Robust Line Balancing on Mixed-Model Assembly Line under Processing Time Uncertainty

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Abstract—Recently, customers’ demands are more personalized and heterogeneous. Therefore, manufacturing firms pay more attention to customization and responsive production system, like Mixed-Model assembly line (MMAL), which can produce in small lots and diverse kinds of products on one line simultaneously. To obtain efficient productions, line balancing method, a scheme for assigning tasks to workstation to meet takt time and balance workload among workstations with minimum idle time, is important. However, Uncertainties, such as uncertain processing time, exist in the reality, and cause higher production costs. Thus, this work focuses on MMAL with uncertain processing time and proposes robust scheduling method which achieve high balancing rate, while confronting with uncertain processing time perturbation. Some robustness measures are considered to define the schedule’s ability to keep its initial solution against uncertain environment. Furthermore, the sensitivity analysis is used to study the effect of processing time uncertainty at different degree of conservatism by comparing robustness and makespan.

Keywords—Mixed-Model assembly line, line balancing, robust optimization, processing time uncertainty

I. INTRODUCTION

Due to the globalization, international integration has been expanded continuously and customers can access to real-time and worldwide information which cause high market competition. To compete with their rivals in this environment, many firms have to provide customers more custom-made alternatives to increase their competitiveness. Customers are able to select and modify their choices rely on their financial budget. Thus, customization and responsive production system are necessary. To face this problem, firms have to consider customization by producing small lot size, but various kinds of products, which causes higher cost of production. In addition, there are uncertainties exist in the reality, such as uncertain processing time. Most of conventional methods assumed that processing times are precisely known and deterministic, which leads to the unrealistic drawback. Because, in the practice, it is rarely in that manner due to the random nature and measure error.

Moreover, there are very few conventional researches that concern with robustness in this problem, which are still not enough in this problem solving. With this reason, a responsive production, like Mixed-Model assembly line (MMAL), is required. MMAL can produce a small lot size and diverse kinds of products on one line simultaneously. Also, the most robust schedule with high balancing rate, while confronting with the uncertain processing time perturbation is set as our goal.

To achieve the productive production, line balancing problem is essential. The assembly line balancing is a scheme for assigning tasks to workstation in order to meet production cycle and balance workload among workstations with minimum idle time. To cope with line balancing problems of MMAL, there are two common problems must be solved. The first one is to deal with how to distribute tasks among workstations, in order to balance workload of assembly line as much as possible. The Second one is how to create a production task sequence for minimizing production makespan. Moreover, to make the Assembly line more reliable under the processing time uncertain environment, the robustness should be considered to define the ability of the schedule to remain its initial solution against uncertain environment.

In this research, there are two approaches to solve the problems. The first approach, we proposed an algorithm, which can be divided into two phases. In phase I, deterministic scheduling is studied and a near-optimal initial solution is generated by modified Ranked Positional Weight method. In phase II, deterministic processing times are relaxed to be uncertain which take values in symmetric and bounded interval and has a uniform distribution. Robust counterpart optimization based on Bertsimas and Sim formulation is applied to transform uncertain processing times to be deterministic processing times and check the takt time violation. Furthermore, in the experiment plan, the effect of uncertainty on processing time is analyzed by using a sensitivity analysis.

II. PROBLEM DESCRIPTION & FORMULATION

A. Basic Constraints [1]

There are 2 basic constraints must be satisfied.

1) Takt time: this constraint controls how often finished products must be produced and come out at the end of assembly lines. We can find takt time by the following formula:
2) **Precedence Relation:** this constraint controls the relation between tasks. For example, in fig. 1, task 1 and 5 must be finished before operating task 6.

**Process**

![Fig.1: Precedence Graph](image)

**B. Assumptions of MMAL are as below:**

1. Task times are relaxed to be uncertain and have symmetric and bounded form, which take value in uncertain interval and have a uniform distribution.
2. Tasks among different models have no need to be same.
3. Priority relations of all models are precisely known.
4. WIP is not allowed between workstations.
5. Common tasks of different models must be assigned to the same stations.
6. All models have the same number of workstation.
7. Parallel stations are not allowed.

