Performance Analysis of DS-CDMA Systems on Nakagami Fading Channels with Impulsive Noise

Srinivasa Rao Vempati, Soma Umamaheshwar, Tula Santhosh Kumar, Kolloju Spandana, Habibulla Khan, and Anil Kumar Tipparti

Abstract—Performance analysis of maximal-ratio combining receive diversity scheme over Nakagami-0.5 fading channels and its application to the multiuser detection for direct sequence-code division multiple access systems is presented in this paper. A new M-estimator, which performs well in the heavy-tailed impulsive noise, is proposed to robustify the detector. Further, the performance of Eigen mode transmission over a multiple-input–multiple-output Nakagami channel under the channel inversion power allocation scheme is also investigated. The extension to Rician and Rayleigh fading scenarios is also outlined. Simulation results are provided to show the efficacy of the proposed M-estimator, under severe fading conditions of Nakagami-m channel.

Keywords—Code-Division Multiple-Access (CDMA), Channel inversion (CI), Maximal Ratio Combiner, Multiple-Input Multiple-Output (MIMO), M-Estimator; Multiuser Detection; Nakagami-m Fading.

I. INTRODUCTION

Over the past decade, Nakagami-m distribution received considerable attention as it better fits to measured data in different fading environments best fits to indoor mobile multipath propagation as well as ionospheric radio links [1]. Beaulieu and Rabiei considered Nakagami-0.5 distribution as a special case of the Nakagami-m distribution with m = 0.5 and proved it as a limiting worst-case with severe fading and also as the one-sided Gaussian case [2].

Performance analysis of a wireless communication system in Nakagami-0.5 fading channels becomes more crucial when a high level of quality of service (QoS) is required [3]. Multiuser detection (MUD) for direct-sequence code-division multiple-access (DS-CDMA) systems over Nakagami-m channels has been thoroughly investigated in [4] and references therein.

M-estimator [5-6] based Robust MUD for synchronous DS-CDMA systems with MRC receive-diversity over a single-path Nakagami-m fading channel was presented in [7], recently.

Hence, this paper presents the performance analysis for DS-CDMA systems over Nakagami fading channels in the impulsive noise environment including MRC receive diversity. Channel ambient noise is assumed to be a sequence of independent and identically distributed (iid) random variables with a non-Gaussian distribution and the probability density function (PDF) of this noise model is assumed as Middleton Class A model [10] (the commonly used two-term Gaussian mixture model for the additive noise samples).

Further, multi-input multi-output (MIMO) channel can be reduced to a set of non-interfering spatial channels (virtual channels) with appropriate transmitter and receiver signal-processing techniques [8]. The ability to independently code, modulate, and power allocate for these virtual channels facilitates the use of single antenna signal processing techniques in MIMO [8]. Application of the channel inversion (CI) power allocation scheme [9] across the virtual channels realized through eigen-mode transmission is also studied in this paper. Finally, simulation results are provided to prove the effectiveness of the proposed method in non-Gaussian fading channels.

II. NAKAGAMI-0.5 FADING CHANNEL

A wireless communication system with D-branch antenna diversity operating at signal-to-noise ratio (SNR) per bit γ in each branch is considered. Assuming that the received envelope X is Nakagami-0.5 distributed, the PDF and the cumulative distribution function (CDF) of X are given by [11]

\[ f_X(x) = \frac{2}{\sqrt{2\pi\Omega}} \exp\left(-\frac{x^2}{2\Omega}\right), \quad x \geq 0 \]  

and

\[ F_X(x) = \text{erf}\left(\frac{x}{\sqrt{2\Omega}}\right), \quad x \geq 0 \]

respectively, where the total average multipath received power, Ω = E[X^2] for a single channel. In MRC, the output of the
combiner is a weighted sum of all diversity branch signals. Hence, the combiner output is given by

\[ X = \sum_{d=1}^{D} X_d^2 \]  \hspace{1cm} (3)

The PDF of the envelope \( X \) in MRC is given by [2]

\[ f_{X_{\text{MRC}}} (x) = \frac{2}{\Gamma \left( \frac{D}{2} \right)} \frac{1}{x^{D-1}} \exp \left( - \frac{x^2}{2\Omega} \right), \quad x \geq 0 \]  \hspace{1cm} (4)

where \( \Gamma(\cdot) \) is the gamma function [11].

