Natural Convection of Nanofluids in a Square Cavity in the Presence of Horizontal Magnetic Field

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***Abstract*—**Convective flow and heat transfer characteristics of nanofluid in a two dimensional square cavity is investigated in the presence of uniform horizontal magnetic field. The vertical walls of the cavity are isothermal whereas the horizontal walls are adiabatic. The finite volume method is used to discretize the governing equations and solved by iterative technique. Two different models are considered for calculating the effective viscosity of the nanofluid. Numerical simulations are carried out for various parameters involving in the problem and the results are presented graphically. It is found that convection heat transfer is affected significantly in presence of horizontal magnetic field. A small change is observed in temperature field between two models when increasing the volume fraction and Hartmann number. When increasing the volume fraction of the nanofluid, the heat transfer rate is increased monotonically for model I and decreased for model II for weak magnetic field, whereas heat transfer rate of both models increased for moderate and strong magnetic field.

***Keywords*—**Natural convection, MHD, Nanofluids, Cavity, Viscosity.

I. Introduction

CONVECTIVE heat transfer cooling enhancement technique has been major problem in engineering and technological applications. In past, several researchers in various aspects to get better heat transfer medium have conducted research on convective heat transfer technique. Recently, fluids with nano-meter sized particles suspended into them called nanofluids, is considered in research. There are several models used for calculation of physical properties of nanofluid by several researchers [1-3]. Considerable amount of experimental and numerical studies has been conducted the effect of improvement of the thermal conductivity of nanofluids, because most of the researchers is thinking that thermal conductivity plays a dominant role on convective transport. However, less number of studies devoted on the effect of viscosity of the nanofluids. Since the viscosity of the fluid is increased when nano-particles suspended into them, we must consider the effect of viscosity on convective flow and heat transfer.

Recently, Polidori et al. [4] have investigated the problem of natural convection flow and heat transfer of alumina-water nano-particles over a vertical semi-infinite plate under uniform wall temperature and heat flux. They concluded that not only effective thermal conductivity but also the viscosity of the nanofluid characterizes natural convection heat transfer. Prasher et al. [5] reported the experimental results on the viscosity of alumina based nanofluids. Their data showed that increase in the viscosity is higher than the enhancement in the thermal conductivity as reported in literature. Nguyen et al. [6] experimentally investigated the influence of both temperature and particle size on the dynamic viscosity of two nanofluids. They found that the dynamic viscosity of nanofluid increases with an increase of particle volume fraction but decreases on increasing temperature. Lee et al. [7] experimentally investigated the viscosities and thermal conductivities of water-based nanofluids with low concentration of nano-particles. They found that the measured viscosity of nanofluids behaves nonlinearly with concentration of nano-particles.

Convection heat transfer of nanofluids in enclosure is analyzed by some researchers with considering different models of nanofluid properties. Khanafer et al. [8] numerically investigated buoyancy driven convection in a two dimensional enclosure using nanofluids. They found that heat transfer rate increases with an increase in the nano-particle volume fraction. They also analyzed a comparative study of different models based on the physical properties of nanofluids. Jou and Tzeng [9] performed a numerical study for the effect of small aspect ratio of enclosures on natural convection of nanofluids in a rectangular enclosure. They found that the average Nusselt number is increased as the aspect ratio decreases. Hwang et al. [10] theoretically studied the thermal characteristics of natural convection of water based Al_2O_3 nanofluids in a rectangular cavity. They used two different models to calculate the effective viscosity of nanofluid and compare the results obtained from these models. They showed that the ratio of heat transfer coefficient of nanofluid to that of base fluid is decreased as the size of nano-particle increases.

Ho et al. [11] numerically studied the effects of uncertinities in the effective viscosity and thermal conductivity of nanofluids on convection in enclosure. They concluded that

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the effective dynamic viscosity should be taken into account when studying the heat transfer efficacy for natural convection in enclosures. Sivasankaran et al. [12] numerically studied the natural convection of nanofluids in a cavity with linearly varying wall temperature. They found that the increment in average Nusselt number is strongly dependent on the nanoparticle chosen. Ho et al. [13] experimentally investigated the natural convection of Al2O3-water nanofluid in enclosures of three different sizes. The effect of sinusoidal temperature distribution on natural convection of nanofluid in a cavity is examined numerically by Sivasankaran and Pan [14]. They found that the heat transfer is enhanced by non-uniform heating of both walls as compared to the case of uniform heating on one wall.

