A New Integrated Approach for Enhancing the System Reliability in Facility Location-Network Design Problems

Ahmad Jafarnejad, Farzad Bahrami, and Davood Shishebori

Abstract— The combined facility location and network design is an attractive practical problem in locating public and private facilities. Besides, considering the reliability in modeling of facility location problems is one of the most effective ways to hedge against failures of system from time to time. In reality, the combined facility location network design problem with respect to the reliability of system has a number of applications in industries and services such as locating health care service centers, locating gas compressor stations, and designing water tubing networks. Accordingly, in this paper, for the first time, we consider the combined facility location network design problem with respect to the reliability of system and propose a mixed integer nonlinear programming formulation to model it .Then, the proposed model is linearized by suitable techniques. Moreover, a practical case study is presented in detail to illustrate the application of the proposed mathematical model. Finally, a sensitivity analysis is done to provide an insight into the behavior of the proposed model in response to changes of key parameters of the problem.

Keywords— Facility location, Network design, Reliability, Linearization, Case study, Health care.

I. Introduction

FACILITY location and allocation to customers is one of the practical and strategic problems in the today's industrial and competitive world. With increasing the competition in industries and services, it is evident that manufacturers try to introduce better products or services and decrease their expenses to endure in the business market. Facility location and allocation is an effective tool that can easily facilitate such goals by reducing transportation costs and accelerating the rate of return of investment (reducing the time of return of investment). On the other hand, the worldwide economic downturn in recent years which has brought about financial and economical crisis in almost all countries has provoked the government of these countries to reduce their investment costs in all sort of municipal and parochial plans, yet to pay even more attention to the subject of facility location and allocation and to keep the cost of the plans under budget.

One important matter in modeling of facility location problems is to propose an efficient mathematical model which can display an effective description of the problem and therefore considerably reduce its related costs. Two significant topics that can help to reach such goal are network design and reliability of system. The importance of these topics in modeling of facility location problems will be explained further.

As we know, the classical facility location problems includes p-median and p-center problems [1], the uncapacitated facility location problems [2], the maximum covering location problems [3] and the set covering location problems [4], have been widely utilized to analyze and determine the locations of public and private facilities.

All of the above classical models locate facilities on a predetermined network. However, the topology of the underlying network may profoundly impact upon the optimal facility locations and can have many applications in industries and services. In the literature review, it is evident that Daskin et al. in 1993 introduced the first initial model of facility location-network design problem (FLNDP) [5]. They presented some preliminary results which showed the effect of network design topic in mathematical modeling of facility location problems and their optimal solution. Later, Melkote [6] in his doctoral thesis researched three models for the FLNDP including UFLNDP, the capacitated facility location-network design problem (CFLNDP), and the maximum covering location-network design problem (MCLNDP). The results of the thesis were published in [7]-[8]. Drezner and Wesolowsky [9] proposed a new network design problem with potential links, each of which could be either constructed at a given cost or not. Moreover, each constructed link could be constructed as either a one-way or two-way link. They developed four basic problems subject to two objective functions. In another doctoral thesis, Cocking [10]-[11] expanded some efficient approaches to solve the static budget constrained (FLND) problem. Recently, Bigotte et al. [12] have proposed a mixed-integer optimization model for integrated urban hierarchy and transportation network planning. The model simultaneously determines which urban centers and which network links should be transferred to a new

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level of hierarchy in order to improve availability of all groups of facilities.

Reliability is another significant subject that can affect facility location and allocation. The importance of the reliability of system is recognized when a set of facilities has been constructed, but one or some of facilities occasionally become unavailable in situations such as inclement weather, labor actions, sabotage, or changes in ownership. There are different types of such catastrophic plight, many of which caused facilities to shut down including a series of mail-based anthrax attacks in the United States in 2001-2002 [13]-[16] and SARS outbreak in Toronto, Canada, in the summer of 2003 [16]. It is observed that when a facility failure occurs, customers may have to be reassigned from their original facilities to the other available facilities, a condition that surely requires higher transportation costs.

