A Mathematical Approach for 2nd tier Supplier Selection when Considering Tolerance Design

Jirarat Teeravaraprug

***Abstract*—**The problem of supplier selection is one of the most important supply chain design topics. Normally, the problem can be divided into two types: local optimization and global optimization. To solve the problem by using local optimization, only 1st tier suppliers are determined. This paper assumes a two-tier supply chain where both 1st tier and 2nd tier suppliers have to be selected. Further, a concept of tolerance is applied to the selection of both tiers suppliers. A mathematical approach is provided to solve the problem and a numerical example is utilized to illustrate the mathematical model.

***Keywords*—**Supplier selection, tolerance design, mathematical approach.

I. Introduction

RECENTLY, many researchers have paid attention to total optimization in the supply chain. It is proved that the effective supply chain would bring high customer satisfaction, low costs, high product variety, high quality, and short lead times [1-6]. One of the important supply chain design decisions is supplier selection. Supplier selection is a multi-criteria decision-making problem mainly involves evaluating a number of suppliers according to a set of common criteria.

Two types of optimization have been defined in [7]: local optimization and global or total optimization. Local optimization supply chain deals with determining the optimization of a company whereas global optimization is about determining the optimization of the whole supply chain. In the global optimization of the supply chain, selecting the right upstream suppliers is a key success factor that will significantly reduce purchasing cost, increase downstream customer satisfaction, and improve competitive ability.

This paper considers a two tier supply chain and integrates the concept of tolerance design in the supplier selection problem. Based on the review, only local optimization has been considered [8-9] in the integration of tolerance design and the problem of supplier selection. This paper then gives an attempt to obtain global optimization of supplier selection when considering tolerance design.

II. Problem statement

In manufacturing a product, manufacturers have treated tolerance as a very important topic. Tolerance is defined as the range between a specification limit and the nominal dimension. A careful analysis and assignment of tolerances can significantly reduce manufacturing costs [8]. The tolerance considered in this paper is called tolerance stackup (accumulation). Normally, the tolerance stackup problems are about determining of process selection and it can be applied to the problem of obtaining the 1st tier suppliers for each component as shown in Fig. 1.

The product having an acceptable tolerance limit of T comprises components A, B, and C. Each component has three suppliers. 1st tier suppliers $A1$, $A2$, and $A3$ are suppliers of component A. 1st tier suppliers $B1$, $B2$, and $B3$ are suppliers of component B. Similarly, 1st tier suppliers $C1$, $C2$, and $C3$ are suppliers of component C. The objective of the problem is to select only one supplier for each component in order to minimize total costs whereas the total stackup tolerance does not exceed its tolerance limit T. Therefore, by selecting suppliers, both tolerance and component cost have to be considered.

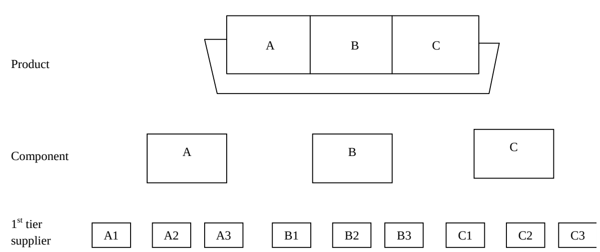

Fig. 1 the problem of obtaining the 1st suppliers

Based on the raise of supply chain, the concept discussed above can be viewed as a local optimization of supplier selection when considering tolerance design. This paper extends the situation discussed above to a two-tier supply chain. Fig. 2 shows the 2nd tier supplier of component A when $A1$ is selected as the supplier of component A. It can be seen that by producing component A, three elements has to be stacked-up. They are called as $A1a$, $A1b$, and $A1c$ when $A1$ is selected as the supplier of component A. Each element has three suppliers and these suppliers are called 2nd tier suppliers. Element $A1a$ has $A1a1$, $A1a2$, and $A1a3$ as element A1a's suppliers. Element $A1b$ has $A1b1$, $A1b2$, and $A1b3$ as element $A1b$'s suppliers. Element $A1c$ has $A1c1$, $A1c2$, and $A1c3$ as element $A1c$'s suppliers. So, it can be seen that the number of

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2^{nd} tier suppliers of component *A* when *A1* is the 1^{st} tier supplier is 9. Similarly, the suppliers of *A2a*, *A2b*, and *A2c* are *A2a1*, *A2a2*, and *A2a3*; *A2b1*, *A2b2*, and *A2b3*; and *A2c1*, *A2c2*, and *A2c3*. The number of 2^{nd} tier suppliers of element *A* when *A2* is the 1^{st} tier supplier is also 9. The total number of 2^{nd} tier suppliers is then 81 suppliers. The first and second characters stand for component type and its supplier whereas the third and last characters represent element type and its supplier.

This paper proposes to integrate the concept of supply chain by considering 2^{nd} tier suppliers and gives an attempt to find global optimization when considering tolerance design. Hence, in the means to select 2^{nd} tier supplier, total tolerance does not exceed the tolerance stackup limit.

