Optimization of Economic Load Dispatch Problem by Linear Programming Modified Methodology

Ahsan Ashfaq, and Akif Zia Khan

***Abstract*—**High costs for fossil fuels and escalating installations of alternate energy sources are daunting main challenges in power systems by making the economic operation and planning of power system as high tasks among the major assignments in the power generation. In this paper we solve one of the main featured economic problems of power system by optimizing the economic load dispatch problem by linear programming and compared it with firefly algorithm and lambda iteration method. We have applied the proposed algorithm on different systems of generation units and the results proved the attainment of optimal point when compared with different algorithms.

***Keywords*—** Economic dispatch, transmission losses, modified methodology, linear programming.

NOMENCLATURE

i	*No. of gen units where (i=1-N)*
$F(t)$	*Fuel cost*
P	*Power generated*
a_i,b_i,c_i	*Fuel cost curve coefficients.*
P_i	*Power generated by i no. of generators (MW).*
P_r	*Power required to meet load (MW).*
P_l	*Power losses in transmission (MW).*
P_x,P_y	*Vectors of generation units outputs.*
B_{xy}	*Loss coefficients*
B''	*Loss constant*
$P_{i,min}$	*Min gen level of generator i.*
$P_{i,max}$	*Max gen level of generator i.*
q	*No. of points taken on each unit's fuel cost curve.*
S_l	*Slope between two points on the fuel cost curve*
$\Delta f(c)$	*Cost increment of generation unit i at point q.*
P_{iq}	*Power level of gen unit i at point q.*
C_iP_{imin}	*Cost of min power gen of unit i*
P_{ij}	*Power generation between two points on a curve*
lb	*Lower bound of generator i*
ub	*Upper bound of generator i*

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I. INTRODUCTION

Economic dispatch orders per minute demand of load that is connected to generating plant so that the cost of generation is a minimum by considering the transmission losses and all the other essential constraints. Attaining the optimal point of generation in economic dispatch problem can result in the significant amount of fuel saving and cost reduction in power systems so a lot of work has been done in this area. As the fuel cost curve of modern generation units are highly non linear so we have to apply such algorithm that can deal with this non-linearity. Previously different mathematical methods have been used by different researchers as lambda iteration methods and Langrange Multiplier method to solve the problem [1]. In the modern era Artificial Intelligence has taken the interests of the researchers that include GA technique[2], Hopfield networks[3], Particle Swarm optimization[4], dynamic programming[5], simulated annealing[6], firefly algorithm[7], are applied on different scenarios to get the results better and more appropriate but the problem with these methods is that they behave stochastically, their computational time is more and they can generate random population that give the local minima or maxima but we need the definite optimal point of generation.

In this research paper we will show how we use the methodology to convert the quadratic and non linear behavior of the fuel cost curve into linear function and accordingly modify the objective function and constraints. In section 2 we will describe the problem formulation of ELD problem mathematically and the constraints and bounds applied on the problem. In section 3 we will describe the methodology use to transform the quadratic function into linear function and how the linear programming is applied. The algorithm is being discussed in section 4 and in section 5 there will be the results of the algorithm applied on 3 and 6 unit systems and compared with the other algorithms. The section 6 comprises of the conclusion and the future work.

II. PROBLEM FORMULATION

In this section we will formulate the problem by expressing the objective function, equality and in-equality constraints mathematically of economic load dispatch.

A. *Objective function*

The objective function of the problem is to find the optimal point of generation for each generation unit at which we can minimize the total cost of generation by meeting the necessary

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constraints and bounds. The above statement makes the problem as the constrained optimization problem that can be expressed mathematically as in [8]:

$$minF(t) = \sum_{i=1}^{N} F_i(P_i) \quad (1)$$

As the fuel cost curve is known for every generation unit so the objective function can be turned as[8];

$$minF(t) = \sum_{i=1}^{N}((a_i \times P(i)^2) + (b_i \times P(i)) + c_i) \quad (2)$$

The optimization problem has some constraints and bounds that include:

B. Power balance equality constraint:

For the concerned optimization problem power balance of the system must be fulfilled. As the total power generation in a system must be equal to the power demand added with the transmission losses in a system [9].

$$\sum_{i=1}^{N} P_i = P_r + P_l \quad (3)$$

Here P_l is the power loss in transmission lines and one of the methods to calculate the transmission power loss is B coefficients method that can further be expressed mathematically as in [9]

$$P_l = \sum_{x=1}^{N}\sum_{y=1}^{N}(P_x \times B_{xy} \times P_y) + \sum_{x=1}^{N} B_x \times P_x + B' \quad (4)$$

By putting the value of (4) in (3) we get the equality constraint as

$$\sum_{i=1}^{N} P_i = P_r + \sum_{x=1}^{N}\sum_{y=1}^{N}(P_x \times B_{xy} \times P_y) + \sum_{x=1}^{N} B_x \times P_x + B' \quad (5)$$

C. Generation limit inequality constraints

This constraint holds the inequality as the generation of each generator should remain between the upper and lower limits and mathematically it can be expressed as in [8]:

$$P_{i,min} < P_{i,gen} < P_{i,max} \quad (6)$$

III. LINEAR PROGRAMMING METHODOLOGY

As the objective function is highly non linear in nature so have to adopt such a methodology that turn the objective function into linear. For this we have taken "q" no of points on the fuel cost curve and by proceeding point to point strategy we have attained the linearity.