In the traditional locational analysis literature, Snyder and Daskin in 2003 were the first to propose an implicit formulation of the stochastic P-median and fixed charge problems based on level assignments, in which the candidate sites are subject to random disruptions with equal probability [13]-[15]. Shen et al. [17] and Berman et al. [16] relaxed the assumption of uniform failure probabilities, formulated the stochastic fixed-charged facility location problem as a nonlinear mixed integer program, and expanded several heuristic solution algorithms. Berman et al. [16] concentrated on an asymptotic property of the problem and verified that the solution to the stochastic P-median problem coincides with the deterministic problem as the failure probabilities approach zero. They also presented some efficient heuristics with bounds on the worst-case performance. Lim et al. [18] suggested a reliability continuum approximation (CA) approach for facility location problems with uniform customer density. For simplification, a specific form of failure-proof facility was supposed to exist; a customer was always reassigned to a failure-proof facility after its nearest regular facility failed, regardless of other regular facilities. With respect to the huge investment for facility location and network design, the attention to the failures of system based on facility disruptions in facility locating and network design has been increased recently [19]-[20].

As it is obvious, study of facility location problems with considering network design and reliability of system is relatively rare. Moreover, the existing studies have not considered both network design and reliability of system altogether on facility location. However, there are numerous practical instances such as locating health care service centers, locating gas compressor stations, and designing water tubing networks, in which simultaneously consideration of network design and reliability of system can lead to a more realistic and practical mathematical modeling of the problem. As a result, proposing a new mathematical model formulation, which can obtain optimum facility location and link constructing under some special conditions such as predetermined maximum failure cost, can lead decision makers to more accurate solutions for the considered problem.

In this paper, we develop a new integrated approach as it can be named reliable facility location network design problem (RFLNDP). In our approach, the goal is to determine:

- ✓ the optimum locations of new facilities based on the predetermined maximum allowable failure cost,
- ✓ the primary facility and backup facility of every demand node,
- ✓ the transportation links that should be constructed or improved in the proposed network,
- ✓ the amount of demands of nodes that should be transported by the transportation links, and,
- ✓ The fraction of every demand that should be supplied by new and exciting facilities.

The main contributions that differentiate this paper from the existing ones in the related literature can be summarized as follows:

- ✓ Introducing a new mathematical optimization model to consider simultaneously facility location and allocation, network design and reliability of system as a mixed-integer, nonlinear programming (MINLP) problem. Proposing such mathematical modeling can present a more accurate and integrated description of the problem and eliminate the obstacles of using stochastic optimization models [13], [15], [21]-[23]; besides, some studies have recently emphasized on integrating strategic and tactical decisions to obtain more accurate improvement on considered practical problems [22], [24]-[25].
- ✓ Our new mathematical formulation not only takes into account the facility location costs, link construction or improvement costs, and transportation costs, but also considers the maximum allowable failure cost of the system.
- ✓ Specifically, we consider and explain the RFLNDP in a practical case study which exactly shows the application of new proposed mathematical model.

The rest of the paper is organized as follows: In section 2, the mathematical model formulation of RFLNDP is developed. In section 3, the linearization of proposed model is presented, and a case study that exactly shows the application of the model formulation is demonstrated and solved by the model in section 4. Sensitivity analysis of the model parameters is reported in section 5. Finally, conclusions and future works are presented in section 6.

II. Mathematical Model Description

A. Definition

In this section, the general structure of mentioned problem is exactly described. Suppose that a set of demand nodes exists in a geographical region and a set of transportation links that contains existing and new candidate links is defined to construct a transportation network on the mentioned region. A set of facilities exists in the region and it is clearly desired to locate a set of new facilities, to construct new candidate links, and to improve existing links so that the total investment costs (including locating facilities, constructing links, and improving existing link) and total operational costs (including transportation costs) are minimized. One point that should be considered is that all of the mentioned facilities (containing existing and new facilities) are not reliable and due to some unexpected events such as inclement weather, labor actions, sabotage, or changes in ownership, they occasionally fail and become unavailable. Accordingly, the demand nodes of failed facility must be assigned to the nearest active facility. Therefore, increasing in the traveled distances by demand nodes raise transportation costs. If the increase in transportation costs is considered as failure cost [21, 22], an upper bound can be defined as "maximum allowable failure cost" in which failure costs cannot exceed.

