Fuzzy Neural IOSS Filter Design for State Estimation of T-S Fuzzy Dynamic Neural Networks

Prof. Choon Ki Ahn

***Abstract*—**This paper is concerned with the input/output-to-state stable (IOSS) filtering problem for Takagi-Sugeno (T-S) fuzzy dynamic neural networks. A new set of linear matrix inequality (LMI) conditions is proposed such that the filtering error system of T-S fuzzy dynamic neural networks is input/output-to-state stable and asymptotically. The gain matrix of the proposed fuzzy neural filter can be determined by solving the feasibility problem for the proposed LMI conditions.

***Keywords*—**Input/output-to-state stability (IOSS), Takagi-Sugeno (T-S) fuzzy neural network, filtering.

I. Introduction

Over the past decade, a lot of attentions have been received on dynamic neural networks due to their practical applications in many areas such as function approximation, parallel computation, pattern recognition, and computer vision [1]. In recent years, Takagi-Sugeno (T-S) fuzzy model has provided an important method to represent complex nonlinear systems using local linear systems [2, 3]. Based on the T-S fuzzy model concept, T-S fuzzy neural networks are also considered as an efficient tool to describe complex systems or networks. Recently, Ahn investigated many features of T-S fuzzy neural networks such as stability, learning, and robust performance [4, 5, 6, 7, 8, 9, 10, 11, 12].

The input/output-to-state stability (IOSS) approach, first introduced in [13], is an indispensable method to deal with robust performance and stability analysis for many complex dynamic systems using relations between input, state, and output information. During the last decade, some results on the IOSS have been presented in the literature [13, 14, 15, 16, 17, 18]. However, to the best of our knowledge, there have been no results published on the IOSS filtering for T-S fuzzy neural networks in the literature so far.

In this paper, we consider the IOSS filtering problem for T-S fuzzy neural networks. We propose a new set of conditions such that the filtering error system of T-S fuzzy neural networks is input/output-to-state stable based on linear matrix inequality (LMI) approach. Without disturbance, the conditions can guarantee the asymptotic stability of the filtering error system of T-S fuzzy neural networks.

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II. IOSS Filter Design for T-S Fuzzy Neural Networks

Consider the following neural network:

$$\dot{x}(t) = Ax(t) + W\phi(x(t)) + J(t) + Gw(t), \tag{1}$$

$$y(t) = Cx(t) + Ew(t), \tag{2}$$

where $x(t) = [x_1(t) \ldots x_n(t)]^T \in R^n$ is the state vector, $y(t) = [y_1(t) \ldots y_m(t)]^T \in R^m$ is the output vector, $w(t) = [w_1(t) \ldots w_k(t)]^T \in R^k$ is the disturbance vector, $A = diag\{-a_1, \ldots, -a_n\} \in R^{n\times n}$ $(a_k > 0, k = 1, \ldots, n)$ is the self-feedback matrix, $W \in R^{n\times n}$ is the delayed connection weight matrix, $\phi(x(t)) = [\phi_1(x(t)) \ldots \phi_n(x(t))]^T : R^n \to R^n$ is the nonlinear function vector satisfying the global Lipschitz condition with Lipschitz constant $L_\phi > 0$, $G \in R^{n\times k}$, $C \in R^{m\times n}$, $E \in R^{m\times k}$ are known constant matrices, and $J(t) \in R^n$ is an external input vector. In this paper, we consider the following T-S fuzzy Hopfield neural networks:

Fuzzy Rule i:

IF ω_1 *is* μ_{i1} *and* $\ldots$ ω_s *is* μ_{is} *THEN*

$$\dot{x}(t) = A_i x(t) + W_i\phi(x(t)) + J_i(t) + G_i w(t), \tag{3}$$

$$y(t) = C_i x(t) + E_i w(t), \tag{4}$$

where ω_j $(j = 1, \ldots, s)$ is the premise variable, μ_{ij} $(i = 1, \ldots, r, j = 1, \ldots, s)$ is the fuzzy set that is characterized by a membership function, r is the number of the IF-THEN rules, and s is the number of the premise variables. Using a fuzzy inference method, the system (3)-(4) is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\omega)[A_i x(t) + W_i\phi(x(t)) + J_i(t) + G_i w(t)], \tag{5}$$

$$y(t) = \sum_{i=1}^{r} h_i(\omega)[C_i x(t) + E_i w(t)], \tag{6}$$

where $\omega = [\omega_1, \ldots, \omega_s]$, $h_i(\omega) = w_i(\omega) / \sum_{i=1}^{r} w_j(\omega)$, $w_i : R^s \to [0,1]$ $(i = 1, \ldots, r)$ is the membership function of the system with respect to the fuzzy rule i. h_i is the normalized weight of each IF-THEN rule and it satisfies

$$h_i(\omega) \geq 0, \qquad \sum_{i=1}^{r} h_i(\omega) = 1. \tag{7}$$

We propose the following fuzzy neural filter:

Fuzzy Rule i:

IF ω_1 *is* μ_{i1} *and* $\ldots\omega_s$ *is* μ_{is} *THEN*

$$\dot{\hat{x}}(t) = A_i\hat{x}(t) + W_i\phi(\hat{x}(t)) + J_i(t) + L(y(t) - \hat{y}(t)), \quad (8)$$

$$\hat{y}(t) = C_i\hat{x}(t), \quad (9)$$

where $\hat{x}(t) = [\hat{x}_1(t) \ldots \hat{x}_n(t)]^T \in R^n$ is the state vector of the filter, $\hat{y}(t) = [\hat{y}_1(t) \ldots \hat{y}_m(t)]^T \in R^m$ is the output vector of the filter, and $L \in R^{n\times m}$ is the gain matrix of the filter. Using a fuzzy inference method, the fuzzy neural filter (8)-(9) is inferred by

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(\omega)[A_i\hat{x}(t) + W_i\phi(\hat{x}(t)) + J_i(t) + L(y(t) - \hat{y}(t))], \quad (10)$$

$$\hat{y}(t) = \sum_{i=1}^{r} h_i(\omega)C_i\hat{x}(t). \quad (11)$$

Define the filtering error $e(t) = x(t) - \hat{x}(t)$. Then, the filtering error system is given by

$$\dot{e}(t) = \sum_{i=1}^{r} h_i(\omega)\{(A_i - LC_i)e(t) + W_i\tilde{\phi}(x(t)) + (G_i - LE_i)w(t)\},$$

(12)

$$y(t) - \hat{y}(t) = \sum_{i=1}^{r} h_i(\omega)[C_ie(t) + E_iw(t)], \quad (13)$$

where $\tilde{\phi}(x(t)) = \phi(x(t)) - \phi(\hat{x}(t))$. Now, we introduce the following definitions:

Definition 1 *[13] A function* $\gamma : R_{\geq 0} \to R_{\geq 0}$ *is a* K *function if it is continuous, strictly increasing and* $\gamma(0) = 0$. *It is a* K_∞ *function if it is a* K *function and* $\gamma(s) \to \infty$ *as* $s \to \infty$. *A function* $\beta : R_{\geq 0} \times R_{\geq 0} \to R_{\geq 0}$ *is a* KL *function if, for each fixed* $t \geq 0$, *the function* $\beta(\cdot,t)$ *is a* K *function, and for each fixed* $s \geq 0$, *the function* $\beta(s,\cdot)$ *is decreasing and* $\beta(s,t) \to 0$ *as* $t \to \infty$.

In this paper, we design a fuzzy neural filter (8)-(9) such that the filtering error system (12)-(13) satisfies

$$\|e(t)\| \leq \max\{\beta(\|e(0)\|,t), \gamma_1(\sup_{0\leq\tau\leq t}\|w(\tau)\|),$$
$$\gamma_2(\sup_{0\leq\tau\leq t}\|y(\tau) - \hat{y}(\tau)\|)\}, \quad (14)$$

where $\gamma_i(s)$ $(i = 1,2)$ is a K function and $\beta(s,t)$ is a KL function.

In the following theorem, we obtain the gain matrix of the fuzzy neural filter (8)-(9).

Theorem 1 *Assume that there exist matrices* $P = P^T > 0$, $S_1 = S_1^T > 0$, $S_2 = S_2^T > 0$, $S_3 = S_3^T > 0$, *and* M *such that*

$$\begin{bmatrix} (1,1)_i & (1,2)_i & PW_i \\ (1,2)_i^T & -S_2 - E_i^TS_3E_i & 0 \\ W_i^TP & 0 & -I \end{bmatrix} < 0, \quad (15)$$

where $(1,1)_i = (PA_i - MC_i)^T + PA_i - MC_i + L_\phi^2I + S_1 - C_i^TS_3C_i$ and $(1,2)_i = PG_i - ME_i - C_i^TS_3E_i$ for $i = 1,2,\ldots,r$. Then the filtering error system (12)-(13) is input/output-to-state stable and the gain matrix of the fuzzy neural filter (8)-(9) is given by $L = P^{-1}M$.

Next, we investigate the asymptotic stability of the filtering error system (12)-(13).

Corollary 1 *Assume that there exist matrices* $P = P^T > 0$, $S_1 = S_1^T > 0$, $S_2 = S_2^T > 0$, $S_3 = S_3^T > 0$, *and* M *such that*

$$\begin{bmatrix} (1,1)_i & (1,2)_i & PW_i \\ (1,2)_i^T & -S_2 - E_i^TS_3E_i & 0 \\ W_i^TP & 0 & -I \end{bmatrix} < 0, \quad (16)$$

$$S_1 - C_i^TS_2C_i > 0 \quad (17)$$

for $i = 1,2,\ldots,r$. Then, the filtering error system (12)-(13) is asymptotically stable with $w(t) = 0$.

III. Conclusion

This paper has considered the IOSS filtering problem for T-S fuzzy neural networks. A new set of LMI based conditions was proposed such that the filtering error system of T-S fuzzy neural networks is input/output-to-state stable. In addition, we can guarantee the asymptotic stability of the filtering error system of T-S fuzzy neural networks without disturbance. The gain matrix of the proposed fuzzy neural filter can be obtained by solving the LMIs.

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