

# Radar Dwelling Time Design Method Based on Wind Driven Optimization

Zhenkai Zhang, and Sana Salous

**Abstract**—It is necessary to dynamically control the radar's dwelling time for radar's performance of low probability of intercept (LPI). A novel radar dwelling time control method based on wind driven optimization (WDO) algorithm is presented in this paper, which optimizes the dwelling time of radar during the tracking. Firstly, the predicted covariance matrix is computed. Secondly, the influence of the dwelling time is considered in the tracking performance of the radar. Finally, the dwelling time of the radar is optimized by WDO method. The tracking accuracy and LPI performance are demonstrated in the Monte Carlo simulations. The simulation results show that the proposed method not only has more excellent tracking performance but also saves more dwelling time.

**Keywords**—Dwelling time, Target tracking, Wind driven optimization, Low Probability of Intercept

## I. INTRODUCTION

As we know, the less emitted time of the radar, the more excellent performance of the LPI. It is necessary to dynamically control the radar's dwelling time for radar's performance of LPI. The work in [1] develops a generalized framework for the radar task scheduling problem as an optimization model, and all radar task parameters are treated as variables, thereby allowing greater scheduling flexibility and the ability to handle more targets using single radar. The scheduling of track dwells to minimize radar energy and time with an agile beam radar is considered in [2], where the trade between higher energy waveforms and radar time is further investigated. The paper [3] introduces time-windows that specify allowable earliness and lateness of radar tasks, and proposes a chaining process that combines the dwell times and the time-windows of tasks with consecutive priorities. A distributed, consensus-based approach to optimize radar resource management for ballistic missile surveillance and tracking is presented in [4].

Almost all of those works concern the radar's tracking performance instead of the radar's LPI ability. In this paper, a novel scheduling algorithm of radar's dwelling time is proposed. The remainder of this paper is organized as follows. Section II describes the wind driven optimization method. Section III presents the dwelling time scheduling method in details. Simulations of the proposed algorithms and

comparison results with other methods are provided in Section IV. The conclusions are presented in section V.

## II. WIND DRIVEN OPTIMIZATION TECHNIQUE

Building on the successful record of the existing nature-inspired optimization algorithms, the work in [5] introduces and utilizes an entirely new optimization method which is called Wind Driven Optimization (WDO). In the WDO, each air parcel's velocity and position are updated at every iteration as its exploration of search space progresses. Thus, the change in velocity  $\Delta \mathbf{u}$ , can be written as  $\Delta \mathbf{u} = \mathbf{u}_{new} - \mathbf{u}_{cur}$ , where  $\mathbf{u}_{cur}$  is the velocity at the current iteration and  $\mathbf{u}_{new}$  is the velocity in the next iteration. As described in [8], the influence of the Coriolis force is replaced by the velocity influence from another randomly chosen dimension of the same air parcel,  $\mathbf{u}_{cur}^{other\ dim}$ , and all other coefficients are combined into a single term  $c$ , e.g.,  $c = -2|\Omega|U_g T_e$ . The  $\mathbf{u}_{new}$  can be written as:

$$\mathbf{u}_{new} = (1 - \alpha)\mathbf{u}_{cur} - g\mathbf{x}_{cur} + \left( U_g T_e \left| \frac{1}{i} - 1 \right| (\mathbf{x}_{opt} - \mathbf{x}_{cur}) \right) + \left( \frac{c\mathbf{u}_{cur}^{other\ dim}}{i} \right) \quad (1)$$

Where  $\Omega$  represents the rotation of the earth,  $i$  represents the ranking among all air parcels,  $\mathbf{x}_{opt}$  and  $\mathbf{x}_{cur}$  represent the current location and optimum location respectively,  $\alpha$  is the friction coefficient,  $g$  is the earth's gravitational field,  $U_g$  is the universal gas constant and  $T_e$  is the temperature.

Once the new velocity is calculated the position can be updated by utilizing the following equation,

$$\mathbf{x}_{new} = \mathbf{x}_{cur} + \mathbf{u}_{new}\Delta t \quad (2)$$

Where  $\Delta t$  is a time step.

A population of air parcels starts at random positions in the search space with random velocities. Each air parcel's velocity and position are adjusted at every iteration, as the parcels move toward an optimum pressure location and the optimum solution at the end of the last iteration. In this manner WDO offers a simple and effective way to solve complex optimization problems. The implementation of WDO is illustrated in Figure 1. As seen in the flowchart, the algorithm starts with the initialization stage, where all parameters related to the WDO as well as the other parameters related to the optimization problem have to be defined. Also, one must define a pressure

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function as a fitness function, and establish parameter boundaries. Once the optimization problem is set up, the population of air parcels are randomly distributed over the  $N$ -dimensional search space and assigned random velocities. The next step is to evaluate the pressure values of each air parcel at its current position.

