Developing a Computational Modeling Algorithm for Thermostressed Condition of Rod Made of Heat-resistant Material ANB-300 Type

Zhuldyz Tashenova, Elmira Nurlybaeva, and Anarbay Kudaykulov

***Abstract*—**Bearing components of jet propelled and hydrogen engines, atomic and heat power plants, technological lines of processing industries, as well as internal combustion engines work in complicated thermal area. Reliable operation of these structures will depend on thermo-stress condition of bearing components. Therefore this research is dedicated to numerical study of thermo-stress condition of bearing components of structures in the form of limited length rods constrained from both ends. Herewith the rod is under influence of local temperature and heat exchange. Apart from this the rod under study is made of heatproof material ANV-300. Particularity of this material is that temperature expansion coefficient of the rod material depends on the temperature. The offered computational algorithm is based on the energy conservation principle. Herewith all types of integrals in energy functional formulas are integrated analytically. Where upon the acquired numerical solutions will be of high accuracy.

***Keywords*—**heatproof material, local temperature, heat exchange, movement, deformation, stress, forces

I. Introduction

In many strategic constructions rods made of heat resistant alloys are used as supporting elements. Nowadays for this purpose special ANB-300 type heat resistant alloys are widely used. In these alloys the value of thermal expansion coefficient $\alpha\left(\frac{1}{^\circ C}\right)$ strictly depends on temperature. In research paper [1] α -values are provided at various temperatures. Based on experimental materials a corresponding functional relationship may be developed:

1) $at \quad 20^\circ C \le T \le 100^\circ C,$

$$\alpha = 0{,}0225 \cdot 10^{-6} \times T + 9{,}65 \cdot 10^{-6} \quad \left(\frac{1}{^\circ C}\right)$$

2) $at \quad 100^\circ C \le T \le 200^\circ C,$

$$\alpha = 0{,}013 \cdot 10^{-6} \times T + 10{,}6 \cdot 10^{-6} \quad \left(\frac{1}{^\circ C}\right)$$

3) $at \quad 200^\circ C \le T \le 300^\circ C,$

$$\alpha = 0{,}015 \cdot 10^{-6} \times T + 10{,}2 \cdot 10^{-6} \quad \left(\frac{1}{^\circ C}\right)$$

4) $at \quad 300^\circ C \le T \le 400^\circ C,$

$$\alpha = 0{,}023 \cdot 10^{-6} \times T + 7{,}8 \cdot 10^{-6} \quad \left(\frac{1}{^\circ C}\right)$$

5) $at \quad 400^\circ C \le T \le 500^\circ C,$

$$\alpha = 0{,}013 \cdot 10^{-6} \times T + 11{,}6 \cdot 10^{-6} \quad \left(\frac{1}{^\circ C}\right)$$

6) $at \quad 500^\circ C \le T \le 600^\circ C,$

$$\alpha = 0{,}02 \cdot 10^{-6} \times T + 8{,}3 \cdot 10^{-6} \quad \left(\frac{1}{^\circ C}\right)$$

7) $at \quad 600^\circ C \le T \le 700^\circ C,$

$$\alpha = 0{,}017 \cdot 10^{-6} \times T + 10{,}1 \cdot 10^{-6} \quad \left(\frac{1}{^\circ C}\right)$$

8) $at \quad 700^\circ C \le T \le 800^\circ C,$

$$\alpha = 0{,}012 \cdot 10^{-6} \times T + 13{,}6 \cdot 10^{-6} \quad \left(\frac{1}{^\circ C}\right)$$

Now let's consider a limited length horizontal rod made of ANB-300 type heat resistant alloy. Let's designate the rod length as $L(cm)$, and cross section areas as $F(cm^2)$. Let's designate rod material thermal expansion coefficient as $\alpha = \alpha(T(x))\left(\frac{1}{^\circ C}\right)$. Rod material heat conductivity coefficient as $K_{xx}\left(\frac{W}{cm \cdot ^\circ C}\right)$. Let's assume the left end of the rod under consideration rigidly fixed whereas the right end as loose. Axial tensile force P (kg) is applied on the free end. Let's direct axis Ox from the left to the right. It coincides with the rod axis. Heat exchange with the environment takes place across the rod entire length throughout the lateral area as well as cross section area of the rod right end. Herewith environment temperature $T_{oc}(^\circ C)$, whereas heat exchange coefficient $h\left(\frac{W}{cm^2 \cdot ^\circ C}\right)$. Constant temperature $T(x=0)=T_1$ is set on the rod left fixed end. A computational model for solving the problem under consideration is provided in the Fig. 1.