C. The notation used for Robust Mixed-Model Assembly Line Balancing Problem Constraints:

- **N**: Number of different assembly tasks
- **M**: Number of Model to be assembled on MMAL
- **K**: Number of workstations
- **i**: Task index
- **j**: Job Index
- **k, l**: Station Index
- **TT**: Takt time
- **x\(_{ik}\)**: Binary variable, equal to 1; if task i is assigned to station k and equal to 0; otherwise
- **IP\(_m\)**: Set of immediate predecessors \(IP_m = \{(task1, task2), (task1, task5) \ldots\}\)
- \(\bar{t}_{ij}\): Deterministic processing time of task i of model j
- \(\hat{t}_{ij}\): Uncertain Processing Amplitude
- \(\tilde{t}_{ij}\): Uncertain Processing Time
- **NT\(_{kj}\)**: Number of tasks in subset of all tasks that can be assigned to station k of model j
- **\(\hat{\xi}_j\)**: Binary variable, equal to 1; if station k is utilized for model j and equal to 0; otherwise
- **\(A_k\)**: Binary variable, equal to 1; if station k is utilized by all models and equal to 0; otherwise

D. **Throughput-related performance measure**

In order to measure solutions' ability to keep its original solution, in this work, we consider balancing rate.
G. Robustness Measure

Balancing Rate Regret: the deviation of deterministic balancing rate and realized balancing rate, or we call regret and this solution is often called Robust.

\[ \text{BRR} = \text{DBR}(X^0) - \text{RBR}(X) \]  

Where X is scenario solution and X^0 is deterministic and a low value of BRR indicates more robustness.

H. Line Balancing Problem of MMAL with uncertainty:

\[ \text{Min} \quad \text{BRR} \quad \text{Max} \quad \text{RBR} \]  

Subject to: \[ \sum_{k=1}^{M} x_k = 1 \]  
\[ \sum_{k=1}^{M} k \times x_k - \sum_{i=1}^{l} i \times x_k \leq 0; \forall (a, b) \in IP \]  
\[ \sum_{i=1}^{N} t_i \times x_k \leq TT \]  
\[ x_k - NT_k \times \zeta_k \leq 0 \]  
\[ \sum_{j=1}^{M} \zeta_k - M \times A_k = 0 \]

The formulation set (3)-(8) and the formulation set (11)-(16) are similar. The differences are the objective function and takt time constraint. For the objective function (3) is to maximize deterministic balancing rate, but the objective function (11) is to maximize balancing rate regret or it is equal to maximize realized balancing rate (RBR) value.

For the takt time constraint, the processing time variable (\( \tilde{t}_{ij} \)) in (6) is the deterministic data, which we assume that deterministic processing times is precisely known. However, the processing time variable (\( t_{ij} \)) in (14) represents the uncertain processing times after we relaxed deterministic processing times (\( \tilde{t}_{ij} \)), which has a uniform distribution and takes value in uncertain range [\( \tilde{t}_{ij} - \bar{t}_{ij}, \bar{t}_{ij} + \bar{t}_{ij} \)].

III. SOLVING METHOD

Phase I: Deterministic Scheduling

1. Collect and Calculate input data
2. Generate initial solution
3. Add some uncertainties and Perturbation analysis
4. Workstation’s Takt time Constraint (upper limit)
   - Yes Changing sequence or increasing no. of workstation
   - No Optimal Robust Solution

Phase II: Robustness Optimization

\[ \text{Product Demand Weight:} \quad P_{Wj} = \frac{P_j}{\sum_{j=1}^{m} P_j} \]  
\[ \text{Weighted Task Time (WPT)} = \frac{\sum_{j=1}^{m} P_i \times P_{Wj}}{\sum_{j=1}^{m} P_{Wj} \times u_{ij}} \]

Where
- \( i \): Task index
- \( j \): Product index
- \( P_j \): Demand of model j
- \( \tilde{t}_{ij} \): Task time of job i of model j
- \( P_{Wj} \): Product demand weight of model j
- \( u_{ij} \): Binary variable; equal to 1, when task i is operated by model j; otherwise 0.