Convection heat transfer in the presence of a uniform magnetic field has been studied extensively due to wide range of applications include process of manufacturing materials and cooling in a nuclear reactor. Sivasankaran and Ho [15] numerically investigated the effects of variable fluid properties on MHD convection of cold water in a square cavity. They found that the average Nusselt number decreases with an increase of the Hartmann number. Sivasankaran et al. [16] numerically examined the MHD convection of cold water near its density maximum in an open cavity with variable fluid properties. They observed that convection heat transfer is enhanced by thermo-capillary force when buoyancy force is weakened. Sivasankaran et al. [17] numerically investigated MHD mixed convection in a square cavity with sinusoidal boundary temperatures at the sidewalls. The flow field and heat transfer inside the cavity are strongly affected in the presence of the magnetic field. Sivasankaran and M. Bhuvaneswari [18] numerically studied the effect of thermally active zones and direction of the external magnetic field on hydro-magnetic convection in an enclosure. The flow field is altered when changing the direction of the magnetic field in the presence of strong magnetic field. Malleswaran et al. [19] investigated the effects of various lengths and different locations of the heater on magneto convection in a lid-driven cavity. It is found that existence of the magnetic field suppresses the convective heat transfer and the fluid flow.

Convective heat transfer analysis of nanofluid in the presence of magnetic field has been attracted a wide range of applications, such as preparation of magnetic fluid, data storage and biomedical science. Ghasemi et al. [20] numerically examined the natural convection of water-Al_2O_3 nanofluid in an enclosure under the influence by a magnetic field. They found that the effect of the solid volume fraction on heat transfer rate strongly depends on the values of the Rayleigh number and the Hartmann number. Teamah and El-Maghlany [21] numerically studied the natural convection in a square cavity filled with different nanofluids in the presence of magnetic field. They found that the addition of nanoparticles is necessary to enhance the heat transfer for weak magnetic field applications, but for strong magnetic field applications there is no need for nanoparticles because the heat transfer will decrease. Mahmoudi and Abu-Nada [22] numerically examined the natural convection of CuO-water nanofluid in the presence of a magnetic field.

The present study aims to investigate the effect of viscosity on MHD natural convection characteristics of Al_2O_3-water nanofluids in a square cavity.

II. Mathematical Analysis

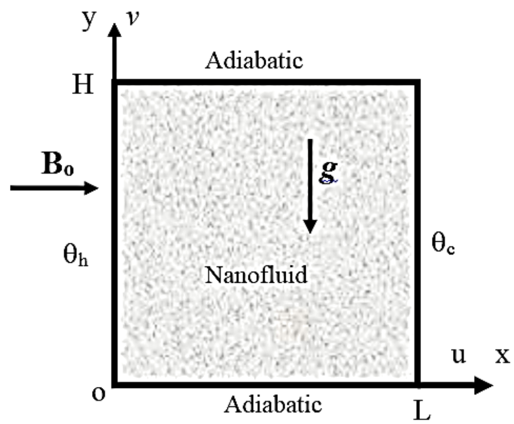


Fig. 1 Physical configuration

Consider a two-dimensional square cavity of size L filled with nanofluids as shown in Figure 1. The cavity is heated differentially between the two isothermal vertical sidewalls at different temperature of θ_h and θ_c respectively, with $\theta_h > \theta_c$. The horizontal walls are assumed to be adiabatic, non-conducting and impermeable to mass transfer. The nanofluid in the enclosure is a solid-liquid mixture with uniform volume fraction ϕ, shape and size of nano-particles (Al_2O_3) dispersed within a base fluid (water) and it is also Newtonian. The flow is assumed to be incompressible and laminar. It is assumed that both nano-particles and base fluid are in thermally equilibrium. The Boussinesq approximation is valid in the buoyancy term and all other thermo-physical properties are assumed to be constant. A uniform magnetic field of constant magnitude B_o is applied in the horizontal direction. The electric current J and the electromagnetic force F are defined by $J = \sigma_e (V \times B)$ and $F = \sigma_e (V \times B) \times B$, respectively. The induced magnetic field due to the motion of the electrically conducting fluid is very small compared to the applied magnetic field. Therefore the magnetic Reynolds number is too small and it is neglected. Moreover, it is assumed that the viscous dissipation, radiation and Joule heating are assumed to be negligible.