Once the pressure values have been evaluated, the population is ranked based on their pressure, and the velocity update according to (1) is applied with the restrictions given in (3).

$$u_{new}^* = \begin{cases} u_{max} & \text{if } u_{new} > u_{max} \\ -u_{max} & \text{if } u_{new} < -u_{max} \end{cases} \quad (3)$$

The positions for the next iteration are updated by utilizing (2), and the boundaries are checked to prevent any air parcel from exiting the search space. Once all the updates are carried out, the parcel pressures at the new locations are evaluated. This procedure continues until the maximum number of iterations is reached. Finally, the best pressure location at the end of the last iteration is recorded as the optimization result and, hence, the best candidate solution to the problem.

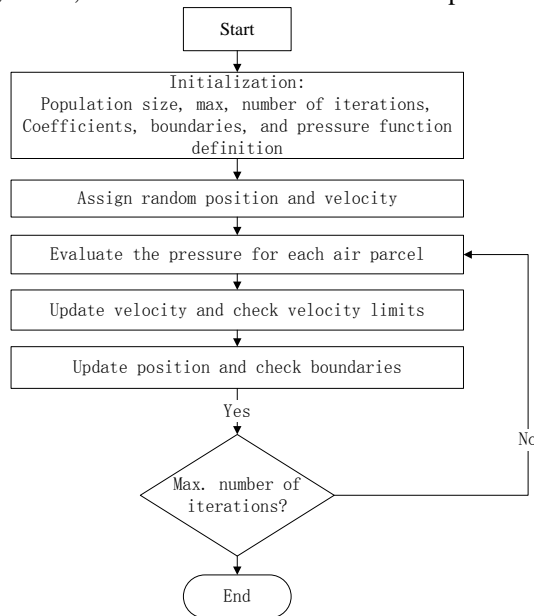


Fig. 1 Implementation of WDO

### III. DWELLING TIME SCHEDULING ALGORITHM BASED ON WDO

#### A. Computation of predicted tracking covariance matrix

Let  $X(k)$  and  $Z(k)$  represent the state vector and the observation vector respectively, the state equation and transfer equation at time  $k$  are:

$$X(k+1) = \phi_j(k+1)X(k) + w_j(k) \quad (4)$$

$$Z(k) = H_j X(k) + v_j(k) \quad (5)$$

Where  $w_j(k)$  and  $v_j(k)$  are stationary white noise processes with covariance matrices  $Q_j(k)$  and  $W(k)$  of model  $j$ ,  $\phi_j(k)$  is the transition matrix and  $H_j$  is the observation matrix.

The prediction of covariance for model  $j$  at time  $k$  can be represented as:

$$P_j'(k+1) = \phi_j(k+1)P_{0j}(k)(\phi_j(k+1))^T + Q_j \quad (6)$$

Where, the covariance  $P_{0j}(t_{k-1})$  of model  $j$  is obtained from all the states and model probabilities of last recurrence.

Then the variance matrix  $S_j(k+1)$  and filtering gain  $K_j(k+1)$  can be written respectively:

$$S_j(k+1) = H_j P_j'(k+1)(H_j)^T + W_j(k+1) \quad (7)$$

$$K_j(k+1) = P_j'(k+1)(H_j)^T (S_j(k+1))^{-1} \quad (8)$$

The covariance estimation for every model can be represented as:

$$P_j^{pre}(k+1) = (I - K_j(k+1)H_j) P_j'(k+1) \quad (9)$$

At last, the predicted covariance matrix is given as:

$$P^{pre}(k+1) = \sum_{j=1}^r \mu_j(k)(P_j^{pre}(k)) \quad (10)$$

Where  $\mu_j(k)$  is model probability at time  $k$ .