The fuel cost curve of generator i (i=1…N) starts from the min. point of generation limit and ends at the max point of generation limit. We will generate "q" no of points on the curve. The starting point will be P_{i1} and the points will increase as (P_{i1}, P_{i2}….P_{iq}). The distance between the two consecutive points can be calculated by two point formula given by (7)

$$S_{liq} = \frac{(f_{cq} - f_{c(q-1)})}{(lb_q - lb_{(q-1)})} \quad (7)$$

The slope between each point on the curve is represented by (S_{li1},S_{li2},S_{liq}). The increase in cost function f(C) consequent to each line segment is specified by

$$\Delta f(C) = S_{liq} P_{iq} \quad (8)$$

These points symbolize the increments in generation range from 0 to max limit of generation for each unit. The linear form of the curve after taking points will turn out to be

$$f(Pi) = starting\ point + \sum_{j=1}^{q} \Delta S_{lij} \times P_{ij} \quad (9)$$

Therefore the cost function can be estimated using the set of line segments, and that estimation may get better to any wanted level by escalating the number of line segments used. So by putting the value of (8) in (9) the approx linear cost curves are given as

$$f_i(P_{i1}, P_{i2}, \dots, P_{iq}) = \left(C_i(P_{i,min})\right) + (S_{li1} \times P_{i1}) + (S_{li2} \times P_{i2}) + \cdots + (S_{lij} \times P_{iq}) \quad (10)$$

A. Modified Objective Function

By the two point method we calculate the slope between the points and this will leave only 1 variable to be calculated as P_{iq} where (i=1 to N and q=no. of points on curve). This makes our objective function as

$$min \sum_{i=1}^{n} f(P_i) = min[\sum_{i=1}^{n} C_i(P_{imin}) + \sum_{i=1}^{n}\sum_{j=1}^{q}(S_{liq} \times P_{ij})] \quad (11)$$

B. Modified Power Balance Equality Constraint

The power balance equality constraint can be expressed by (12)

$$\sum_{i=1}^{N} P_i = P_r + P_l \quad (12)$$

It may appear that the sum of the new "q" no of variables should also equivalent the total electrical load plus losses to be supplied as a load, but this is not relatively correct. The reason why is that the "q" no of variables are turns as additions, and therefore do not comprise the each unit's minimum generation level. As a result, we have to subtract the each unit's total minimum generation level from the total load plus losses to be supplied as a load. So in terms of "q" no of new variables the equality constraint is:

$$[(P_{11} + P_{21} + \cdots + P_{i1}) + (P_{21} + P_{22} + \cdots + P_{i2}) + \cdots .. + (P_{i1} + P_{i2} + \cdots + P_{i1})] = [P_l + P_r - (P_{1,min} + P_{2,min} + \cdots + P_{i,min})]$$

C. Modified Generation Limit Inequality Constraint

The generation limit inequality constraint can be transformed as:

1) *(1) Modified lower bounds*

It is distinguished that the value of all of the new variables must be nonnegative entities. This is realizes from the fact that we are unable to have a negative amount of electrical generation increment. But the amount that we can have is the generation increment between zero and the upper bound. Consequently the lower bound on all "i×q" variables is zero.

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(2) Modified upper bounds

For the consequent linear segments the maximum possible increment on the variable is the upper bounds.

As we have taken "q" points on a curve and we set all the lower bounds as zero. So the upper bounds can be calculated as

For the "i" generation unit where i=1 to N. The cost curve has "q" no of points.

$$Ub(i_q) = P_{i,gen}(q) - P_{i,gen}(q1) \tag{13}$$

Continuing in this fashion, we obtain the upper bounds for all of the "i×q" decision variables according to the below:

$$\begin{bmatrix} 0 \\ 0 \\ . \\ . \\ 0 \\ 0 \\ 0 \\ . \\ . \\ 0 \\ . \\ . \\ 0 \\ 0 \\ . \\ . \\ 0 \end{bmatrix} \leq \begin{bmatrix} P_{11} \\ P_{12} \\ . \\ . \\ P_{1i} \\ P_{21} \\ P_{22} \\ . \\ . \\ P_{2i} \\ . \\ . \\ P_{q1} \\ P_{q2} \\ . \\ . \\ P_{qi} \end{bmatrix} \leq \begin{bmatrix} Ub_{11} \\ Ub_{12} \\ . \\ . \\ Ub_{1i} \\ Ub_{21} \\ Ub_{22} \\ . \\ . \\ Ub_{2i} \\ . \\ . \\ Ub_{q1} \\ Ub_{q2} \\ . \\ . \\ Ub_{qi} \end{bmatrix} \tag{14}$$