III. SYSTEM MODEL

The received signal during \( i \)th symbol interval of DS-CDMA system is given by [7]

\[ r(t) = \sum_{j=0}^{\infty} \sum_{l=1}^{L} \sqrt{\frac{2E_b}{T}} \alpha_i e^{j\theta_i(l)} b_l(t) s_l(t) + n(t) \]  \hspace{1cm} (5)

where \( \alpha_i(l) \) is the fading gain of the \( l \)th user’s channel during the \( i \)th symbol interval, \( b_l(i) \) is the \( i \)th bit of the \( l \)th user, \( s_l(t) \), \( \int_{0}^{T} s_l^2(t) dt = 1 \), is the spreading waveform of the \( l \)th user and \( n(t) \) is assumed as a zero-mean complex two-term Gaussian mixture noise. The PDF of this noise model has the form [10]

\[ f_{\epsilon}(\epsilon) = (1 - \epsilon) N(0; \nu^2) \ast N(0, \nu^2) \]  \hspace{1cm} (6)

with \( \nu > 0 \), \( 0 \leq \epsilon \leq 1 \), and \( \kappa \geq 1 \). \( N(0, \nu^2) \) represents the nominal background noise, \( N(0, \kappa \nu^2) \) represents impulsive component and \( \epsilon \) represents the probability that impulses occur.

The received signal \( r(t) \) is passed through a matched filter bank and its output at the \( j \)th sampling instant can be represented as a column vector of length \( L \) as

\[ r[j] = R[b][j] + [n][j] \]  \hspace{1cm} (7)

Where \( R \) is the signature cross-correlation matrix with elements \( \rho_{lm} = \int_0^T s_l(t)s_m(t) dt \), \( l, m = 1, 2, \ldots, L \), with unity diagonal elements, \( b \) is the data vector with components \( b_l \), and the vector \( n \) contains the corresponding samples of the non-Gaussian noise process. The channel matrix \( W[b] \) is the diagonal matrix with diagonal elements \( W_{ll} = \sqrt{E_b} C_l(i) \) with \( C_l(i) = \alpha_l(i) e^{-j\theta_l(i)} \). It is also assumed that \( \alpha_l(i) \) are independent and identically distributed (iid) Nakagami-0.5 random variables with PDF given by (4).

IV. M-ESTIMATOR

An M-estimator estimates unknown parameters \( \theta_1, \theta_2, \ldots, \theta_L \) (where \( \theta = W[b] \)) by minimizing a sum of \( \rho(\cdot) \) function of the residuals [12]

\[ \hat{\theta} = \arg \min_{\theta \in \mathbb{R}^L} \sum_{j=1}^{N} \rho \left( r_j - \sum_{l=1}^{L} s_l^j \theta_l \right) \]  \hspace{1cm} (8)

where \( \rho \) is a symmetric, positive-definite function with a unique minimum at zero, and is chosen to be less increasing than square and \( N \) is the processing gain. Suppose that \( \rho \) has a derivative with respect to the unknown parameters \( \theta \) \( (\psi\rho=\psi') \), called the influence function, since it describes the influence of measurement errors on solutions. The solution to Eq. (8) satisfies the implicit equation [12]

\[ \sum_{j=1}^{N} \psi \left( r_j - \sum_{l=1}^{L} s_l^j \theta_l \right) b_k = 0, \quad k = 1, \ldots, L \]  \hspace{1cm} (9)

The influence function (Fig. 1) of proposed M-estimator is given by [12]

\[ \psi_{\text{PRO}}(x) = \begin{cases} x & \text{for } |x| \leq a \\ \text{sgn}(x) & \text{for } a < |x| \leq b \\ \frac{a}{b^3} \exp \left( 1 - \frac{b^2}{|x|^2} \right) & \text{for } |x| > b \end{cases} \]  \hspace{1cm} (10)

Where the constants \( a = \kappa \nu^2 \) and \( b = 2\kappa \nu^2 \).
V. ASYMPTOTIC PERFORMANCE OF M-ESTIMATOR

Asymptotic probability of error for the class of decorrelating detectors described by (8) for large processing gain \( N \), is given by [9]

\[
P_{e}^{l} = \Pr(\hat{d}_{j} < 0 | \theta_{j} > 0) = Q \left( \frac{W_{j}}{\sqrt{N} R^{-1}_{ll}} \right) \tag{11}
\]

Where \( Q(x) \) is Gaussian \( Q \) - function and

\[
V^{2} = \left[ \frac{\nu^{2}(u)f(u)du}{[\nu^{2}(u)f(u)du]} \right] .
\]

\( W_{j} \) is a Nakagami random variable.