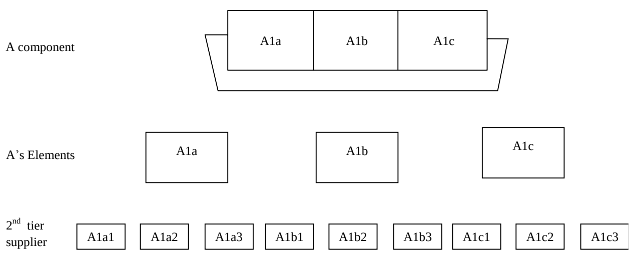

Fig. 2 A component and its suppliers when A1 is selected as A's component supplier

III. Mathematical model

The local optimization given by [8] is the means to select processes or suppliers. The worst case model which is widely used in tolerance stackup analysis is applied to the model. Kusiak and Feng [8] presented the model shown in Eq. (1-4).

$$\text{Minimize } TC = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} X_{ij} \tag{1}$$

Subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij} X_{ij} \le T \tag{2}$$

$$\sum_{j=1}^{n} X_{ij} = 1 \quad \forall i \tag{3}$$

$$X_{ij} = 0,1 \quad \forall i, j \tag{4}$$

where

C_{ij} =manufacturing cost of process j used to produce dimension i

t_{ij} =three sigma normal variation of process j used to produce dimension i

T =single side tolerance stackup limit for dimensional k

X_{ij} = 1 if process j is selected for dimension I, and
0 otherwise

By applying the above model to two-tier supply chain, elements' costs and their tolerances have to be considered instead of component costs and their tolerances. The objective is then to minimize total costs (*TC*) which are elements' costs. The main constraint is the product's tolerance. Assume that C_{ijkl} be cost of element *ijk* when the 2^{nd} tier supplier is *ijkl* and X_{ijkl} be 1 if the 2^{nd} supplier *ijkl* is selected and 0 otherwise; where *i* and *ij* stand for component and its supplier, and *ijk* and *ijkl* stand for element and its supplier respectively. The objective function is then:

$$\text{Minimize } TC = \sum_{i=A}^{C}\sum_{j=1}^{3}\sum_{k=a}^{c}\sum_{l=1}^{3} C_{ijkl} X_{ijkl} \tag{5}$$

The main constraint is total tolerance does not exceed the tolerance stackup limit as shown in Eq. (6).

$$\sum_{i=A}^{C}\sum_{j=1}^{3}\sum_{k=a}^{c}\sum_{l=1}^{3} t_{ijkl} X_{ijkl} \le T, \tag{6}$$

Where, t_{ijkl} is the element tolerance of the 2^{nd} tier supplier *ijkl* and *T* is the desired tolerance range. Assuming that Y_{ij} is 1 if supplier *ij* is selected for component *i* and 0 otherwise. So, Eq. (7) is used to ensure that exactly one supplier is selected for each component and Eq. (8) is used to ensure that exactly one supplier is selected for each element. Lastly, the constraint in Eq. (9) ensures the integrality of X_{ijkl} .

$$\sum_{j=1}^{3} Y_{ij} = 1 \quad \forall i \tag{7}$$

$$\sum_{l=1}^{3} X_{ijkl} = Y_{ij} \quad \forall i, j, k \tag{8}$$

$$X_{ijkl} = 0,1 \quad \forall i, j, k, l \tag{9}$$

IV. Numerical Example

This paper extends the numerical example presented by [8]. Fig. 3 shows the example. The product has three components: components *A*, *B*, and *C*. Each component has three suppliers. Table 1 shows the component tolerances and their costs. Kusiak and Feng [8] gave a solution to the above example as X_{13}, X_{23}, and X_{32} equal to one and the other are zero. Based on the literature, we proposed the problem extension to the level of elements. Each component is composed of three elements: elements *a*, *b*, and *c* and each component has three suppliers. Table 2 provides the element

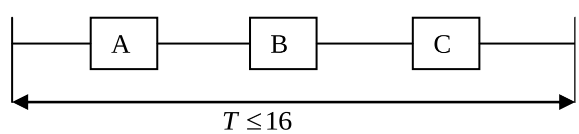

Fig. 3 Numerical example

tolerances and their costs. By reapplying the local optimization model to determine the 2nd tier suppliers, the result shows that 2nd tier suppliers *A3a*3, *A3b*2, *A3c1*, *B3a3*, *B3b2*, *B3c*1, *C3a*1, *C3b3*, and *C3c3* are selected. The total tolerance is also 16 and its cost is 148.

Considering the global optimization model for selecting suppliers in this paper, the data in Table 2 are inserted into the models (Eqs. 5-9). The model is then solved by using Lingo. Fig. 4 shows the Lingo program. The results are X_{A1a1}, X_{A1b1}, X_{A1c1}, X_{B3a3}, X_{B3b2}, X_{B3c3}, X_{C3a3}, X_{C3b3}, X_{C3c3} equal 1 and others equal zero. It means that 1st tier suppliers *A1*, *B3*, and *C3* are selected and 2nd tier suppliers *A1a1*, *A1b1*, *A1c1*, *B3a3*, *B3b2*, *B3c3*, *C3a3*, *C3b3*, and *C3c3* are selected. The total tolerance is 16 and total cost is 129.

V. Conclusion And Discussion

This paper proposes a mathematical approach for supplier selection in a two-tier supply chain when considering tolerance. The result of the proposed approach is compared to the one with local optimization given in [8]. It is shown that the proposed approach provides lower cost whereas the tolerances are equal and stay within its range. It can be concluded that using the global optimization for selecting suppliers would reduce costs comparing to the local optimization.

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