Zhuldyz Tashenova, L.N. Gumilyov Eurasian National University, 01008, Astana, Mirzoyana Str., 2, Kazakhstan
(87022155293; zhuldyz_tm@mail.ru)
Elmira Nurlybaeva, KazNTU after K.I. Satpaev, Almaty, Kazakhstan (87016118468; NurEmuk@mail.ru)
Anarbay Kudaykulov, L.N. Gumilyov Eurasian National University, 01008, Astana, Mirzoyana Str., 2, Kazakhstan (87017114687; kudaykulov2006@mail.ru)

$T(x=0)=T_1$

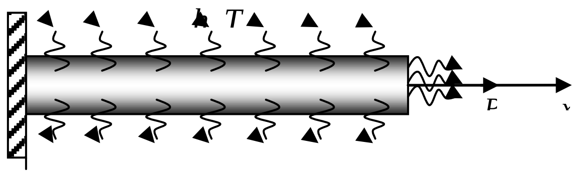

Fig.1 Computational model for solving the problem under consideration.

Now it's required to determine temperature distribution law across the tested rod length, as well as how the rod stretching value depends on the set temperature value $T(x=0)=T_1$. At the same time one should consider natural dependency of the rod material thermal expansion coefficient on temperature. For this purpose to begin with let's discretize the rod under study on n-elements with the same length $\ell=\frac{L}{n}\,(cm)$. Let's consider one discrete element. Let's approximate temperature distribution field within each local element using a complete polynomial of the second kind , that is

$$T(x)=ax^2+bx+c,\quad 0\le x\le \ell,\ 5 \qquad (1)$$

If we assume within one local element limits:

$$T_i=T(x=0);\quad T_j=T\left(x=\frac{\ell}{2}\right);\quad T_k=T(x=\ell) \qquad (2)$$

Then we may rewrite (1) within the given element in the following form

$$T(x)=\varphi_i(x)\cdot T_i+\varphi_j(x)\cdot T_j+\varphi_k(x)\cdot T_k,\quad 0\le x\le \ell \qquad (3)$$

Where $\varphi_i(x)$, $\varphi_j(x)$ and $\varphi_k(x)$ are approximate spline functions which are called form functions for a quadratic discrete element with three knots. They have the following appearance [2].

$$\varphi_i(x)=\frac{\ell^2-3\ell x+2x^2}{\ell^2};\ \varphi_j(x)=\frac{4\ell x-4x^2}{\ell^2};\ \ \varphi_k(x)=\frac{2x^2-\ell x}{\ell^2};\ 0\le x\le \ell$$

(4)

Then the temperature gradient within each element limits shall be determined as follows:

$$\frac{\partial T}{\partial x}=\frac{\partial\varphi_i(x)}{\partial x}T_i+\frac{\partial\varphi_j(x)}{\partial x}T_j+\frac{\partial\varphi_k(x)}{\partial x}T_k,\qquad 0\le x\le \ell \qquad (5)$$

Now we shall write down a functional formula for (n-1) elements which specifies its heat energy in full extent:

$$J_i=\int_{V_i}\frac{K_{xx}}{2}\left(\frac{\partial T}{\partial x}\right)^2 dV+\int_{S_{\delta\Pi i}}\frac{h}{2}(T-T_{oc})^2 dS \qquad (6)$$

Where $i=1\div(n-1)$; V_i - volume of i - discrete element; $S_{\delta\Pi i}$ - lateral area of i - element.

Finally let's write down functional analogous formula for n- discrete element [3].

$$J_n=\int_{V_n}\frac{K_{xx}}{2}\left(\frac{\partial T}{\partial x}\right)^2 dV+\int_{S_{\delta\Pi n}}\frac{h}{2}\left(T-T_{oc}\right)^2 dS+\int_{S_{x=L}}\frac{h}{2}\left(T-T_{oc}\right)^2 dS$$

(7)

where $S(x=L)$ - cross section area of the rod right end through which heat exchange with the environment also takes place. Then for the entire rod under study the functional formula which specifies its full heat energy shall have the following form:

$$J=\sum_{i=1}^{n}J_i \qquad (8)$$

Minimizing J by temperature pivotal values we obtain a resolving system of linear algebraic equations:

$$\frac{\partial J}{\partial T_i}=0,\qquad i=2\div(2n+1) \qquad (9)$$

Where i- starts changing beginning from 2, because $T_1=T(x=0)$ is deemed already set.

Temperature pivotal values shall be obtained through resolving the system (9) using Gauss method. Based on them for each local discrete element next integral is calculated the essence of which is the element's elongation due to heat expansion effect:

$$\Delta\ell_i=\int_0^{\ell}\left[\varphi_i(x)\cdot\alpha_i+\varphi_j(x)\cdot\alpha_j+\varphi_k(x)\cdot\alpha_k\right]\left[\varphi_i(x)\cdot T_i+\varphi_j(x)\cdot T_j+\varphi_k(x)\cdot T_k\right]dx,\quad 0\le x\le \ell$$

(10)

Then total elongation of the rod under study will be calculated using the following formula:

$$\Delta\ell=\sum_{i=1}^{n}\Delta\ell_i \qquad (11)$$

Here we shall notice that at different values of set temperature $T(x=0)=T_1$, a corresponding value $\Delta\ell$ is acquired.

