Optimizing Location of Power Transformers Using Queuing Theory and Linear Programming

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Abstract**—** Electric column network is progressively becoming more and more complex, consuming an enormous amount of resources, and affecting the environment. The design of an electric column network system consists of various steps among which the location of the electric columns and related maintenance costs due to weathering effects. In this paper, it is proposed a model that helps users choose sites where to locate power transformers in a residential town as well as choose loads of these transformers to be serving the customers. This model also helps in assessing strategic decisions through constraints that force each transformer to be capacitated enough to fit the expected required load at its serving area, while taking into account quality of service constraints from the citizens' point of view. The model is applied to a city with existing electric columns to show how replacing electric columns by transformers minimize the overall cost and helps with preventive maintenance system.

Keywords**—** Set covering problem, transformers, location.

I. Introduction

ELECTRIC power systems are complex systems that include thousands of components such as generators, transformers, transmission lines, control and protection equipment, etc. Constructing, operating and maintaining power systems is very expensive.

Operating conditions are very dynamic, new customers and power system objects appear, prices vary and legislation changes. Even the constantly changing weather conditions effect significantly the operation of such systems. The costly components that have a finite life of several decades and the high cost related to construction and maintenance forced local municipality to replace the electric columns with underground power supplying networks. These networks contain central power transformer connected to the main power source to transform the required voltage to the customers.

Location are long-term strategically decisions and almost impossible to reverse. Therefore, locations problems are critical managerial decision. These strategic decisions have a big influence on the network's flows and clients' satisfaction. Basically, these problems are concerned with locating a number of facilities to supply a set of clients at the minimum cost with respect to a number of constraints and involve various purposes which naturally are in conflict. For example, in some locations there might be restrictions on amount of available budget, number of facilities, facilities' capacity, coverage distance, etc., while the objective function maximizes total covered demand and minimizes total construction and transportation costs [1]. Set covering and network optimization problems lay in the heart of the research field known as Combinatorial Optimization. In particular, set covering is a well-known special case of the general integer linear programming problem, while it also serves as a general model for network optimization problems. Network optimization has received special attention mainly due to the vast evolution of telecommunication and computer networks, along with the variety of applications built upon them.

Yun Wu et al. [2] presented an extension of the capacitated facility location problem (CFLP), in which the general setup cost functions and multiple facilities in one site are considered. The setup costs consist of a fixed term (site setup cost) plus a second term (facility setup costs). The facility setup cost functions are generally non-linear functions of the size of the facility in the same site. Two equivalent mixed integer linear programming (MIP) models are formulated for the problem and solved by general MIP solver. A Lagrangian heuristic algorithm (LHA) is also developed to find approximate solutions for this NP-hard problem. Extensive computational experiments are taken on randomly generated data and also well-known existing data (with some necessary modifications).

Nagy et al. [3] introduced an optimization model to minimize the total number of collection sites to be located, chosen among a set of candidate locations. Such an objective ensures not only the reduction of the visual impact due to the presence of collection sites, but also the reduction of the overall cost related to the collection phase. They considered the minimization of the number of sites as a proxy for minimizing the collection cost. The location of the collection places is intertwined with the subsequent collection routes' determination, and should be cast as a location-routing problem.

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Ghiani et al. [4] developed a model, which determines the optimal allocation of citizens to collection sites, as well as the allocation of waste bins to collection sites such that the demand is satisfied. The bins allocated to such sites may be of different types, with differences in length and capacity. Finally, additional constraints impose that each citizen is serviced by a collection site, which is within a threshold distance from his/her home.

The present work is about optimization in designing power transformers terminals and distributors to replace existing electric columns while respecting required voltage and load. A mathematical model was developed using queuing theory and linear programming. The model deals with the trade-offs between both location and facility costs at terminals and aims to minimize the total of these two logistics costs. Therefore, the model described here adopts capacitated location as a solution procedure to obtain an approximate optimal solution. Heuristic methods using lingo is useful in practice to obtain approximate optimal solutions of large-scale optimization problems that cannot be solved exactly by conventional methods. Based on location planning and covering set problem, the optimal location of the power transformers will be introduced considering the requirements of the customer needs and minimum overall costs.

II. Problem Description and Formulation

The objective is to minimize the total number of needed transformers to be located and chosen among a set of candidate locations as shown in figure 1. The objective function in this case must ensure the reduction of the visual impact due to the presence of collection sites and the reduction of the overall cost related to the collection phase. The minimization of the number of sites as a proxy for minimizing the construction cost. Indeed, the location of the transformers is intertwined with the set covering model.

Moreover, the model determines the optimal allocation of citizens to collection sites, as well as the allocation of bins to collection sites such that the demand is satisfied. The transformers allocated to such sites may be of different types and different capacity. Finally, additional constraints impose that each citizen is serviced by a collection site, which is within a threshold home.

