge triggers problematic outcomes when uncertainty bound is large. Therefore, multiple model adaptive control approach is exerted in the dynamic model of space manipulator that switches over manifold controllers to handle the uncertainty issue. In order to subside switching deficiency an aggregation of weighted control signals, as the main control law, is used in the model. Simulation results show desirable outcomes.

**Index Terms**- uncertainty, space manipulator, free floating, multiple model adaptive control, switching.

## I. INTRODUCTION

Nowadays Space manipulator's usage is being increasing. Space manipulators have been substituted instead of astronauts. Therefore, versatile performance of the space manipulator is a significant requirement. One of the main source of unsuitable performance is uncertainty. Parlaktuna and Ozkan [1] utilized an adaptive control method to space manipulator using dynamically equivalent manipulator (DEM) model. In this approach, unknown parameters such as mass and inertia tensor are estimated by adaptive approach. DEM is a fixed base manipulator that models the dynamic of free floating space manipulator. This approach transfers nonlinear parameters of free floating space manipulator to linear parameters. Gu and Xu [2] used an extended robot model to handle the parameters of space manipulator with high nonlinearity. In this method, it is presumed that the position and orientation of base, its velocity and acceleration are available. Shin and Lee [3] proposed an adaptive method in joint space. They utilized the extended manipulator model's dynamics. On the other hand, they used an off-line adaptive identification scheme for these dynamics.

The uncertainty issue is more significant when the bound of uncertainty is large causing unstable behavior. In this case, handling of system, in order to acquire perfect performance, is a big challenge. Multiple Model Adaptive

Control (MMAC) is proposed to solve large uncertainty bound issue. In this approach, a multiple control system is designed to control a system with large uncertainty bound. Also a switching logic system alters those controllers sequentially to select the most appropriate controller for system. In this approach, the large bound of uncertainty is divided into several smaller bounds. Every controller is designed to handle the system with one of the small bounds of uncertainty. An estimation block provides an estimation of the uncertain parameter in every small bound. An appropriate controller is selected according to estimation error of uncertain parameter. In other words, controller with the smallest appropriate estimation error is selected as appropriate controller for the system. In this paper, the DEM approach is used to model the dynamic of the system.

This paper is organized as follows: section 2 presents dynamic of space manipulator. In the section 3, Multiple Model Adaptive Control (MMAC) is designed for the system. Stability proof for the system is rendered in section 4. Simulation results are shown in section 5. Finally, conclusion is presented in the section 6.

## II. DYNAMIC OF FREE FLOATING SPACE MANIPULATOR

A free floating space manipulator has complicated and nonlinear inertia parameters because of its free base. Therefore, dynamic representation of this intricate system is difficult. DEM model is efficient map to transfer the parameters of free floating space manipulator to a fixed base manipulator. Hence, inertia parameters of system is represented linearly. Considering an n-DOF free floating space manipulator with rigid links, the system includes an n-link manipulator and its base. The configuration of free floating space manipulator is shown in Fig. 1. In Fig. 1,  $(\phi, \theta, \omega)$  are Euler angles representing the position of the base,  $J_i$  is the joint connecting  $(i-1)$ th link and  $i$ th link,  $\theta_i$  is the rotation of  $i$ th link of space manipulator around joint  $J_i$ .  $C_o$  is total center of mass of the system,  $C_i$  is center of mass of space manipulator's  $i$ th link,  $l_i$  is vector connecting  $C_o$  to  $C_i$ ,  $u_i$  is the rotation axis of  $J_i$ . The total kinetic energy of the space manipulator's system is written as

$$T = \sum_{i=1}^{n+1} \left( \frac{1}{2} m_i \dot{\rho}_i^T \dot{\rho}_i + \frac{1}{2} \omega_i^T R_i^o I_i R_i^{oT} \omega_i \right), \quad (1)$$

where  $\dot{\rho}_i$  is the translational velocity of the center of mass of the  $i$ th link,  $\omega_i$  is the angular velocity of  $C_i$ ,  $R_i^o$  is the rotation matrix that denotes the coordinate frame  $l$  relative to frame  $o$ ,  $I_i$  is inertia tensor of  $i$ th link. The vector of coordinates of space manipulator is represented as