For the purposes of numerical investigation of dependency $\Delta\ell=\Delta\ell(T(x=0)=T_1)$ let's take the following reference data:

$K_{xx}=72\ W/(cm\cdot^\circ C)$; $h=10\ W/(cm^2\cdot^\circ C)$; $T_{oc}=40\,C$;
$T(x=0)=T_1=(100\div 800)^\circ C$; $L=30\,cm$; $n=300$;
$\ell=\frac{L}{n}=0{,}1cm$; $r=1cm$; $F=\pi r^2=\pi$; $P=2\pi r=2\pi$

Based on these initial data temperature distribution field across the rod length is depicted in the Fig. 2.

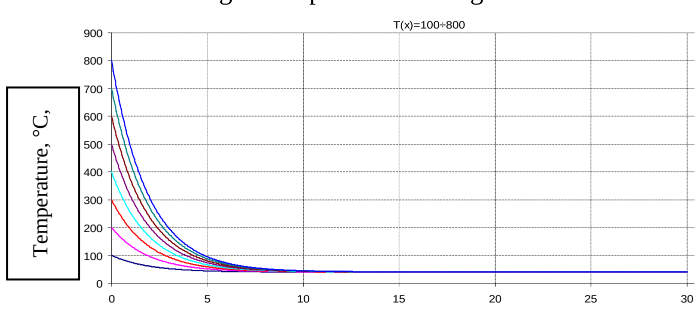

Fig. 2 Dependency $T=T(x)$

TABLE I. DEPENDENCY BETWEEN T_1 AND $\Delta\ell_T$, R, σ

No.	$T_1(^\circ C)$	$\Delta\ell_T(cm)$	Equivalent "stretching" force $R(\kappa g)$, at which such elongation would be obtained	Equivalent "stretching stress" $\sigma(\kappa g/cm^2)$	$\overline{\Delta\ell}_T(cm)$ at $\alpha = const = 10,1\cdot 10^{-6}(1/^\circ C)$	Specific elongation in %	$k = \frac{\Delta\ell_T}{\overline{\Delta\ell}_T}$ (times)
1.	100	0,014	2930,66	933,33	0,0133	0,047	1,052
2.	200	0,0165	3454	1100	0,0152	0,055	1,085
3.	300	0,0193	4040,1	1286,66	0,0171	0,064	1,129
4.	400	0,02247	4703,72	1498	0,0190	0,075	1,183
5.	500	0,0259	5432,2	1730	0,0209	0,086	1,239
6.	600	0,0297	6217,2	1980	0,0228	0,1	1,303
7.	700	0,03388	7092,2	2258,66	0,0247	0,113	1,372
8.	800	0,038	7954,66	2533,33	0,0267	0,127	1,423

Fig. 2 shows temperature distribution field across the rod length at various T_1 values, whereas Table 1 shows $\Delta\ell_T$ values at various T_1 values, that is dependency between T_1 and $\Delta\ell_T$, R, σ. As you one see from Fig. 2 temperature distribution field across the rod length will be a smooth curve. Graphical dependency between temperature source value (T_1) and corresponding rod elongation value $(\Delta\ell_T)$ against heat expansion is shown in Fig. 3.

At $T_1 = 100(^\circ C)$, beginning from $x = 15,5(cm)$, that is within section $15,5 \le x \le 30(cm)$ constant temperature equaling to $\approx 40(^\circ C)$ is observed. In this case due to heat expansion the rod elongates for $\Delta\ell_T = 0,014(cm)$. To compare with it worth to mention that this elongation is equivalent to the rod stretching if it's stretched out applying force $R = 2930,66(\kappa g)$. Naturally based on Hooke's law in this case tension stress to the extent of $\sigma = 933,33(\kappa g/cm^2)$ would appear in the rod cross section.