The problem is to determine: (1) the optimum locations of new facilities based on the predetermined maximum allowable failure cost; (2) the primary facility and backup facility of every demand node; (3) the transportation links that should be constructed or improved in proposed network; (4) the amount of demands of nodes that should be transported by transportation links, and, (5) the fraction of every demand that should be supplied by new and exciting facilities.

B. Assumptions

The assumptions for RFLNDP can be described as follow:

1. Each node of network shows a demand point.
2. The facilities and links are uncapacitated.
3. Facilities can only be located on the nodes of the network and may not be located on the links of network.
4. At most one facility can be located on each node.
5. The general structure of the network is planned based on a customer-to-server system, which means that the demands themselves travel to the relevant facilities in order to be served.
6. All travel costs are symmetric.
7. All network links are directed.
8. Locating facilities and allocating demand nodes are considered so that the facility location costs, link construction or improvement costs, and transportation costs are minimized, subject to a constraint that if any facility fails, then, by re-assigning the demand nodes to the available facilities, the resulting cost will not be more than a pre-specified upper bound. In other words, the facility failure costs cannot exceed a predetermined value namely "maximum value failure cost".
9. At most one facility fails at a time.
10. In order to simplify the calculation of the total costs and control the complexity of the problem, neither the probability nor the duration of a failure will be considered; In fact, our goal is simply to hamper the cost that results from a failure, regardless of how frequently this cost incurs.All tables and figures you insert in your document are only to help you gauge the size of your paper, for the convenience of the referees, and to make it easy for you to distribute preprints.

C. Notifications

Parameters:

N : set of nodes in the network

di : demand at node i∈N

D : $\sum_{i\in N} d_i$ = total demands of network

fi : fixed cost of locating a facility at node i∈N

M : set of links in the network (including existing and new candidate links)

cij :cost of constructing or improving link (i, j)

P : number of facilities to open, (P > 2)

V* : maximum allowable failure cost

tij0 : transportation cost of a unit flow on link (i,j)

tijl : transportation cost of a unit flow of demand node l on link (i,j) = tij0 dl

We assume all parameters are integer-valued except all kinds of costs. As an important point, it is mentioned that $t_{ij}^{0}(t_{ij}^{l})$ presents a link-specific transportation cost, not an origin-destination transportation cost and we have to utilize link-specific transportation cost as an initial parameter of RFLNDP model because in RFLNDP, unlike RFLP, the network is not known in advance. Hence, we cannot calculate origin-destination transportation costs.

As another important point, it is mentioned that the maximum allowable failure cost *V** may vary from facility to facility (V*) in practical conditions, but for simplicity, it is assumed that the value of *V** is unique for all existing and new facilities. Determining a suitable *V** in practice may be a disputable factor, because industrial factories and service centers may find it difficult to quantify the maximum failure cost they could tolerate. However, the problem can be solved iteratively with different values of *V** to obtain a tradeoff curve from which decision makers may choose a solution between operating cost and failure cost, based on their preference. The method for generating this tradeoff curve is discussed in Sections 4 and 5.

Variables:

Zik :1 If a facility is located at node i as a primary facility and the facility located at node k is the i's back up facility, and 0 Otherwise.