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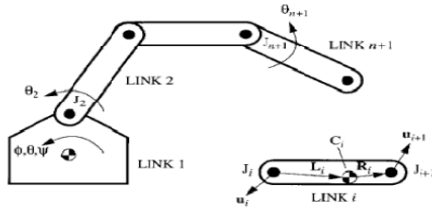


Fig. 1. Free floating space manipulator.

$$q = [\phi \ \theta \ \psi \ \theta_2 \ \dots \ \theta_{n+1}]^T = [q_1 \ \dots \ q_{n+3}]^T. \quad (2)$$

Therefore, dynamic equation of space manipulator is rendered as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} = \tau, \quad (3)$$

where  $M(q) \in R^{(n+3) \times (n+3)}$  is inertia matrix,  $\tau$  is torque exerted on the space manipulator's joints and  $C(q, \dot{q}) \in R^{(n+3)}$  is vector of the Coriolis and centrifugal forces.

To obtain a simpler representation of system dynamic, DEM model is used. DEM is a fixed base manipulator that preserves the dynamic characteristics of the free floating space manipulator. The first joint of DEM is identical to the base of space manipulator and they have the same dynamical behaviors. The first joint of DEM is passive spherical joint and no torque is exerted on that. The space manipulator and its corresponding DEM is shown in Fig. 2.

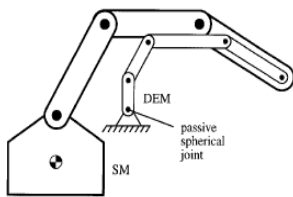


Fig. 2. Space manipulator and its corresponding DEM.

In Fig. 3, the DEM with dynamical parameters is shown. In Fig. 3  $(\phi', \theta', \omega')$  are Z-Y-Z Euler angles representing the position of the first joint,  $J'_i$  is the joint connecting the DEM's  $(i+1)$ th link and  $i$ th link,  $l_{ci}$  is the vector connecting  $J'_i$  to  $C'_i$ ,  $W_i$  is the length of  $i$ th link,  $\theta'_i$  is the rotation of the DEM's link around joint  $J'_i$ ,  $\omega'_i$  is the angular velocity of  $C'_i$ . The lagrangian of DEM model is as bellow

$$\frac{d}{dt} \left( \frac{\partial T'}{\partial \dot{q}_i} \right) - \frac{\partial T'}{\partial q_i} = Q'_i. \quad (4)$$

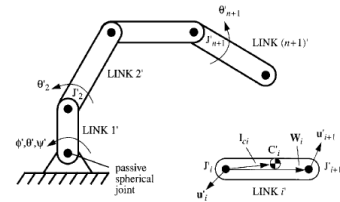


Fig. 3. DEM with dynamical parameters.

The first joint is passive, therefore, no torque is exerted upon that joint. Hence, the dynamic equation of DEM model is obtained as

$$M'(q)\ddot{q}' + C'(q, \dot{q})\dot{q}' = \tau', \quad (5)$$

where  $M'(q) \in R^{(n+3) \times (n+3)}$  is inertia matrix,  $\tau' = [0 \ 0 \ 0 \ \tau'_2 \ \dots \ \tau'_{n+1}]$  is torque exerted on the DEM's joints and  $C'(q, \dot{q}) \in R^{(n+3)}$  is vector of the Coriolis and centrifugal forces. In Equation 5,  $M'(q)$  and  $M'^1(q)$  are uniformly bounded. The specific map that transfers the parameters of free floating space manipulator to DEM model is represented as

$$\begin{aligned} m'_i &= \frac{m_i \left( \sum_{k=1}^{n+1} m_k \right)^2}{\sum_{k=1}^{i-1} m_k \sum_{k=1}^i m_k}, \quad i = 2, \dots, n+1, \\ I'_i &= I_i, \quad i = 1, \dots, n+1, \\ W_1 &= \frac{R_1 m_1}{\sum_{k=1}^{n+1} m_k}, \\ W_i &= R_i \left( \frac{\sum_{k=1}^i m_k}{\sum_{k=1}^i m_k} \right) + L_i \left( \frac{\sum_{k=1}^{i-1} m_k}{\sum_{k=1}^{n+1} m_k} \right), \quad i = 2, \dots, n+1, \\ l_{c1} &= 0, \\ l_{ci} &= L_i \left( \frac{\sum_{k=1}^{i-1} m_k}{\sum_{k=1}^{n+1} m_k} \right), \quad i = 2, \dots, n+1. \end{aligned} \quad (6)$$