A. *Proposed Solution & Assumptions*

- The targeted region is already constructed, so the suggested transformers locations is specified.
- The number of served people is given and determined.
- The various load required form region to another.

Before presenting the optimization model, we introduce the following further notation:

Table I
Parameters and Index Sets

Symbols	**Indexed by**	**Description**
j	$j \in \{1, 2, \ldots \lvert j \rvert\}$	Set of potential locations of transformers
k	$k \in \{1, 2, \ldots \lvert k \rvert\}$	Set of regions
Parameter	Description	
c	Transformer construction cost	
T	Transformer	
m	Maintenance cost	
n	Population number in specific area	
Q	Transformer capacity	
a	A constant number	

Solution Approach

Objective function

a.
$$Construction\ Cost = \sum_j \sum_k c_k T_{jk} \tag{1}$$

b.
$$Maintenance\ Cost = \sum_j \sum_k m_j T_{jk} \tag{2}$$

Total minimization

$$minimize\ z = \sum_j \sum_k c_k T_{jk} + \sum_j \sum_k m_j T_{jk} \tag{3}$$

Subjected to:

$$\sum_j \sum_k T_{jk} a_{ij} \geq 2 \quad \forall i \tag{4}$$

$$\sum_k Q_k T_{jk} \leq \sum_i n_i a_{ij} \quad \forall j \tag{5}$$

$$\sum_k T_{jk} = 1 \quad \forall j \tag{6}$$

$$T_{jk} = \{0,1\} \quad \forall j, k \tag{7}$$

Non-negativity, Integer

Constraints Explanation

1. Each region must be linked to at least two transformers.
2. The total load of transfers must be less or equal than the total load required by the region.
3. Only one transformer to be located in capacitated location, assignment constraint.

4. A transformer either serving (located) or not serving, binary constraint.
5. Non-negativity
6. Integer Variables

Illustrated Example using Lingo

The model has been applied on a constructed as shown in figure 1.

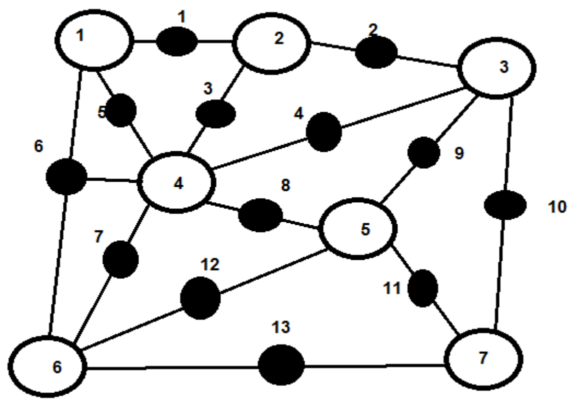

Fig. 1 The tested city, a) black areas: transformers allowed locations, b) white regions: the serviced area.

The code:

1. Sets

```
sets:
types /1..3/: Q, cost;
districts /1..7/: n;
potential_locations /1..13/: mcost;
link_jk(potential_locations,types): T;
link_ij(districts, potential_locations): a;
end sets
```

2. Data:

```
data:
cost = 3000  2000  1500;
n =  25  68  23  32  18  55  53;
mcost =
200  200  150  160  350 450  200  270  300  400 420  150  300;
a =
1  0  0  0  1  1  0  0  0  0  0  0  0
1  1  1  0  0  0  0  0  0  0  0  0  0
0  1  0  1  0  0  0  0  1  0  1  0  0
0  0  1  1  1  1  1  1  0  0  0  0  0
0  0  0  0  0  0  1  1  0  0  1  1  0
0  0  0  0  0  1  1  0  0  0  0  1  1
0  0  0  0  0  0  0  0  0  1  0  1  1;
end data
```

3. Objective function

```
min = @sum(potential_locations(j): @sum(types(k):
cost(k)*T(j,k))) + @sum(potential_locations(j):
@sum(types(k): mcost(j)*T(j,k)));
```

4. Subjected to

```
1. @for(districts(i): @sum(potential_locations(j):
@sum(types(k): T(j,k)*a(i,j))) >= 2);
2. @for(potential_locations(j): @sum(types(k): T(j,k)) = 1);
3. @for(potential_locations(j): @sum(types(k): Q(k)*T(j,k))
<= @sum(districts(i): n(i)*a(i,j)));
4. @for(potential_locations(j): @for(types(k): @bin(T(j,k))));
5. Non-negativity
6.  integer
```

III. Conclusions

- Computational results showed that the proposed solution provide consistently solutions than that currently implemented, resulting in minimization total cost.
- Set covering model is valid as an optimization method to minimize the total cost of transformer construction and maintenance.
- Optimizing of the system before implementing it will give the best solution as shown in the illustrated example.
- Replacing the electric columns by transformers minimize the overall cost and helps in preventive maintenance system.
- Routing model should be implemented for more convenient results in order to obtain the connection cost between the region and the supplying transformers.

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