Xij :1 If link (i,j) is constructed or improved, 0 Otherwise

y_{ij}^{l} : demands of node l on link $(i,j) \in M$

$y_{ij}^{i} = X_{ij} \quad (i,j)\in M$

W_i^{l} : demands of node l served by a facility at node $i\in N$

$W_i^{i} : Z_i \quad i\in N$

D. Model Formulation

Using these notations and assumptions, the mathematical formulation of the RFLNDP is shown below:

$$Min\ C = \sum_{i\in N}\sum_{k\in N} f_i Z_{ik} + \sum_{(i,j)\in M} C_{ij} X_{ij} + \sum_{(i,j)\in M} t_{ij}^{i} X_{ij} + \sum_{(i,j)\in M}\sum_{l\in N: l\neq i} t_{ij}^{l} Y_{ij}^{l} \tag{1}$$

s.t.
$$\sum_{k\in N} Z_{ik} + \sum_{j\in N} X_{ij} = 1 \quad \forall i\in N \tag{2}$$

$$X_{li} + \sum_{j\in N: j\neq l} Y_{ji}^{l} = \sum_{j\in N} Y_{ij}^{l} + W_i^{l} \quad \forall i,l \in N: i\neq l, \forall (l,i)\in M \tag{3}$$

$$\sum_{j\in N: j\neq l} Y_{ji}^{l} = \sum_{j\in N} Y_{ij}^{l} + W_i^{l} \quad \forall i,l \in N: i\neq l, \forall (l,i)\notin M \tag{4}$$

$$\sum_{k\in N} Z_{lk} + \sum_{i\in N: i\neq l} W_i^{l} = 1 \quad \forall l \in N \tag{5}$$

$$Y_{ij}^{l} \le X_{ij} \quad \forall (i,j)\in M \;,\; \forall l \in N: i\neq l, \tag{6}$$

$$X_{ij} + X_{ji} \le 1 \quad \forall (i,j)\in M \tag{7}$$

Model (I):

$$\left[\sum_{m\in N}\sum_{n\in N} f_m Z_{mn} + \sum_{(m,j)\in M} C_{mj} X_{mj} + \sum_{(m,j)\in M} t_{mj}^{m} X_{mj} + \sum_{(m,j)\in M}\sum_{l\in N: l\neq m} t_{mj}^{l} Y_{mj}^{l}\right] + Z_{ik}\left[C_{ik} + \sum_{j\in N} C_{jk} X_{ji} + \sum_{j\in N}(t_{jk}^{0} - t_{ji}^{0}) d_j X_{ji} + \sum_{j\in N}\sum_{l\in N: l\neq j}(t_{jk}^{l} - t_{ji}^{l}) Y_{ji}^{l}\right] \le V^{*} \quad \forall i, k\in N \tag{8}$$

$$\sum_{i\in N}\sum_{k\in N} Z_{ik} = P \tag{9}$$

$$\sum_{k\in N} Z_{ik} \le 1 \quad \forall i \in N \tag{10}$$

$$\sum_{i\in N} Z_{ik} \le \sum_{m\in N} Z_{km} \quad \forall k \in N \tag{11}$$

$$\sum_{i\in N} Z_{ik} \le \sum_{m\in N} Z_{km} \quad \forall k \in N \tag{12}$$

$$Z_{ii} = 0 \quad \forall i \in N \tag{13}$$

$$Y_{ij}^{l} \ge 0 \quad \forall i,j,l \in N \tag{14}$$

$$W_i^{l} \ge 0 \quad \forall i,l \in N \tag{15}$$

$$X_{ij} \in \{0,1\} \quad \forall (i,j)\in M \;, \forall l \in N : l \neq i \tag{16}$$