The resulting governing equations in non-dimensional form are as follows.

$$\frac{\partial \Omega}{\partial \tau} + \frac{\partial U\Omega}{\partial X} + \frac{\partial V\Omega}{\partial Y} = \left(\frac{Pr_f\, \mu^*_{nf} Cp^*_{,nf}}{k^*_{nf}}\right)\left[\frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2}\right] + \left\{Ra_f\, Pr_f\, \beta^*_{nf} \left(\frac{\rho^*_{nf}\, Cp^*_{,nf}}{k^*_{nf}}\right)^2\right\} \frac{\partial T}{\partial X} + \left(Ha_f^2\, Pr_f\, \frac{Cp^*_{,nf}}{k^*_{nf}}\right)\frac{\partial V}{\partial Y} \tag{1}$$

$$\frac{\partial^2\Psi}{\partial X^2}+\frac{\partial^2\Psi}{\partial Y^2}=-\Omega \tag{2}$$

$$\frac{\partial T}{\partial \tau}+\frac{\partial UT}{\partial X}+\frac{\partial VT}{\partial Y}=\frac{\partial}{\partial X}\left(k_{en}^{*}\frac{\partial T}{\partial X}\right)+\frac{\partial}{\partial Y}\left(k_{en}^{*}\frac{\partial T}{\partial Y}\right) \tag{3}$$

The ratio of physical properties presents in equations (1) and (3) are as follows. $k_{nf}^{*}=\frac{k_n}{k_f}$, $k_{en}^{*}=\frac{k_e}{k_n}$, $\mu_{nf}^{*}=\frac{\mu_n}{\mu_f}$, $c_{p,nf}^{*}=\frac{c_{p,n}}{c_{p,f}}$, $\rho_{nf}^{*}=\frac{\rho_n}{\rho_f}$ and $\beta_{nf}^{*}=\frac{\beta_n}{\beta_f}$, where the subscripts n and f denote, respectively, the nanofluid and the base fluid. Moreover, k_e denotes the effective thermal conductivity associated with possible heat transfer enhancement mechanisms of the nanofluid such as Brownian motion, liquid layering at liquid/particle interface, and phonon movement in nano-particles. The formulae selected for the thermo-physical properties of the nanofluid in the present model are as follows.

Density: $$\rho_n=(1-\phi)\,\rho_f+\phi\,\rho_p \tag{4}$$

Thermal expansion coefficient:
$$\beta_n=\frac{1}{\rho_n}(1-\phi)\,\rho_f\beta_f+\phi\,\rho_p\beta_p \tag{5}$$

Specific heat: $$c_{p,n}=(1-\phi)\,c_{p,f}+\phi\,c_{p,p} \tag{6}$$

Thermal conductivity: Thermal conductivity is evaluated from the well-known Maxwell formula as
$$k_n=k_f\left[\frac{2+k_{pf}^{*}+2\phi(k_{pf}^{*}-1)}{2+k_{pf}^{*}-\phi(k_{pf}^{*}-1)}\right] \tag{7}$$
with $k_{pf}^{*}=\frac{k_p}{k_f}$

Dynamic viscosity: For the effective dynamic viscosity of nanofluid, Brinkman's formula [23] and an empirical correlation obtained by Maiga et al. [24] are considered here. They are expressed as
$$\mu_n=\mu_f(1-\phi)^{-2.5} \text{ (Model I)} \tag{8}$$
and
$$\mu_n=\mu_f\,(1+7.3\,\phi+123\,\phi^2) \text{ (Model I)} \tag{9}$$

The above two equations (8-9) are referred as Model I and Model II in this study. The thermo-physical properties of the base fluid (water) and nano-particles (Al_2O_3) are available in literature [11].