The average probability of error for decorrelating detector over single path Nakagami-0.5 fading channel can be obtained over the PDF (4) as [7]

\[
P_{e}^{l} = \left( \frac{D}{2} \right)^{\frac{1}{2}} \sum_{j=0}^{D-1} \left( \frac{D - 1}{2} \right)^{j} \left( \frac{D - 1 - j}{2} \right)^{j} \eta_{j} \left( \frac{1}{\sqrt{N} R^{-1}_{ll}} \right)^{j} \tag{12}
\]

\[
= \left( \frac{D}{2} \right)^{\frac{1}{2}} \sum_{j=0}^{D-1} \left( \frac{D - 1 - j}{2} \right)^{j} G^{j}, D = 2, 4, ...
\]

Where \( \binom{n}{r} \) is the Binomial coefficient,

\[
F = 1 - (\sigma^{2} + 1)^{-1/2} \text{ and } G = 1 + (\sigma^{2} + 1)^{-1/2} \text{ with } \sigma = \sqrt{\frac{\nu^{2}(u) R^{-1}_{ll}}{\eta}} .
\]

The average symbol error rate (SER) is given by [8]

\[
P_{s} = \frac{\mu}{(4^{2n} \kappa_{2,n}^2)} \sum_{k=0}^{2n} \binom{2n}{k} \left( \frac{4}{v P} \right)^{2(n-k)} G_{1}^{2} 2 \left( \frac{4}{v P} \right)^{0.51,2n-0.5} G_{2}^{2} 2 \left( \frac{4}{v P} \right)^{0.51,2n+0.5} G_{3}^{2} 2 \left( \frac{4}{v P} \right)^{0.51,2n+0.5} G_{4}^{2} 2 \left( \frac{4}{v P} \right)^{0.51,2n+0.5}
\]

(13)

Where \( P \) denote the transmit SNR, \( \mu, v \) are constants dependent on the modulation scheme, \( \kappa_{m,n} = \prod_{k=0}^{m-1} (m-k)!/(n-k)! \) and \( G(.) \) is Meijer G function.

VI. SIMULATION RESULTS

In this section, the performance of M-decorrelator is presented by computing (12) for \( D = 2 \) and 4 with different influence functions. In Fig. 2 and 3, the average probability of error versus the SNR corresponding to the user 1 under perfect power control of a synchronous DS-CDMA system with six users (\( L = 6 \)) and a processing gain of 127 (\( N = 127 \)) is plotted for \( D = 2 \) and 4 with \( \varepsilon = 0.01 \) and 0.1. Probability of error of the decorrelator for an asynchronous DS-CDMA system with \( L = 6, N = 127 \) and \( \varepsilon = 0.1 \) is in Fig. 4. Finally, Average SER for BPSK for \( \mu = v = 1 \) over a MIMO DS-CDMA System is plotted in Fig. 5. These computational results reveal that the increase in diversity order (from 2 to 4) improves the detector performance. Simulation results also reveals that the proposed M-estimator provides better gains in highly impulsive noise under severe fading conditions of the channel. Moreover, this performance gain increases as the SNR increases.
In this paper, the study of worst-case fading conditions of Nakagami-$m$ fading channel is considered and multiuser detection technique for DS-CDMA systems over Nakagami-0.5 (worst-case fading condition of Nakagami-$m$ fading) fading channels in impulsive noise environment is presented. Simulation results show that the proposed $M$-estimator based robust multiuser detector offers significant performance gains over Huber and Hampel $M$-estimator based detectors when the fading severity is more under highly-impulsive noise.

**References**