Modified Helgeson-Birnie Method Procedure:

Step 1: Draw precedence graph.
Step 2: Calculate takt time and Weight Task Time by demand volume.
Step 3: For each task, determine the positional weight. It is the total time on the longest path from the beginning of the operation to the last operation of the network.
Step 4: Rank the work elements in descending order of ranked positional weight (R.P.W).
Phase II – Robustness Optimization

Assume that the general linear optimization problem is as formulation below:

\[
\begin{align*}
\text{Maximize} & \quad c'x \\
\text{Subject to} & \quad Ax \leq b \\
& \quad l \leq x \leq u
\end{align*}
\]

Let’s assume that uncertainty affects the elements in matrix A. Let \( J \) is a set of coefficients \( a_{ij}, j \in J \) that are subject to uncertainty. \( \tilde{a}_i, j \in J \) has symmetric and bounded form and take values in uncertain interval \([a_i - \tilde{a}_i, a_i + \tilde{a}_i]\) with mean is equivalent to nominal value \( a_{ij}, j \in J \). Bertsimas and Sim [4]–[7] introduced a parameter \( \Gamma \), or call budget of uncertainty of constraint i, which is not necessary to be integer and takes values in \([0, |J|]\) to adjust robustness against the level of conservatism. However, it is unlikely that all \( a_{ij}, j \in J \) will change, so the goal is to control that up to \([\Gamma_i]\).

They consider the following formulation:

\[
\begin{align*}
\text{Maximize} & \quad c'x \\
\text{Subject to} & \quad Ax \leq b \\
& \quad l \leq x \leq u \\
& \quad y \geq 0 \\
& \quad 0 \leq \sum_j a_{ij} + \sum_{j \in J} \hat{a}_{ij} y \leq b, \forall i
\end{align*}
\]

(19)

(20)

And let \( \beta(x^*, \Gamma) = \max_{[a_i] \in [\Gamma]} \left\{ \sum_{j \in J} \hat{a}_{ij}, \forall i \right\} \)

which is called protection function.

(21)

If \( \Gamma_i \) is integer, the \( i \)th constraint will be protected by

\[
\beta(x^*, \Gamma) = \max_{[a_i] \in [\Gamma]} \left\{ \sum_{j \in J} \hat{a}_{ij} \right\}.
\]

If \( \Gamma_i = 0 \), there is no protection against uncertainty and equal to nominal problem.

If \( \Gamma_i = |J| \), highest protection and the most conservative.

If \( \Gamma_i \in [0, |J|] \), the decision maker can adjust the robustness against the level of conservatism of the solution.

However, the formulation (20) is still non-linear formulation. To reformulate (20) to be linear optimization problem, we need to do as follows:

Let a vector \( x^* \) be the optimal solution of formulation (21)

\[
\beta(x^*, \Gamma) = \max_{[a_i] \in [\Gamma]} \left\{ \sum_{j \in J} \hat{a}_{ij}, \forall i \right\} + \left( \Gamma_i - \sum_{j \in J} \hat{a}_{ij} \right)
\]

(22)

And it equals to the objective function of the following linear optimization problem:

\[
\begin{align*}
\text{Min} & \quad \sum_{j \in J} \hat{a}_{ij}, \forall i, \forall j \\
\text{Subject to} & \quad 0 \leq \sum_{j \in J} \hat{a}_{ij} + \Gamma_i, \forall i, \forall j
\end{align*}
\]

(23)

It is clearly that the optimal solution value of (23) consists of \([\Gamma_i]\) variables at 1 and another variable at \(\Gamma_i\) and the duality of (23) is as (24):

\[
\begin{align*}
\text{Maximize} & \quad \sum_{j \in J} \hat{a}_{ij} + \Gamma_i z_i, \forall i, \forall j \\
\text{Subject to} & \quad z_i + r_i \leq \Gamma_i, \forall i, \forall j \\
& \quad r_i \geq 0, \forall j \\
& \quad z_i \geq 0, \forall i
\end{align*}
\]

(24)

Where \( r_i \) is the dual variable of the inequality \( z_i \leq 1 \) and \( z_i \) is the dual variable of the inequality \( \sum_{i \in J} z_i \leq \Gamma_i \).