In this paper, MMAC method is used to handle the uncertain dynamic. Uncertainty is considered to be in inertia tensor of joint 2 with large bound. The system with estimated uncertain parameter is obtained as

$$\hat{M}'(q)\ddot{q}' + \hat{C}'(q, \dot{q})\dot{q}' = \tau'. \quad (7)$$

### III. MULTIPLE MODEL ADAPTIVE CONTROL

The dynamic model of system has large uncertainty bound in inertia tensor of joint 2. The multiple model adaptive control (MMAC) provides a multiple controller system includes several controllers appropriated for the system with corresponding minor uncertainty bound. In other words, there is a switching block that alters controllers to choose the

suited controller with minimum uncertainty estimation error. In switching block, estimation error signals are compared with each other and the minimum one recognizes the appropriate controller designed for that minor uncertainty bound. The configuration of the MMAC is depicted in Fig. 4 (uncertainty bound is divided into N minor bounds).

For control of system, sliding mode control is used. According to sliding condition the control law for every controller block is designed. In order to handle uncertainty bounds, control law is considered as

$$\tau_j = s - k_j \operatorname{sgn}(s), \quad (8)$$

$$(s = \lambda_1 \dot{e} + \lambda_2 e, e = q_d - q, j = 1, 2, \dots, N)$$

where  $s$  is sliding surface,  $e$  is error,  $q_d$  is desired joint trajectory,  $\lambda_{1,2} \in R^{(n+1) \times (n+1)}$  are diagonal positive-definite matrices and  $k_j$  is a constant matrix chosen for the controller blocks according to inequality as below

$$k_j(i, i) \geq |\Delta f_i|, \quad k_j(i, i) = \operatorname{diag}\{k_j(i, i)\} \\ i = 1, 2, \dots, n+1, \quad (9)$$

and

$$\Delta f = [\Delta f_1 \quad \Delta f_2 \quad \dots \quad \Delta f_{n+1}]^T, \quad (10)$$

where

$$\Delta f = (\hat{M}' - M')\ddot{\sigma}_r + (\hat{C}' - C')\dot{\sigma}_r. \quad (11) \\ (\dot{\sigma}_r = \lambda_1 \dot{q}_d + \lambda_2 e', \ddot{\sigma}_r = \lambda_1 \ddot{q}_d + \lambda_2 \dot{e}')$$

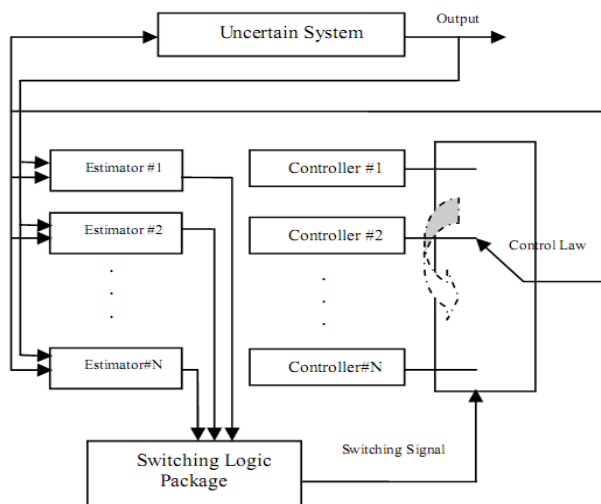


Fig. 4. Configuration of MMAC.

In order to select the appropriate controller, the switching block switches over the controllers. This switching obligation causes tendency to unstable behavior. Avoid having switching, a combination of weighted controllers provide the total control law as

$$\tau_T = \sum_{j=1}^N \alpha_j \tau_j, \quad (12)$$

where  $\alpha_j$  is coefficients of controllers weighted according to estimation error,  $\tau_j$  is the appropriate control law for the minor bound and  $\tau_T$  is total control law. The scheme of combinational controller is shown in Fig. 5. In this case,  $\alpha_j$  is weighted as Equation (13).

