Investigation of Weight Matrix and the Average Probability of Observability and System Observability Redundancy Indices in Optimal Placement of PMU

Majid Saeidi¹, Mohammad Zohour Attar², Farhad Namdari³, and Sattar Aghayee poor⁴

Abstract— Phasor measuring units (PMU) is a completely new technology to estimate the general state of power system, the use of which leads to terminating most nature delays existing in estimation of state. In this article an attempt has been made to determine the best placement of PMU in minimum numbers for total observability of network. Since the PMU with more measuring directions costs more, the investigation of weight matrix is dealt with here. To choose the optimal answer in equal conditions of weight matrix, the indices of the observability redundancy and average probability of system observability are also suggested. The solution of the problem of this article is simulated with linear integer program on MATLAB software in IEEE standard 9- and 57-bus system. The results purport the placement of PMU with less cost and effective performance of suggested scales in ranking the answers of placement problem.

Keywords— Optimal Placement, Weight Matrix, System Observability Redundancy Index, Average Probability of Observability, Phasor measuring unit.

I. INTRODUCTION

One of the most basic elements of systems of energy management is state estimates. State estimates calculate the variables of state system i.e. voltages and relative angles of buses by using the solution of nonlinear equations and according to the sent measurements of the surface of system [1]. Phasor measurement unit (PMU) by transforming usual nonlinear state estimation equations of network to linear ones and with no need for complex computing methods to solve them leads to improvement of the speed of control, protection and managing systems which use the results of state estimations[2,3]. But because of the high cost of this equipment and the fact that installing PMU on each bus enables voltage phasor of that bus and current phasor of all attached branches to it to measured, the amount of voltage and the angle of phase of attached buses to the bus equipped with PMU can also be computed and indirectly observed. The optimal specification of these measuring instruments in power networks according to their place and number for observability of the whole network and also the least amount of cost are remarkable [4,5,6,7]. The cost of installation and operating PMU in different places is, however different, as the installed PMU in the bus which is connected to more branches and has more measuring directions, is higher than other PMUs [3].

In reference [8] placement to gain a complete set of answers by ILP method is introduced. In the reference [3] for a sample network with PSO, the investigation of weight matrix in placement of PMU in normal stage has been done without the comparison and ranking of answers. In the reference [9] evaluation of the measuring performance of phasor system and the introduction of bus observability index (BOI) and system observability redundancy index (SORI) are dealt with. In reference [10] with algorithm of mixed integer programming (MIP) operating PMU in system has been done by using the index of average probability of observability (APO).

In this article, based on the more optimality of the answer of ILP method according to the need of workers of power industry who risk to have a technical and economic optimal choice, a complete answer set of placement with minimum number of PMU considering weight matrix are specified and with indices of APO and SORI the answer set for the IEEE standard 9- and 57-bus systems are compared.

II. THE FORMULATION OF PMU OPTIMAL PLACEMENT

The formulation of the optimal placement of PMU in a system with n buses is presented as equation (1) [2,9,10,11,12]:

\[
\text{min } \sum_{i=0}^{n} w_i x_i \\
\text{s.t. } y = Ax \geq b
\]

In equation 1, w is the cost function for the installed PMUs, and in normal stage, placement equal to matrix of unit nxn is considered. A is connection matrix of nxn reveals how buses are connected and is defined as (2) [2,9,10,11,12]:
The discrete nature of the optimal placement of PMU make it necessary for X vector to be defined as equation (3) such that the elements of that position show the installation of this equipment in each bus [2,9,10,11,12,13]:

\[
A_{n \times n}(i,j) = \begin{cases} 
1 & \text{if buses } i \text{ and } j \text{ are connected} \\
1 & \text{otherwise}
\end{cases} 
\]  

(2)

The optimal placement of PMU is determined for each bus [2,9,10,11,12]:

\[
x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{cases} 
1 & \text{if PMU is installed at bus } i \\
0 & \text{otherwise}
\end{cases}
\]  

(3)