IV. Algorithm

step1) The program starts with reading the generation units data.

step2) The fuel cost curves are generated and read the value of "q" to generate points on the curves.

step3) Program reads the upper and lower limits of the generation units and also the transmission loss coefficient matrix to calculate the P_l.

step4) Load demand is saved in the program to distribute the load between different generation units.

step5) When the program read the load data it generates different points on the fuel cost curve of the generation units.

step6) As the points are being plotted the upper and lower bounds are changed accordingly.

step7) Now the curve is being changed into small segments of straight lines. The beginning of the line is taken as lower bound and the end of the line is taken as upper bound.

step8) After rebuilding the bounds the program begins to compare the sum of power levels of all generating units to meet the load demand by considering the upper and lower limits of genartaion units.

step9) Iteration starts and the program begin to sum up the different line segments generated on the cost curve and add the generation levels to form different combinations to meet the load demand by fulfilling the bounds applied on the upper and lower limits of each generation unit.

step10) When all the combinations are attained that fulfil the purpose of meeting load demand the program checks for the most economical combination.

At this point of the program the combination is attained that

- Meets the load demand
- Fulfil the upper and lower bounds requirements of generation units
- The most economical way of generation

V. Case Study

To apply the above algorithm we select a three and six generation unit systems. The fuel cost coefficients, lower and upper bounds of each generation units and transmission loss coefficients are used as in [7]. In step 1 the problem is being solved by linear programming methodology and in step 2 it is compared with the other algorithms.

A. Case 1

In case 1 we solve three generation unit system. The results of the economic dispatch problem solve by linear programming methodology is given in Table I and comparison with the other algorithms are given in Table II

TABLE I
OPTIMAL LOAD DISTRIBUTION BETWEEN 3 GENERATION UNIT SYSTEMS

S. No.	Load (MW)	P1 (MW)	P2 (MW)	P3 (MW)	Pl (MW)	Total cost ($/hr)
1	350	75.30	150.20	130.20	5.70	185.60
2	400	85.0	170.00	152.55	7.50	208.09
3	450	97.90	190.20	171.50	9.60	231.07
4	500	110.10	205.50	196.30	11.90	254.59
5	550	120.90	225.70	217.80	14.40	278.65
6	600	136.10	245.80	235.40	17.30	303.25

TABLE II
COMPARISON BETWEEN ALGORITHMS

S.No.	Load (MW)	Fuel Cost of Lambda iteration algorithm ($/hr)	Fuel Cost of Firefly algorithm ($/hr)	Fuel Cost of linear Programming technique. ($/hr)
1	350	185.70	185.64	185.60
2	400	208.17	208.12	208.09
3	450	231.46	231.12	231.07
4	500	254.95	254.65	254.59
5	550	278.99	278.72	278.65
6	600	303.59	303.34	303.25

The above table shows the result obtained by using the linear programming technique. The results are fulfilling the equality and in-equality constraints. Now we do comparison with the other algorithms.

B. Case 2

In case 2 we solve six generation unit system. The results of the economic dispatch problem solve by linear programming methodology is given in Table III and comparison with the other algorithms are given in Table IV.

TABLE III
OPTIMAL LOAD DISTRIBUTION BETWEEN 6 GENERATION UNIT SYSTEMS

S.No	Load (MW)	P1 (MW)	P2 (MW)	P3 (MW)	P4 (MW)	P5 (MW)	P6 (MW)	Pl (MW)	Total cost ($/hr)
1	600	22.50	10	95.5	100.70	200.50	185	14.20	320.50
2	650	26	10	105.20	110.60	210.70	204.10	16.70	344.32
3	700	28.50	10	120.90	121.70	230.70	207.60	19.40	368.85
4	750	30.40	12	130.60	125.50	250.20	223.60	22.30	393.56
5	800	35.50	15	145.40	141.30	250.30	237.80	25.30	418.45
6	850	32.5	15	150.30	145.60	270.80	264.30	28.50	444.10

The above table shows the result obtained by using the linear programming technique. The results are satisfying the equality and in-equality constraints. Now we do comparison with the other algorithms. The results of comparison are shown in Table IV

TABLE IV
COMPARISON OF FUEL COST BETWEEN ALGORITHMS

S.No.	Load (MW)	Fuel Cost of Lambda iteration algorithm ($/hr)	Fuel Cost of Firefly algorithm ($/hr)	Fuel Cost of linear Programming technique. ($/hr)
1	600	321.29	320.94	320.50
2	650	345.31	344.80	344.32
3	700	369.46	369.12	368.85
4	750	394.22	393.84	393.56
5	800	419.59	418.96	418.45
6	850	445.08	444.50	444.10

VI. CONCLUSION

We have transformed the objective function into linear and apply the linear programming that always gives us the non varying results instead of using heuristic algorithm that give unpredictable results. We have applied the algorithm on the three and six unit system and verified the results. The results can be made better by increasing the no of points constructed on the fuel cost curve to make it more linear. So we have attained a simple method to compete with the other complex methods to solve the economic dispatch problem.

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