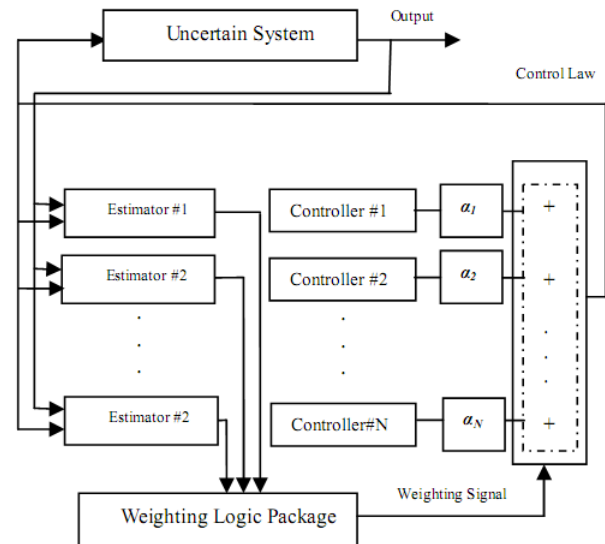


Fig. 5. Configuration of combinational controller in MMAC.

$$\alpha_j = \frac{e^{-|e_{ej}|}}{\sum_{j=1}^N e^{-|e_{ej}|}}, \quad (13)$$

where  $e_{ej}$  is estimation error in the  $j$ th bound.

#### IV. STABILITY ANALYSIS

In order to analyze the stability context, the passivity structure is used. The state variable  $q$  is joint trajectory of the manipulator. To derive a passive mapping, the following presumption (Equation (14) and Equation (15)) are assumed to be true.

$$M(q) = M^T(q) > 0, \quad (14)$$

$$-(\dot{M} - 2C) = (\dot{M} - 2C)^T \\ \Rightarrow x^T (\dot{M} - 2C)x = 0, \quad \forall x \in R^n. \quad (15)$$

The supply rate is considered as

$$S_r(\tau_T(t), \dot{q}(t)) = \dot{q}^T(t) \tau_T(t). \quad (16)$$

Therefore, corresponding storage function is achieved as

$$V(\dot{q}(t)) = \frac{1}{2} \dot{q}^T(t) M(q(t)) \dot{q}(t). \quad (17)$$

The dissipative system condition for space manipulator is as below

$$\int_0^T \dot{q}(t)^T \tau_T(t) dt \geq V(\dot{q}(t)), \quad (18)$$

or

$$\int_0^T \dot{q}(t)^T \tau_T(t) dt - V(\dot{q}(t)) \geq 0. \quad (19)$$

Ascertaining Equation (18) or Equation (19), the manipulator is a passive dissipative system.

## V. SIMULATION RESULTS

In this paper, a 2-DOF free floating space manipulator is considered as the system. Dynamic parameters of the system is mapped to a fixed base manipulator so-called DEM having three joints with the first passive spherical joint and two active joints. Dynamic parameters of space manipulator and DEM are shown in Table I and Table II, respectively. In the dynamic of system, the inertia tensor of joint 2 has a large uncertainty bound. This uncertain parameter is transferred through the map into DEM's dynamic.

TABALE I: DYNAMIC PARAMETERS OF 2-LINK SPACE MANIPULATOR

link	$L_i(m)$	$R_i(m)$	$m_i(kg)$	$I_i(Kg.m^2)$
base	-	0.5	4	0.4
2	0.5	0.5	1	0.1
3	0.5	0.5	1	0.1

TABLE II: DYNAMIC PARAMETERS OF 3-LINK DEM

link	$W_i(m)$	$l_{ci}(m)$	$m_i(kg)$	$I_i(Kg.m^2)$
1	0.333	0	4	0.4
2	0.750	0.333	1.8	0.1
3	0.917	0.417	1.2	0.1

The inertia tensor of joint 2 is laid in the uncertainty bound as below

$$I_{2nominal} - 0.5I_{2nominal} \leq I_{2nominal} \leq I_{2nominal} + 0.5I_{2nominal},$$

where uncertainty bound magnitude is 50% of nominal value of inertia tensor that is considered as a large uncertainty bound.