With strong duality, \( \beta(x^*, \Gamma) \) is equivalent to the objective value of (24), so we can reformulate the robust formulation (20) by substituting the inner optimization problem (protection function) of (24) (the dual problem of (23) and obtain (25) which is equivalent to (20).

\[
\begin{align*}
\text{Maximize} & \quad c'x \\
\text{Subject to} & \quad \sum_j a_{ij} x_j + \sum_{j \in J} \hat{a}_{ij} \gamma_j \leq b, \forall i, \forall j \\
& \quad -y \leq x_j \leq y, \forall j \\
& \quad l \leq x_j \leq u, \forall j \\
& \quad y \geq 0
\end{align*}
\]

(25)

By the above proposition, we can apply to (11)-(16) and get the robust line balancing Problem of MMAL as follow:

\[
\begin{align*}
\text{Min} & \quad BRR(X) \longleftrightarrow \text{Max} \quad RBR(X)
\end{align*}
\]

(26)

\[
\begin{align*}
& \sum_{k \in E} \sum_{i \in J} k \times x_{ai} - \sum_{k \in E} \sum_{i \in J} k \times x_{ai} \leq 0, \forall (a, b) \in IP_w
\end{align*}
\]

(28)

\[
\begin{align*}
& \sum_{i=1}^{N} \sum_{k \in E} \sum_{i \in J} k \times x_{ai} + z_i \times \gamma_i + \sum_{i=1}^{N} r_i \leq TT
\end{align*}
\]

(29)

(30)

(31)
\[ x_e - NT_s \times \xi_e \leq 0 \]  
(32)

\[ \sum_{j=1}^{M} \xi_{e_j} - M \times A_e = 0 \]  
(33)

(26) - (33) can be solved by optimization software for mathematical programming and we utilize ILOG to solve this problem.

IV. EXPERIMENTS AND EVALUATION

In this section, we will evaluate four purposes by using the case study to evaluate the following:

I. Finding the most robust assembly schedule.
II. Sensitivity analysis with changing Budget of Uncertainty (\(\Gamma\))
III. Sensitivity analysis between robustness and makespan
IV. Finding the relation (trade-off) between required robustness and necessary makespan with different variation amplitude (\(\hat{t}_{ij}\)) of nominal processing time by changing value of budget of uncertainty (\(\Gamma\)).

Let’s assume that demand of product A is 600 units, product B is 1,000 units and product C is 400 units. Thus, total demand is 2,000 pcs. The product demand weight of Product A, B and C are 0.3, 0.5 and 0.2 respectively.

Working hours is 8 hours/ day or 28,800 second/ day and the precedence graphs [8] of each product are as fig. 3:

<table>
<thead>
<tr>
<th>Task</th>
<th>WPT</th>
<th>RPW Value</th>
<th>Ranking by RPW</th>
<th>Workstation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4s</td>
<td>35.55</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>6.25s</td>
<td>20.65</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>7s</td>
<td>16.3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>6s</td>
<td>11.4</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>3.9s</td>
<td>9.3</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3s</td>
<td>8.4</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>5.4s</td>
<td>5.4</td>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>

A. Experiment I

The aim is to find the solution with the highest robustness, which has the minimum value of BRR. In order to obtain the most robust schedule, we will follow the proposed algorithm as follows:

Phase I - Deterministic Scheduling

From the case study, we can generate the initial solution by calculating RPW value, ranking all tasks by RPW value in descending order and assigning all tasks to workstations as table I. Moreover, as you can see from Fig.4, the cycle times of the initial solution do not exceed the takt time limit, so this initial solution is feasible with Deterministic Balancing rate (DBR) equal to 82.29%.

Phase II – Robustness Optimization

Let’s assume that the budget of uncertainty is 0.4 and variation amplitude is 15% from deterministic processing times. After we add the uncertainty to the initial solution from Phase I and solve the robust line balancing Problem of MMAL, we obtain the solution as Fig.5.

B. Experiment II

In evaluation II, we study the sensitivity analysis with changing Budget of Uncertainty (\(\Gamma\))
Fig. 6 BRR analysis

In the Figure 6, BRR value lines of 3-station and 4-station solutions are represented by circle and triangle symbols. Max cycle time of 3-station and 4-station are represented by square and cross symbols consecutively.

When we compare each solution with the same number of workstation, if budget of uncertainty increases, the BRR value will decline slightly and max cycle time will increase until it reaches the threshold point which the solution cannot bear more uncertainty perturbation because it will exceed the takt time limit. In order to prevent this happen, we have to regenerate the initial solution again by increasing a number of workstations, which causes the solution point jumps into 4-station line and max cycle time will decrease dramatically due to the task distribution to a new workstation.