The velocity components are defined in terms of stream function $U=\frac{\partial\Psi}{\partial Y}$ and $V=-\frac{\partial\Psi}{\partial X}$. No slip boundary conditions are applied on all four walls of the cavity. The initial and boundary conditions on these equations are specified in the dimensionless form are:

$\tau = 0$; $\Psi = \Omega = T = 0$, at $0 \le X \le 1$, $0 \le Y \le 1$

$\tau > 0$; $\Psi = \frac{\partial T}{\partial X} = 0$, at $Y = 0$ & 1,

$\Psi = 0$, $T = 1$, at $X = 0$,

$\Psi = 0$, $T = 0$, at $X = 1$.

The following non-dimensional variables are used $(X, Y) = (x, y)/L$, $(U, V) = (u, v)L/\alpha_n$, $\tau = t\alpha_n/L^2$, $\Psi = \psi/\alpha_n$, $\Omega = \omega/\alpha_n L^2$, $T = (\theta - \theta_c)/(\theta_h - \theta_c)$. The heat transfer rate at the hot wall of the enclosure is presented by means of the Nusselt number, which is evaluated as follows $Nu_h=\frac{h_n L}{k_f}=-k_{nf}^{*}\left.\frac{\partial T}{\partial Y}\right|_{X=0}$ resulting in the average Nusselt number as $\overline{Nu}=\int_0^1 Nu_h\,dY.$ (10)

III. Method of Solution

The non-dimensional equations subject to the boundary conditions are solved by control volume method. The QUICK scheme is used for the convection terms and central difference scheme is used for diffusion terms. The uniform grid has been selected in both X and Y directions. It is observed from the grid independence test that an 81×81 uniform grid is enough to investigate the problem. The resulting set of algebraic equations for energy, vorticity and stream function are solved by iterative method. The convergence criterion used for the field variables $\phi(=T,\Omega,\Psi)$ is $\left|\frac{\phi_{i,j}^{n+1}-\phi_{i,j}^{n}}{\phi_{i,j}^{n+1}}\right|\le 10^{-6}$.

IV. Results and Discussion

Numerical study is performed to understand the natural convection of nanofluid with two viscosity models in a square cavity in the presence of uniform magnetic field for different volume fraction of nano-particles. The value of Prandtl number is taken to be 6.7. Computations are carried out for Rayleigh number ranging from 10^3 to 10^6, Hartmann number ranging from 0 to 200 and the volume fraction ϕ of nano-particles from 0% - 8%.

Figure 2 shows the isotherms of nanofluids for different volume fractions of Alumina nanoparticles and Hartmann number. There is temperature stratification in the vertical direction and a feeble thermal boundary layer is established along the bottom of the hot wall and top of the cold wall. There is a noticeable change observed in temperature field between two models when increasing volume fraction of nanofluid. The difference in the temperature gradient near the adiabatic walls of the cavity between two models is increased with increasing the volume fraction ϕ of the nanofluid. When increasing magnetic field strength the vertical temperature stratification is affected and thermal boundary layer along the isothermal walls is disappeared. In the presence of strong magnetic field, the isotherms illustrate the conduction type of heat transfer.

ϕ = 0.0 (pure fluid) ϕ = 0.04 (nanofluid)