So if the \( x_{ij} = [x_{i1} \, x_{i2} \, \ldots \, x_{in}] \) is the optimal answer of placement of PMU with m number, to gain w, the optimal answer without the repetition of similar answers, \( A_{n \times m} \) matrix and \( b_{nv1} \) vector will be modified as equation (5)[8].

\[
A_{(n+w) \times n} = \begin{bmatrix} A_{n \times n} & -A_{n \times n} & \cdots & -A_{n \times n} \\
-x_{i1,1} & x_{i1,2} & \cdots & x_{i1,n} \\
-x_{i2,1} & -x_{i2,2} & \cdots & x_{i2,n} \\
\vdots & \vdots & \ddots & \vdots \\
x_{nw,1} & -x_{nw,2} & \cdots & -x_{nw,n} \end{bmatrix}
\]  

(5)

\[
b_{(n+w) \times 1} = [1 \, 1 \, \ldots \, (m-1) \, \ldots \, (m-1)]^T
\]  

(6)

Therefore the optimal placement of phasor measuring units is calculated to achieve a complete set of answers as equation (6)[8].

\[
\min \sum_{i=1}^{n} w_i x_i \\
\text{subject to: } A_{(n+w) \times n} \cdot x_{n \times 1} \geq b_{(n+w) \times 1}
\]  

(6)

In this equation the A matrix and b will be modified vector in each repetition. The repetition will be stopped if the number of PMU is larger than m which is the number of optimal PMUs calculated in the first iteration.

III. OPTIMAL PLACEMENT OF PMU CONSIDERING THE EFFECT OF THE WEIGHT MATRIX (w)

Weight matrix in the equation (1) of PMU placement is related to the number of connected branches to each bus. Equation (7) shows the value of bus with the number of connected branches [7]:

\[
w = \begin{cases} 
1 & \text{if } \text{NUM}_i < 3 \\
1.5 & \text{if } \text{NUM}_i \geq 3
\end{cases}
\]  

(7)

In this equation \( \text{NUM}_i \) reveals the number of connected lines to bus j where, if it is less than 3 the weight of bus j is one, and if it is equal to 3 or more than it, the weight of bus j is 1.5.

IV. BUS OBSERVABILITY INDEX AND SYSTEM OBSERVABILITY REDUNDANCY INDEX IN PLACEMENT OF PMU

Bus observability index (BOI) is proposed as performance indicator on quality of the optimization. BOI for bus i (\( \beta_i \)) is defined as the number of phasor measuring units which are able to observe a given bus [9]. System observability redundancy index (SORI) is defined as the total set of BOIs of system buses. In other words, if bus i with the number of \( \beta_i \) PMU is observable, SORI is obtained as (8) [9].

\[
\text{SORI} = \sum_{i=1}^{n} \beta_i
\]  

(8)

V. AVERAGE PROBABILITY OBSERVABILITY INDEX

The value of average probability of observability of all buses is considered as the system index that makes a value checking on observability of power system for operator and is calculated as the equation (9)[14]:

\[
\text{APO} = \frac{1}{N_b} \sum_{i=1}^{N_b} P_{oi}
\]  

(9)

In which \( N_b \) is the number of system buses and i is a set of buses.

\[
P_{oi} = \text{the probability of observability related to bus } i \text{ which is achieved from equation (10)[14,15]:}
\]  

\[
P_{oi} = 1 - \prod_{j=1}^{m} (1 - \mu_j A_{ij}); \forall i
\]  

(10)

Where \( \mu_j \) is a binary variable and investigates PMU installed in bus j and \( A_{ij} \) examines probability of observability of bus i by installing PMU in bus j. This amount is defined as the equation (11)[14,15]:

\[
A_{ij} = a_{ij} A_{ij}^{\text{PMU}} A_{ij}^{\text{Link}} A_{ij}^{\text{MU}} A_{ij}^{\text{PT}} A_{ij}^{\text{CT}}; \forall i, \forall j
\]  

(11)