$$Z_{ik} \in \{0,1\} \quad \forall i,k \in N : l \neq i \tag{17}$$

The objective function (1) includes the total of facility location costs, link construction or improvement costs, and transportation costs in the network. In general observation, constraints (2-5) consider the rational conditions of the transportation flow between demand nodes and facilities. Specifically, Constraint (2) ensures that demand at *i* is either served by a facility at *i* or by shipping on some link out of *i*. Constraints (3) and (4) state conservation of flow for transshipped demand. Constraint (5) imposes that the demand of node *l* must find a destination, whether it is estimated by node *l* itself (z_{lk}) or by the other nodes *i* (W_i^l). Constraints (6) and (7) guarantee that potential links and facilities are not used if they are not constructed. Constraint (8) is equivalent to ones in RFLNDP that says on any given link, an optimal solution flow will be in only one direction. Therefore, both links (*i*,*j*) and (*j*,*i*) cannot be constructed or improved. Constraint (9) is the reliability constraint and makes the failure cost of facility *i* not be greater than *V**. In this constraint, the first summation at the first bracket shows the whole sentences of objective function (*Z*) and computes the value of *Z* with a considered feasible solution; while the second summation at the second bracket presents the increase in *Z*. If the primary facility *i* is failed and the demands served by *i* are transferred to backup facility *k*, it is mentioned that this constraint applies to all $i \in N$, not just to those facilities that have been opened. If $Z_{ik} = 0$, however, the left-hand side of the constraint reduces to the objective function. Since, with this kind of definition, the failure costs in this state (RFLNDP) are always greater than the total cost in FLNDP, this constraint is non-binding if $Z_{ik} = 0$.

Constraint (10) restricts the total number of newly located facilities to the predetermined facilities of *P*. Constraint (11) represents that the maximum number of the selected facilities as a backup of a facility is equal to 1. Constraint (12) examines the possibility of selection of a facility as a backup of a newly located facility. Constraints (13) emphasize that a primary facility cannot be selected as a backup facility of itself. Constraints (14) and (16) force the flow variables to be non-negative; while, Constraints (15) and (17) enforce the binary restriction on the primary and backup facility location and link decision variables.

As it mentioned, according to the single assignment property, every demand of node is completely assigned to the closest single facility. That is, nothing is gained by "splitting up" a demand and sending parts of it to different facilities. Therefore, the fractions of demands, which served a single facility, are integer-valued, while W_i^l and Y_{ij}^l are integral [6].

III. Linearization of Mathematical Model

The mathematical model (I) of RFLNDP is a mixed-integer non-linear programming (MINLP) model because the proposed model has non-linear terms in constraints (9). However, it can be easily linearized by introducing new binary variables and additional constraints as follows:

$$U_{jik} = Z_{ik} X_{ji} \qquad \& \qquad V_{jik} = Z_{ik} y_{ji}^{l}$$

And the following constraints:

$$U_{jik} \le Z_{ik} \qquad \forall i, j, k \in N$$

$$U_{jik} \le X_{ji} \qquad \forall i, j, k \in N$$

$$U_{jik} \ge Z_{ik} + X_{ji} - 1 \qquad \forall i, j, k \in N$$

$$V_{jik} \le Z_{ik} \qquad \forall i, j, k \in N$$

$$V_{jik} \le Y_{ji}^{l} \qquad \forall i, j, k \in N$$

$$V_{jik} \ge Z_{ik} + Y_{ji}^{l} - 1 \qquad \forall i, j, k \in N$$

Consequently, constraints (9) can be substituted with:

$$\left[\sum_{m\in N}\sum_{n\in N} f_m Z_{mn} + \sum_{(m,j)\in M} C_{mj} X_{mj} + \sum_{(m,j)\in M} t_{mj}^{m} X_{mj} + \sum_{(m,j)\in M}\sum_{l\in N: l\neq m} t_{mj}^{l} Y_{mj}^{l}\right] + \left[\sum_{j\in N} C_{jk} U_{jik} + \sum_{j\in N}(t_{jk}^{0} - t_{ji}^{0}) d_j U_{jik} + \sum_{j\in N}\sum_{l\in N: l\neq j}(t_{jk}^{l} - t_{ji}^{l}) V_{jik} + C_{ik} Z_{ik}\right] \le V^{*} \quad \forall i, k\in N$$

Therefore, the final model (I) of RFLNDP converted to MILP easily.

IV. Describing an Application of the Proposed Model by a Case Study

The application of the mathematical model (I) is described as a practical case study, the goal of which is to improve accessibility to health care centers (facilities) for the urban residence centers in a province of Iran named Yazd.

Yazd, with 131575 km2 of area, is known as the fourth largest province of Iran. With respect to its geographical position, Yazd is one of the leading provinces in the field of medical and health care services. Besides, its inexpensive health care services for patients and proximity to deprived southern provinces have dramatically increased its demand for health care services. As one can see on the map in the Fig. 1, Yazd consists of 19 urban residence centers (cities) with total population of 983252.