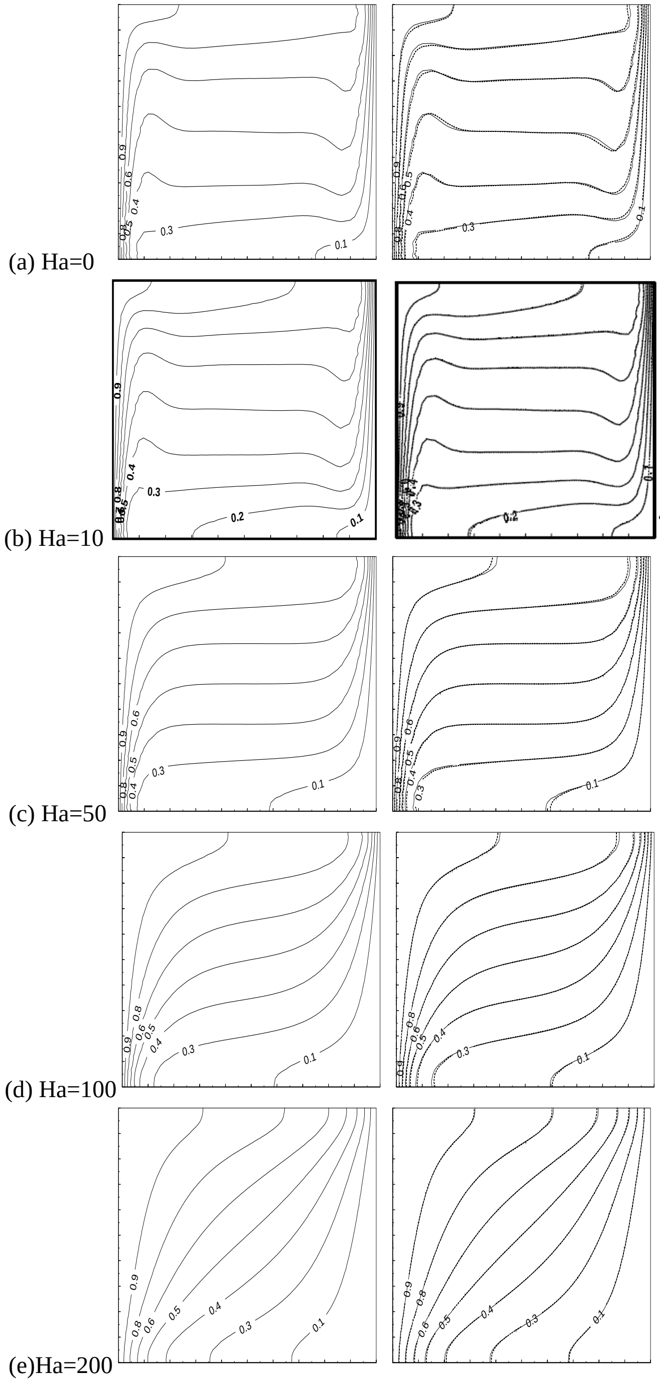


Fig. 2 Isotherms for different volume fraction & Hartmann numbers of model I (solid lines) and model II (dashed lined) with Ra = 10^6.

The streamlines of nanofluids for different volume fractions of nanoparticles and Hartmann number is displayed in Figure 3. It is observed that there is an upward flow near the heated sidewall and downward flow along the cold wall. As the concentration of volume fraction increases, the absolute value of stream function is increased, that is, the velocity increases for model I. It results the high-energy transport through the flow across the cavity. However, the opposite observation is found for model II. That is, circulation rate of the eddy is decreased when increasing volume fraction of nano-particle. Since the viscosity of model II is higher than Model I, it results the enhancement on convection heat transfer for model I. The rate of circulation of the eddy for both models decreases on increasing the magnetic field strength. This is due to the effect of magnetic field on velocity of nanofluid. The eddy is elongated in horizontal direction for weak magnetic field strength. The flow pattern is affected when increasing magnetic field strength. The eddy is elongated from horizontal to vertical direction when increasing the Hartmann number.