At increasing the set temperature value twice as much, that is at $T_1 = 200(^\circ C)$ within the section $19,2 \le x \le 30(cm)$, $40(^\circ C)$ - field temperature is observed. In this case the rod elongation value will be $\Delta\ell_T = 0,0165(cm)$ and will be 17,657% more than at $T_1 = 100(^\circ C)$. This elongation value is equivalent to the rod stretching being under tensile load $R = 3454(\kappa g)$. At the same time tension stress would be $\sigma = 1100(\kappa g/cm^2)$. If we increase the point temperature value three times as much that is at $T_1 = 300(^\circ C)$ value $\Delta\ell_T = 0,0193(cm)$, that exceeds by 37,857% more than in the case at $T_1 = 100(^\circ C)$. It also worth to mention that in this case within the rod section $21,1 \le x \le 30(cm)$ constant temperature close to the rod ambient temperature is observed. In this case value $\Delta\ell_T$ is equivalent to the tested rod stretching under applied force $R = 4040,1(\kappa g)$. Herewith value of tensile stress appeared in cross sections would made up $\sigma = 1286,66(\kappa g/cm^2)$. It should be mentioned that for usual steels this tension have already exceeded the proportionality limit.

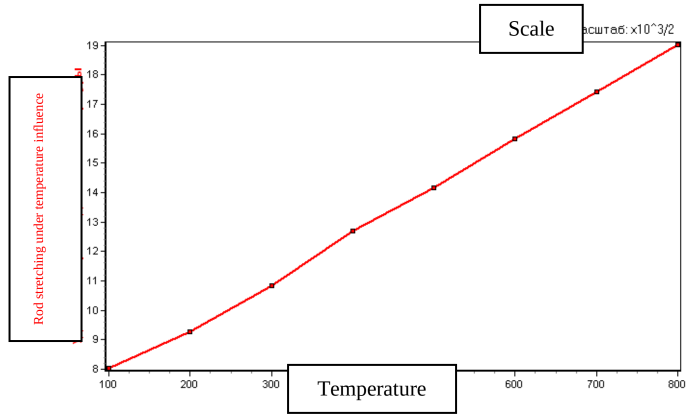

Fig. 3 Graphical dependency between T_1 and $\Delta\ell_T$

Now when we increase value T_1 four times as much, that is at $T_1 = 400(^\circ C)$ we have value $\Delta\ell_T = 0,02247(cm)$. This is equivalent to the rod elongation at stretching it with force equaling to $R = 4703,72(\kappa\Gamma)$. In this case tension stress equaling to $\sigma = 1498(\kappa g/cm^2)$ would appear in the rod sections. Naturally for usual steels such tension deems failure stress. At $T_0 = 500(^\circ C)$ value $\Delta\ell_T = 0,02595(cm)$. This is 85% more than the analogical value $\Delta\ell_T$ at $T_0 = 100(^\circ C)$. Here it should be noticed that in order to acquire the rod elongation at the extent of $\Delta\ell_T = 0,02595(cm)$ when stretching it should have been pulled with force equaling to $R = 5432,2(\kappa g)$. Then tension stress equaling to $\sigma = 1730(\kappa g/cm^2)$ would appear in the rod sections which is quite intense for usual steel structures. It should be noted that at $T_1 = 600(^\circ C)$ value $\Delta\ell_T = 0,0297(cm)$ and it will be 112,14% more than value $\Delta\ell_T$ at $T_1 = 100(^\circ C)$. Equivalent stretching force would be equal to $R = 6217,2(\kappa g)$ and corresponding tension stress will be equal to $\sigma = 1980(\kappa g/cm^2)$. It's interesting that at increasing temperature value T_1 from $T_1 = 100(^\circ C)$ to $T_1 = 600(^\circ C)$, values $\Delta\ell_T$, R and σ will increase to the same extent by 112,14%.

It worth to mention that this approbated method for solving a complicated problem of steady-state thermomechanical conditions of structural elements is relatively universal in term of considering the existing diverse boundary conditions as well as controlling implementation of energy conservation law throughout all computation stages. In this connection the obtained numerical solutions are characterized with high accuracy.

II. CONCLUSIONS

The developed computational algorithm enables to numerically resolve nonlinear problems of stable thermo-mechanical condition of load-carrying structure components at simultaneous presence of local heat insulations, temperatures, heat flows and heat exchanges. Herewith this algorithm enables to take into account natural dependency of the material temperature expansion coefficient against the temperature. The acquired solutions are characterized with increased accuracy.

REFERENCES

[1] Kudaykulov A.K. *Mathematical modeling applications of heat distribution in one-dimensional structural elements*. Baiterek, Turkestan, 2009(6), pp. 163-168.

[2] K.A. Kudaykulovich., N.E. Nurlybaevna. Mathematical modeling installed thermo of the mechanical processes in deformed hard subjects at presence of the local temperature, heat flow, heat exchange and heat to insulates.\\ Book of Abstracts IV Congress of the Turkic World Mathematical society Baku, Azerbaijan, 1-3 July 2011.p. 534

[3] Kenzhegul B.Z., Kudaykulov A.K., Myrzasheva A.N. *Numerical study of stem elongation of heat-resistant alloy based on the availability of all types of sources*. Bishkek, 2009(4). pp. 3-7.