Reliable data were collected, as far as possible, for the problem. There are two available health care service centers scattered throughout the district, including hospitals and large health centers at urban centers. Also other residence centers (19-2=17 residence centers that don't have any health care service centers) are known as potential nodes to open new health care service centers (new facilities).

According to the current conditions, roads in Yazd province are classified into three categories in term of quality: high, medium and low. In fact, depending on the type of the roads, constructing or improving costs vary; as a result, low and medium quality roads can be upgraded to high quality roads with lower constructing costs.

Also, Fig. 1 shows the residence centers and the road network of Yazd province, as well as the existing health care service centers in cities of the province. It can be seen that there are 50 existing and 45 potential links or roads which have three different qualities and picture with various thicknesses in the graph of Fig. 1.

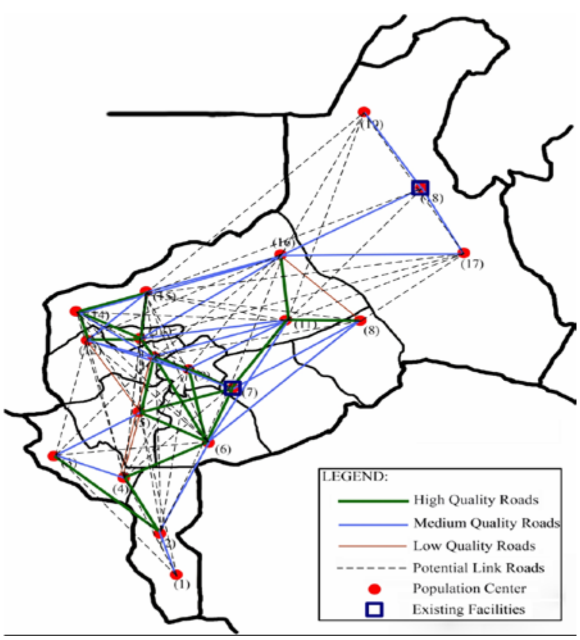

Fig. 1 The map of different roads of Yazd province

The transportation cost for each client in kilometer is randomly calculated subject to a discrete uniform distribution in [0.10, 0.15]. The construction cost of new roads and improvement cost of the existing roads are calculated per kilometer and are between [100000, 400000] as many of transportation cost according to their qualities. Each residence center is a client node with a demand equals its population. The fixed cost of opening facility depends on node demands and varies between $2584010 and $10221186.

In addition, because of some reasons including bad weather conditions, delay in drug supply, lack of specialists or service personnel, staff strikes, and the occurrence of natural disasters such as floods and earthquakes, it is possible that each health service center cannot service its customers, a condition that provoke a center to fail. Since such condition is true for all centers in the province, all of them are unreliable. Based on the geographical, natural, and other important conditions in Yazd province, the maximum value of failure cost is determined to $337500000. Other complementary information contains distances among different cities, transportation cost per unit flow, construction cost of new roads, and improvement cost of existing roads; the rest of complementary information which is about different cities of Yazd province has not been mentioned because of limited volume of the paper.

It is worth mentioning that ministry of health and medical education and ministry of road and transportation are responsible for investment in health care centers and road network construction or improvement, respectively, and they should provide a comprehensive plan to improve the quality of health services in each province. In order to improve the physical access to the health care centers in Yazd, the main goals in the considered case study are to determine:

- ✓ the optimum locations of new health care centers based on the predetermined maximum allowable failure cost,
- ✓ the primary health care center and backup health care centers of every residence center,
- ✓ the transportation links that should be constructed or improved in the proposed network,
- ✓ the amount of demands of residence centers that should be transported by transportation links, and
- ✓ the fraction of every demand that should be supplied by new and exciting health care centers.