#### B. Design for covariance matrix $W_k$ of measurement noise

The covariance matrix  $W_k$  of measurement noise is controlled by the emitted energy. As we know, radar equation at time  $k$  is as follows:

$$R_k^4 = t_B^k \frac{P_{av}^k G_T G_R \lambda^2 \sigma_k}{(4\pi)^3 K T_R S_{NR}^k L} \quad (11)$$

Where  $t_B^k$  is the single dwelling time of the beam from the normal direction at time  $k$ ,  $P_{av}^k$  is the average radiated power,  $G_R$  is the receiver gain,  $\sigma_k$  is the radar cross section(RCS) of the target,  $K$  is Boltzmann constant,  $T_R$  and  $L$  are respectively effective noise temperature and radar system loss,  $R_k$  is the detection range,  $G_T$  is the transmit gain,  $S_{NR}^k$  represents the signal to noise ratio of the system at time  $k$ . Suppose when the target whose range is  $R_0$ , the radar has to emit the power  $P_{av0}$ , and the radar equation become

$$R_0^4 = t_{B0} \frac{P_{av0} G_T G_R \lambda^2 \sigma_0}{(4\pi)^3 K T_R S_{NR0} L} \quad (12)$$

Combined (11) with (12), the emitted signal to noise ratio at time  $k$  can be written as

$$S_{NR}^k = \frac{t_B^k P_{av}^k S_{NR0} R_0^4}{t_{B0} P_{av0} R_k^4} \quad (13)$$

Where, the emitted power is supposed to be a constant parameter in this paper.

Range and range-rate measurements are obtained using the type of linear frequency modulated (LFM) Gaussian pulses [6]. The measurement noise covariance is given by:

$$W_k = \begin{bmatrix} \frac{c^2 p^2}{2S_{NR}^k} & -\frac{c^2 b p^2}{w_0 S_{NR}^k} \\ -\frac{c^2 b p^2}{w_0 S_{NR}^k} & \frac{c^2 b p^2}{w_0 S_{NR}^k} \left( \frac{1}{2p^2} + 2b^2 p^2 \right) \end{bmatrix} \quad (14)$$

Here  $c$  denotes the wave speed (m/s), and  $w_0$  denotes the carrier frequency (Hz),  $p > 0$  denotes the pulse length (s) and  $b$  denotes the sweep rate (Hz/s).  $b$  can be positive (LFM

upsweep), negative (LFM downsweep) or zero. All the waveform parameters are constant except the signal to noise ratio  $S_{NR}^k$ .

We can see that different  $t_B^k$  can lead to different  $W$ . However, during the tracking process,  $R_k$  is unknown before radar detection. So  $R_k$  in (14) is replaced by  $R_k^{pre}$  which is predicted by  $R_{k-1}$  and  $v_{k-1}$ .  $R_k^{pre}$  is presented as

$$R_k^{pre} = R_{k-1} + v_{k-1}T \tag{15}$$

$R_{k-1}$  and  $v_{k-1}$  are the target's range and velocity which are estimated by the IMM tracking algorithm at time  $k-1$ ,  $T$  is the tracking interval.

C. Parcel pressure model for optimization

The desired tracking covariance matrix  $P^{des}$  should be set for the radar firstly. Then the parcel pressure model is presented as:

$$t_B^{k+1*} = \arg \min_{t_B^{k+1}} (\text{trace}(P^{pre}(k+1)) - \text{trace}(P^{des}(k+1))) \tag{16}$$

The WDO method is used to select dwelling time  $t_B^{k+1}$  for the tracking at time  $k+1$ .

IV. SIMULATION RESULTS

In this section, Monte Carlo simulations are performed to analyze the performance of the proposed dwelling time scheduling algorithm based the wind driven optimization (WDO). The IMM filter [7] here is comprised of Constant Velocity model (CV)  $F_{CV}$  and Coordinated Turn rate model (CT)  $F_{CT}$ .

$$F_{CV} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{17}$$

$$F_{CT} = \begin{bmatrix} 1 & \frac{\sin \omega T}{\omega} & 0 & \frac{1 - \cos \omega T}{\omega} \\ 0 & \cos \omega T & 0 & \frac{\sin \omega T}{\omega} \\ 0 & \frac{1 - \cos \omega T}{\omega} & 1 & \frac{\sin \omega T}{\omega} \\ 0 & \sin \omega T & 0 & \cos \omega T \end{bmatrix} \tag{18}$$

Where  $T$  is the sampling interval,  $\omega$  is the turn factor,  $T=2s, \omega=0.1$ .

A. Trajectory design

Fig. 2 shows the target trajectory in 100s.