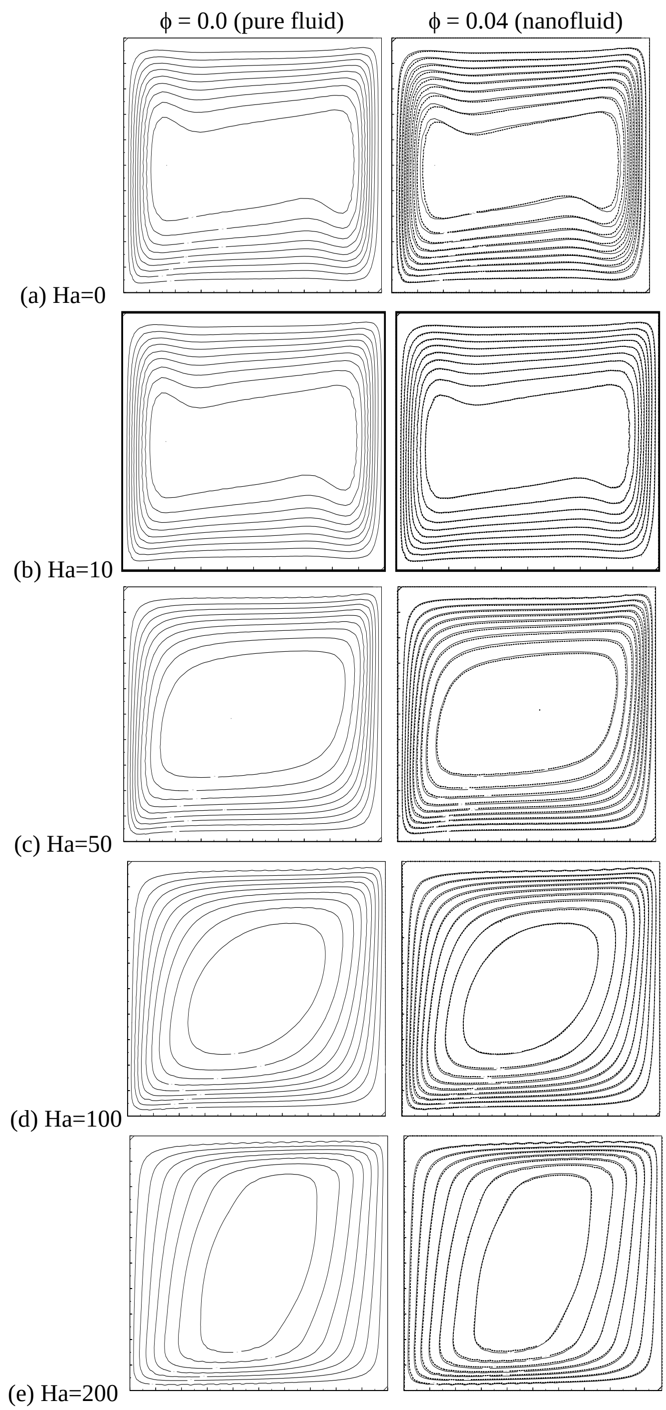


Fig. 3 Streamlines for different volume fraction & Hartmann numbers of model I (solid lines) and model II (dashed lined) with Ra = 10^6.

Figure 4 shows the effect of Hartmann number on local Nusselt number for model I and II with ϕ = 4% and Ra=10^6.

When increasing magnetic field strength the heat transfer is decreased. Rate of heat transfer at the top of the hot wall is very high for small Hartmann number. It reveals that heat transfer is by convection for weak magnetic field whereas heat transfer is by conduction for strong magnetic field strength. In order to find the effect of heat transfer rate on volume fraction of suspended nano-particles for different magnetic field strength, the average Nusselt number is plotted against volume fraction of nano-particles for both models and Hartmann number in Figure 5(a-b). Convective heat transfer rate of model I is increased while increasing the volume fraction of nano-particles for all values of Hartmann number. However, convective heat transfer rate of model II is decreased on increasing the volume fraction of nano-particles for low values of Hartmann numbers and increased while increasing the volume fraction of nano-particles for high values of Hartmann numbers for a given Rayleigh number. Though the viscosity is very high for model II, the magnetic field suppresses the enhancement of viscosity and improves the heat transfer rate across the enclosure by conduction mode. It is seen from the Figure 5 that the deviation on average Nusselt number between two models is increased with increasing the volume fraction of nano-particles for a given Hartmann number. Nanofluid also behaves like pure fluid in the presence of magnetic field that heat transfer rate is decreased with increasing the Hartmann number.