According to the mentioned conditions, it is evident that the case study can be exactly investigated as a reliability facility location–network design problem (FLNDP). As a result, according to the mentioned description, the Model (I) is a suitable mathematical modeling for the case study. Therefore, as a propositional option to decrease total costs, the mathematical model (I) can suggest that new facilities can be established in the nodes in which no facility has located. Also, constructing new roads or improving existing roads as it is shown in Fig. 2 can be suggested as the other propositional options to reduce the total costs of the case study. As a reminder point, the mentioned options to decrease the total costs should be determined in a way that the reliability of system does not exceed a predetermined lower bound. In other words, the mentioned options should be selected so that the failure costs not be more than the predetermined upper bound named maximum failure cost. In short, problem options for improvement consist of building new facilities, constructing new roads, improving existing roads, and determining some backup facilities for new and existing facilities.

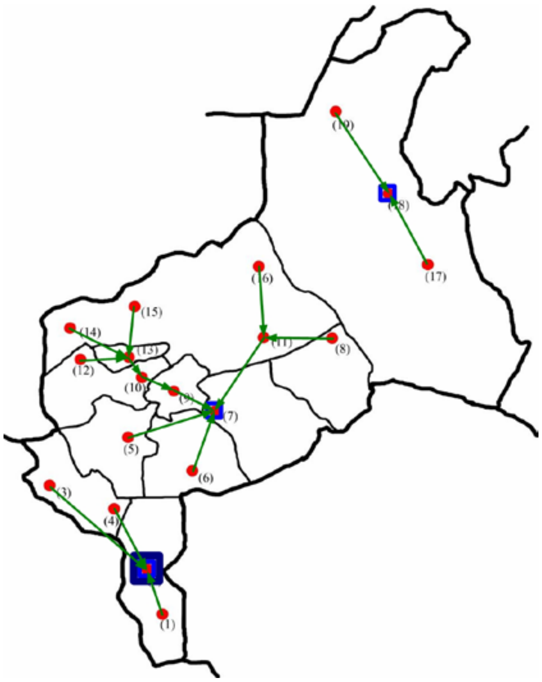

Fig. 2 The optimal solution of the case study

Due to the above description and the value of different parameters, the case study was modeled by the model (I) and coded in GAMS and solved by CPLEX solver. The results are presented in Fig. 3 which visually illustrates the obtained optimal solution. As Fig. 3 shows, the value of $Z_{2,18}$, $Z_{7,2}$ and $Z_{18,7}$ are determined to 1. This means that the optimum locations for new facilities are nodes 2, 7 and 18, and the facilities located in 18, 2 and 7 are determined as the backup facilities located in 2, 7 and 18 respectively. Also with respect to the predetermined value of (V*), constructing new roads is not necessary and only the quality of roads between nodes 1 and 2, nodes 17 and 18, and nodes 18 and 19 should be improved from medium to high. ($X_{1,2}$ =1; $X_{17,18}$=1; $X_{18,19}$=1).

As Fig. 2 presents, the population of some cities should be transferred directly to the health care center located in the identified city. But the population of other cities should be transferred to the health care center located in the determined city via some intermediate cities. The optimal value of objective function is $29789661.020 that in detail, the fixed facility locating cost is $2584011.4, a fixed road constructing or improving cost is $15682500 and the transportation cost is $11523149.62.

The "failure costs" of the three optimal primary health care centers (facilities), as well as their assigned demands, are listed in Table I.

According to table I, the maximum failure cost for the obtained optimal solution is $330329644.5. This means that if the health care center in residence center 7 becomes un-available, it's clients must be served by health care center in residence center 2, ($Z_{7,2}$=1) resulting in a total cost of $330329644.5, i.e, an increase of 1108.873%, which is a huge and incredible increase in the total cost.