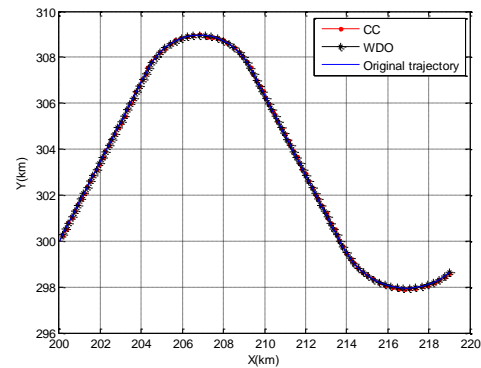


Fig. 2 Trajectory of the target

B. Comparison of tracking performance

The proposed dwelling time design method (WDO), is realized in the simulation, which is also compared with the performance covariance control (CC) in the paper[4]. The Root-mean-square error (RMSE) of time  $k$  can be formulated as:

$$RMSE(k) = \sqrt{\frac{1}{M_c} \sum_{m=1}^{M_c} (x_k - \hat{x}_k^m)^2} \tag{19}$$

Where  $M_c$  is the number of the Monte-Carlo simulation,  $x_k$  is the true state of the system,  $\hat{x}_k^m$  is the estimated vector at the  $m^{th}$  simulation,  $M_c=100$ .

Fig.3 shows the range RMSE of the proposed method during the tracking.

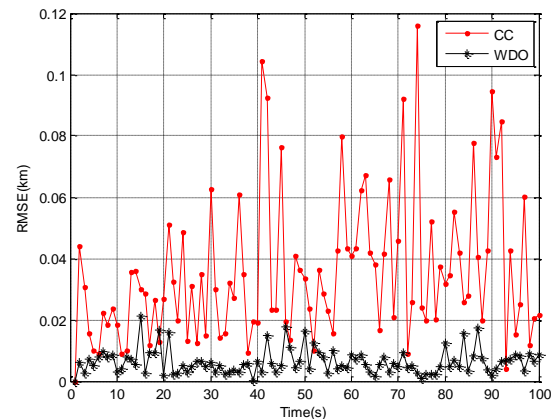
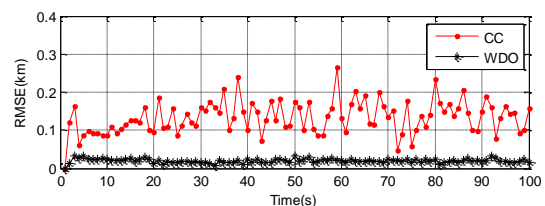
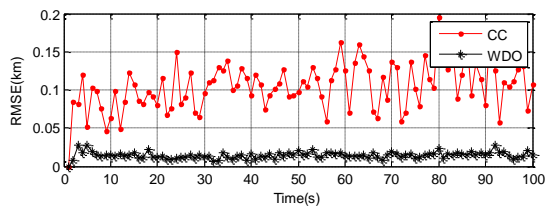


Fig.3 Comparison of tracking performance

Fig. 4(a) and Fig.4(b) show the RMSE of the proposed method in X and Y direction respectively. We can see that the proposed method of WDO presents much more excellent tracking accuracy.



(a) Tracking performance of X direction



(b) Tracking performance of Y direction  
Fig.4 Comparison of tracking performance

C. Comparison of dwelling time

Dwelling time of WDO and CC method are shown in Fig. 5, the mean dwelling time of the two methods are 1.9915ms and 2.8334ms. We can see that the proposed method not only present excellent tracking accuracy, but also reduce more radiated time. As it is shown in [5], the position and velocity updates rules in WDO are similar to those in Particle swarm optimization (PSO), however, the gravitational pull within the velocity update equation in WDO can provide advantages over PSO, where particles occasionally attempt to fly out of and sometimes get stuck at the boundaries, preventing their positions from changing for many iterations.

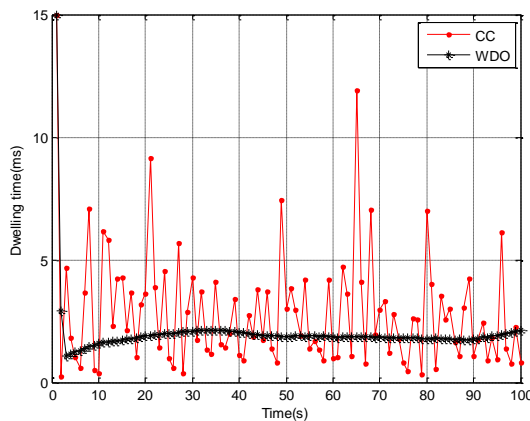


Fig.5 Comparison of dwelling time

V. CONCLUSIONS

In this paper, we have presented a new strategy of dwelling time allocation based the WDO method and predicted covariance theory. The dwelling time is obtained after the WDO at every time in order to meet the requirement of the tracking accuracy. The simulation results show that the proposed method can save much more dwelling time with more excellent tracking accuracy.

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