In which \( A_{ij}^{\text{PMU}} \) and \( A_{ij}^{\text{Link}} \) show availability of voltage measurement respectively by installing PMU in bus j and current measurement with PMU in line ij which can be calculated in availability conditions of voltage transformer (PT) and current transformer (CT)[1,15]. \( a_{ij} \) and \( d_{ij} \) are the parameters of binary connections and are shown as equation (12) [14].

\[
a_{ij} = \begin{cases} 
1 & \text{if buses } i \text{ and } j \text{ are connected} \\
0 & \text{otherwise}
\end{cases}
\]  

(12)

Therefore every answer which has higher APO is selected as an optimal answer.

VI. THE SIMULATION RESULT

In this section to evaluate the capacity of the proposed formulization in the first stage, PMU placement has been done with the unit of taking weight matrix into consideration and in the second level the possible set of answers with the least number is calculated by considering weight matrix based on equation (10). In this stage to rank the answers, two indices of the observability redundancy and average probability of observability system are used.

One-line diagram of the IEEE 9- and 57-bus system are given in figs (1) and (2).
Stage 1) The complete observability in normal stage

In this stage in Table (1) according to the formulization of optimal placement of PMU in equations (2)-(5) optimal placement of PMU in minimum number has been done for system observability by specifying the optimal place and the number. The results of this table show the complete observability of system buses while almost 30% of system buses are equipped with PMU in normal stage of placement.

<table>
<thead>
<tr>
<th>Test system</th>
<th>No. of PMUs</th>
<th>Location of PMUs</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE 9-Bus</td>
<td>3</td>
<td>4,6,8</td>
</tr>
<tr>
<td>IEEE 57-Bus</td>
<td>17</td>
<td>1,4,6,13,20,22,</td>
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<tr>
<td></td>
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<td>25,29,32,36,39,</td>
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<td>41,45,47,51,54</td>
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</table>

Stage 2) The complete observability of network by considering the effect of weight matrix (w)

In this stage in the Tables (4) and (5) according to the equations (6) and (5) the set of answers with the minimum number of PMU is calculated by taking the effect of weight matrix into consideration. Also in order to choose the more optimal answer and the ranking of answers (in conditions in which there is no chance to install PMU on some of the network buses) by prevailing the minimum amount of weight matrix the index APO by using the equations (9)-(12) and the data of Tables (2) and (3) based on reliability and availability of transmission lines for IEEE 9- and 57-bus and the SORI with equation (8) are used. As it can be seen weight matrix has no effect on the minimum number of PMUs.
having the number and w equal with other answers because the indices APO and SORI are higher is the best place to install PMU in the IEEE network 9-bus network for complete system observability. In this way the answers can be ranked with the minimum weight matrix and then high index APO, so if there is a limitation in installing on one row’s answer there will be a better choice among the other answers. Therefore after answer row 1, answer row 2, answer row 4 and answer row 3 are ranked for the selection of optimal answer.

In the same way by evaluating the results of Table (6) the one answer that has less w is more suitable. Therefore the answer rows 1, 2, 3, 4 in this stage with the minimum weight matrix and the equality of observability indices for them are optimal answers. Next to them, answer row 5 and 8 with the higher number of indices compared with row 6 and 7 has priority to be selected and so row 7 is better than row 8.

VII. CONCLUSION

In this paper the answer set of placement PMU in minimum number is calculated for complete observability of the IEEE 9- and 57-bus system IEEE when the weight matrix is taken into consideration and by prevalence of the minimum amount of this matrix the optimal answer is selected. Because there is the probability of answer row in equal weight matrix, to select the optimal answer row and ranking of the answers for conditions in which because of limitations there is no opportunity to select one answer, APO and SORI were proposed to be studied. By using them the answers are ranked with the prevalence of the minimum number of weight matrix and the maximum number for these two indices.

REFERENCES


According to the results of Table (5) the answer row 1 by

<table>
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<tr>
<th>No. of PMU</th>
<th>Rows</th>
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<th>SORI</th>
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<table>
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