The critical point is at the point with 0.4 budget of uncertainty, -6.8125% BRR value and 14.31s max cycle time. When the value of budget of uncertainty increase to 0.5, the max cycle time will be 14.46s, which violates the takt time limit. Thus, the feasible solution point will jump to the 4-station line in order to avoid the takt time violation and the value of BRR and max cycle time will be -5.8008% and 11.09s respectively.

C. Experiment III

In experiment III, we study a sensitivity analysis between robustness and makespan at different value of BRR.

Fig. 7 Sensitivity analysis between BRR value and makespan

As we can see from figure 7, we can say that the solutions with budget of uncertainty ($\Gamma$) higher than 0.4 will be infeasible because their makespans surpass the available working time, 28,800s. The solution with shorter makespan will give lower robustness (values of BRR increase). In other words, we have to compensate for the higher robust solution by accepting the longer makespan. For example, at the budget of uncertainty ($\Gamma$) 0.3, the makespan is 27,715.80s with -3.7031% BRR value, while at the budget of uncertainty ($\Gamma$) 0.4, the makespan is 28,664.17 with -6.8125% BRR Value. Thus, the decision makers have to make a trade-off between granting the higher robust solutions, but accepting the longer makespans, or obtaining the shorter makespans but taking the lower robust solutions.

D. Experiment IV

This experiment purpose is to find the trade-off between the required BRR value/ Robustness level and the necessary makespan at different variation amplitude by changing value of budget of uncertainty ($\Gamma$). Two scenarios are set with 15% and 20% variation amplitude ($\hat{t}_{ij}$) of nominal processing times consecutively.

Fig. 8 trade-off between the BRR value and makespan at different variation amplitude

As shown in figure 8, the Makespan of 15% and 20% variation amplitude ($\hat{t}_{ij}$) scenarios are represented by cross and square symbols. The triangle and circle symbols define BRR value of 15% and 20% variation amplitude ($\hat{t}_{ij}$) scenarios respectively.

From the figure 8, we can notice that at the budget of uncertainty is 0, all scenarios have the same makespan and BRR value because there is no any uncertain perturbation in any solutions. When we compare the set of solutions with the same budget of uncertainty ($\Gamma$) except 0, the scenarios with greater variation amplitudes ($\hat{t}_{ij}$) have longer makespan and higher robustness (less BRR values) than the scenarios with lower variation amplitude ($\hat{t}_{ij}$). For example, in the figure 8, the solutions at 0.2 budget of uncertainty, the 20% variation amplitude solution has a longer makespan, 27,583.19s, and higher robustness,-3.2917% (less BRR value) than the 15% variation amplitude solution, which makespan and BRR value are 27,317.97 and -2.4688%. Because, the solutions with the
higher variation amplitude \( (T^*_{ij}) \) guarantee that they can absorb the higher oscillation than the solutions with lower variation amplitude, but they also have to recompense for higher robustness with longer makespan.

For decision makers, they can utilize this graph by specifying their desired range of BRR value, how robust of a solution they want, and estimate the budget of uncertainty to show the degree of conservatism. After that, choosing a solution from the feasible candidate solutions within the limited range, which does not exceed the available working time. For example, if a decision maker set the required BRR value range between [-6%, -10%] and they think that the budget of uncertainty is 0.3, as we can see from the figure 8, there is only one candidate solution at the point with BRR value -6.8125% which its makespan does not exceed the available working time. Thus, the decision makers will choose this solution as their desired solution.

V. CONCLUSION AND FUTURE WORK

In this work, we proposed two approaches. The first approach, we proposed the two-phase algorithm. Phase I is to find the near-optimal solution for MMAL by using modified Helgeson-Birnie Method, or called ranked positional weight method, in order to generate an initial solution. After that, in phase II, we add some uncertainty into the initial solution and make a tractable trade-off by applying robust counterpart optimization based on Bertsimas and Sim formulation. Then, we check the takt time constraint violation, if takt time constraint is violated, we will regenerate the initial solution again. If not, we will obtain that solution as a robust optimal solution. We measure robustness of solution by using balancing rate regret in order to define the ability to keep its initial solution against processing time uncertainty. Moreover, we use sensitivity analysis method to study the effect of processing time uncertainty at different degree of conservatism by comparing robustness and makespan.

In the future work, we could additionally concern with uncertain demand volume and material consumption. In order to expand the problem scope for stretching to the real-world practice.

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