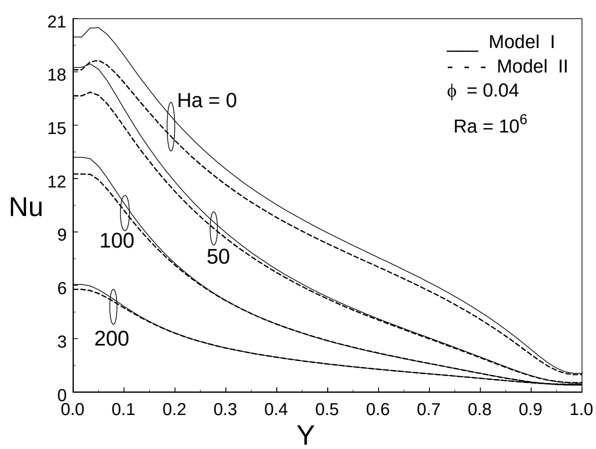
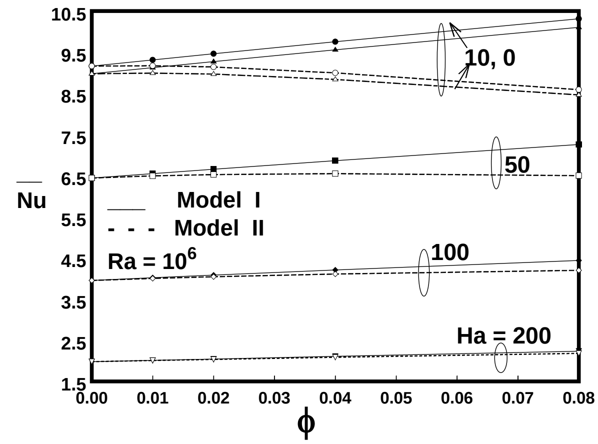
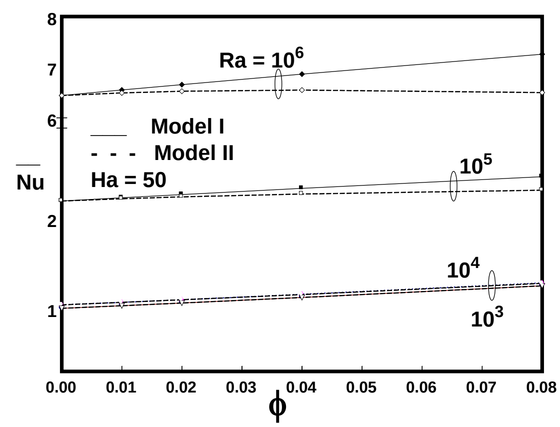


Fig. 4 Local Nusselt number for different Hartmann numbers with Ra=10^6 & ϕ=0.04.

Average Nusselt number of nanofluid for model I of ϕ=1%, 4% and 8% increases about 1.66%, 6.48% and 12.55% respectively, with that of base fluid for hydrodynamic convection case (Ha=0). Average Nusselt number for nanofluid for model I of ϕ=1%, 4 % and 8% increases about 1.68%, 6.54% and 12.63% respectively, with that of base fluid for Ha=50. Average Nusselt number for nanofluid for model I of ϕ=1%, 4% and 8% increases about 1.68%, 6.59% and 12.86% respectively, with that of base fluid for Ha=200. Average Nusselt number of nanofluid for model II of ϕ = 1%, 4% and 8% increases about 0.12% and decreases about 1.79% and 6.23% respectively, with that of base fluid for pure hydrodynamic convection case. Average Nusselt number for nanofluid for model II of ϕ = 1%, 4% and 8% increases about 0.8%, 1.64% and 0.89% respectively, with that of base fluid for Ha = 50. Average Nusselt number for nanofluid for model II of ϕ = 1%, 4% and 8% increases about 1.47%, 5.47% and 10.29% respectively, with that of base fluid for Ha=200.



(a)



(b)

Fig. 5 Average Nusselt number versus volume fraction for different Hartmann numbers and Rayleigh numbers.

It is also observed from these Figures that convection is characterized not only by effective thermal conductivity also by viscosity and external forces. Hence, viscosity of the nanofluid produces an essential effect on convection heat transfer. From the present results, it is concluded that the thermal conductivity not only the significant factor but the viscosity also an important factor on convection heat transfer phenomena.

V. Conclusion

The effect of effective viscosity of nanofluid on MHD natural convection in an enclosure is studied numerically. The following are concluded in the study. It is observed that the boundary layer is disappeared for strong magnetic field for all values of ϕ. The rate of circulation of the eddy decreases when increasing the Hartmann number. When increasing the volume fraction of the nanofluid, the heat transfer rate is increased monotonically for model I for all values of Hartmann number. Heat transfer rate decreases for low values of Hartmann numbers and increases for high values of Hartmann number with increasing volume fraction of nanofluid for model II. An enhancement on heat transfer rate across the enclosure for high viscosity fluid by using magnetic field is observed.

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