Constant  $N$ , matrices of  $\lambda_1$ ,  $\lambda_2$  and  $k_{1,2,3}$  are considered as

$$N = 4, \lambda_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \lambda_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, k_{1,2,3} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}.$$

As with Equation (19), ascertaining the inequality, the system is stable by using the MMAC approach. Simulation result is shown in Fig. 6.

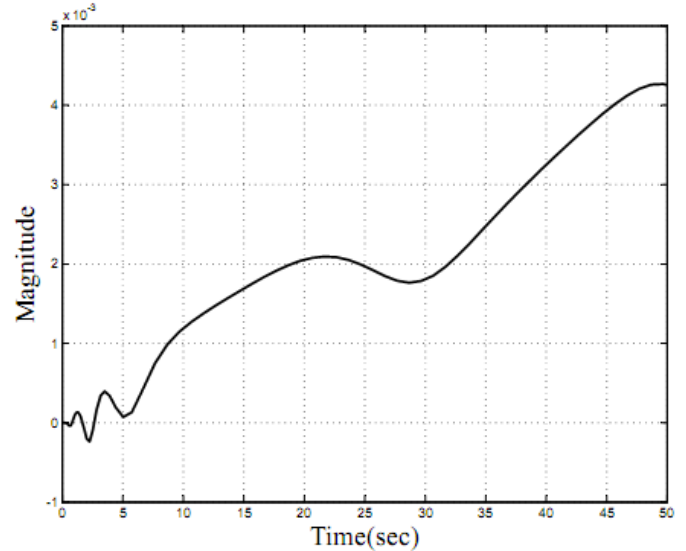


Fig. 6. Simulation for stability analysis of system.

According to Fig. 6, the curve is strictly positive. Therefore, inequality (19) is vindicated and the system is stable.

The combinational control signals, as exerted torques for joint 2 and 3, are shown in Figs. 7 and 8, respectively.

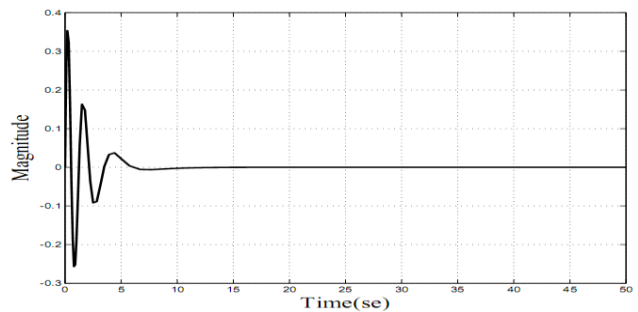


Fig. 7. Exerted torque for joint 2.

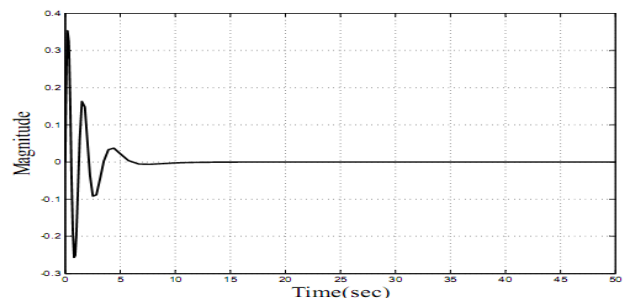


Fig. 8. Exerted torque for joint 3.

## VI. CONCLUSION

In this paper, an uncertain dynamic for a free floating space manipulator was considered as the system. The uncertain parameter with large uncertainty bound is inertia tensor in joint 2 of the space manipulator. In order to handle

this uncertain dynamic and stabilize the system, MMAC approach is proposed. In this control approach, the uncertainty bound is divided into some minor bounds. In every minor bound an estimation is obtained. Then, for every estimation an appropriate controller that in this paper is sliding mode control, is designed so that handles that minor uncertainty bound, obtaining a versatile results. In order to avoid switching, causing unstable behavior tendency, control laws are merged with a proper weighted coefficients. The simulation results show the system is stable by means of this control approach.

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