TABLE I
THE INCREASE OF TOTAL FAILURE COST FOR DIFFERENT PRIMARY AND BACKUP HEALTH CARE CENTERS (FACILITIES)

$Z_{i,k}$	Failure cost ($)	% Increase in total cost	% Demand Served
$Z_{7,2}$	330329644.5	1108.873%	61.92%
$Z_{18,7}$	93742157.6	314.680%	17.55%
$Z_{2,18}$	56739785.1	190.468%	20.53%

This maximum failure cost and the mentioned increase in the total cost may not be accepted under some special conditions and the decision makers may decide to decrease the maximum failure cost instead of a reasonable and desired increase in total cost. Fig. 3 presents the tradeoff curve of RFLNDP for the case study and indicates the trend of increase in objective function for different values of V*.

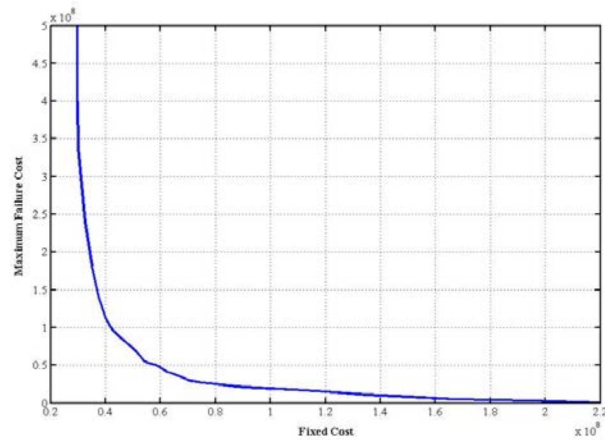

Fig. 3 The changes in optimal value of objective function for different values of maximum failure cost

The optimal FLNDP solution (V*=∞) is the left-most point on the curve. The left part of the tradeoff curve is sharp, indicating that large improvements in reliability may be attained with small increases in FLNDP cost. The smooth right-most portion is of less interest, because it shows a tremendous increase in the optimal value of the total cost compare with a very small decrease in the maximum failure cost.

It is mentioned that the value of objective function for different values of V* is $29789661.020 which is greater than $337500000 (V*>=$337500000); the obtained value of objective function is the minimum value of total cost in our case study. However, if the value of V* decreases, the value of objective function will increase.

This increase is obvious and logical because, with the objective of decreasing the value of maximum failure cost, more costs should be paid as facility location costs, link construction or improvement costs, and transportation costs to enhance reliability of network. On the other hand, the increase in reliability should not cause a noticeable augmentation in mentioned costs of the objective function, but it should increase the reliability of system with a logical and optimized augmentation in mentioned costs of the objective function in order to ameliorate reliability of system with a reasonable cost. A case that is evident in the proposed model (I).

V. CONCLUSIONS AND FUTURE RESEARCH

In this paper, the combined facility location network design problem with respect to the reliability of system, named reliable facility location network design problem (RFLNDP), was considered and a mixed integer nonlinear programming formulation for the mentioned problem was proposed. The basic principal in the proposed formulation is the concept of "backup" assignments, which demonstrate the backup facilities to which clients are assigned when closer facilities have failed and are not available.

The proposed model was linearized by the efficient techniques. Also, a practical case study was presented in detail to illustrate the application of the proposed mathematical model (I). The results show that the model (I) not only can present a more accurate description of RFLNDP but also can propose efficient feasible solutions to use in industries and services. Moreover, a sensitivity analysis was done to provide an insight into the behavior of the proposed model in response to changes of the key parameters. The results show that the changes of the number of facilities (P) has the greatest effect on the changing procedure of the value of Z* and the fewest effect on the changing procedure of the value of Z* is related to the changes of the constructing or improving links cost.

Our findings raise some appropriate questions for future research. First, the size of the case study is small and if the size of the problem increases, a suitable solution procedure should be proposed to obtain optimal or near optimal solution. We are particularly interested in seeking apposite and efficient heuristics and meta heuristics such as tabu search (TS) and particle swarm optimization (PSO) for the mentioned propose. Second, only a single objective function was studied in this paper; however, considering the RFLNDP as a multi objective problem such as minimizing the operating costs and maximizing the reliability of system can have more practical application in industries and services. Finally, we would explore other applications of the proposed model, especially in the fields of integrated facility sitting and supply chain design.

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