The Use of Stochastic Gradient Boosting Method for Multi-Model Combination of Rainfall-Runoff Models

Phanida Phukoetphim, and Asaad Y. Shamseldin

***Abstract*—** In this study the novel stochastic gradient boosting (SGB) combination method is addressed for producing daily river flows from four different rain-runoff models of Ohinemuri catchment, New Zealand. The selected rainfall-runoff models are two empirical black-box models: linear perturbation model and linear varying gain factor model, two conceptual models: Soil Moisture Accounting and Routing model and Nedbør-Afrstrømnings model. In this study, the simple average combination method and the weighted average combination method were used as a benchmark for comparing the results of the novel SGB combination method. The models and combination results are evaluated using statistical and graphical criteria. Overall results of this study show that the use of combination technique can certainly improve the simulated river flows of four selected models for Ohinemuri catchment, New Zealand. The results also indicate that the novel SGB combination method is capable to develop a multi-model combination system of the Ohinemuri river catchment in New Zealand.

***Keywords*—**Multi-model combination, rainfall-runoff modeling, stochastic gradient boosting

I. Introduction

THERE are numerous rainfall-runoff models of varying degrees of sophistication and complexity. However, there is no superior rainfall-runoff model providing river flow forecasts which is better, under all circumstances than those of other competing models [23][24]. Each individual rainfall-runoff model provides many important information sources, which may be different from each model. Thus, an alternative approach for improving the forecasting accuracy and reliability is to combine the information from these different sources [23][24][16][10]. In this study, two types of rainfall-runoff models available were used to simulate river flows as they may make efficiently combine in the combination method. The selected rainfall-runoff models are two empirical black-box models: linear perturbation model (LPM) and linear varying gain factor model (LVGFM), two conceptual models: Soil Moisture Accounting and Routing (SMAR) model and Nedbør-Afrstrømnings Model (NAM).

There are a number of a linear, non- linear neural network and fuzzy based combination methods, which can be used for producing the combined river flows [26]. Nevertheless, none of these methods propose a stochastic gradient boosting combination method. Stochastic gradient boosting (SGB) proposed by Friedman [11]. It is a standard classification tree to improve the accuracy of a predictive function by applying the function repeatedly in a series of trees. However, the use of SGB in hydrology and water resources is limited. Therefore, a novel SGB combination method was used to produce the simulated river flows from four selected rainfall-runoff models for Ohinemuri catchment, New Zealand in this study. The simple average method (SAM) and weighted average method (WAM) are the most popular combining techniques used for combined river flows [25][3][16][12][9]. Therefore, the SAM and the WAM were used as a benchmark for comparing the results of the novel SGB combination method in this study.

However, the research on the combination technique in the context of rainfall-runoff modelling has never been applied to New Zealand catchment in the literatures. Consequently, this study has applied this technique, which can participate in improving the accuracy of river flow simulation in New Zealand and draw guidance about the use of combination technique in New Zealand for future research. The objective of this study is to improve the simulated daily river flows from four selected rainfall-runoff models using the combination technique. The simulated river flows from four models were produced using three combination methods: the SAM, the WAM and the SGB combination methods. The statistic and graphical criteria were used to evaluate the model performance. This paper starts with the details of the study catchment and data used in this paper are described. Secondly, a brief description of the methodology used in this study is described. Thirdly, the details of model efficiency evaluation criteria used for assessment of model efficiency are explained. Finally, the conclusions obtained from the results are discussed.

II. Study catchment and data

In this study, the Ohinemuri river catchment was used for testing multi-model combination system. The catchment area is about 285.39 km^2. It located in the North Island of New Zealand (see Fig. 1). Land use types in the Ohinemuri river catchment are mostly forest (49 %). It includes agriculture (1 %), pasture (48 %), urban and rural residential (1.5 %) and the rest consist of swamp. Eight different soil types within Ohinemuri river catchment consist of clay loam (96 %), urban land (2%), brown clay loam 1% and the rest consist of rock. The topography of the catchment varies from 19 m above sea

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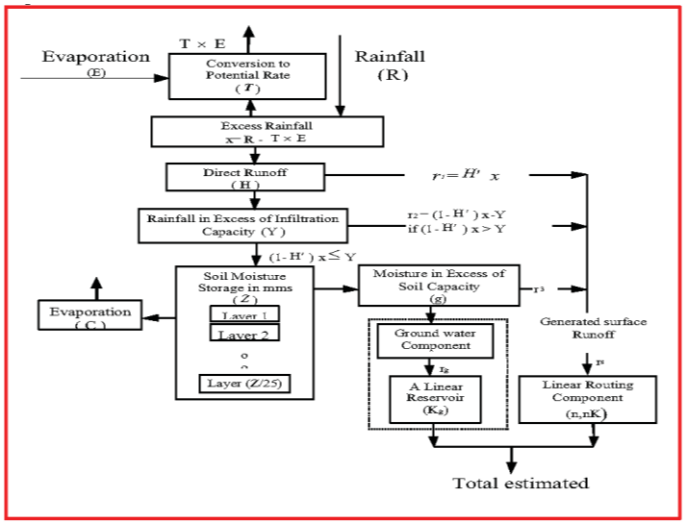
the attenuation and the diffusive effects of the catchment by routing the different generated runoff components of the water balance part through linear storage systems. A schematic diagram of the SMAR model [23] is presented in Fig 4.



Parameter	Description
Z	The combined water storage depth capacity of the layers [>0 in mm].
T	A parameter (less than unity) that converts the given evaporation series to the model-estimated potential evaporation series [0 to 1].
C	The evaporation decay parameter, facilitating lower evaporation rates from the deeper soil moisture storage layers [0 to 1].
H	The generated *'direct runoff'* coefficient [0 to 1].
Y	The maximum infiltration capacity depth [>0 in mm/timestep].
n	The shape parameter of the Nash gamma function *'surface runoff'* routing element; a routing parameter [>0].
nK	The scale (lag) parameter of the Nash gamma function *'surface runoff'* routing element; a routing parameter [>0].
g	The weighting parameter, determining the amount of generated *'groundwater'* used as input to the *'groundwater'* routing element [0 to 1].
K_g	The storage coefficient of the *'groundwater'* (linear reservoir) routing element; a routing parameter [>0].

Fig. 4 Schematic diagram of SMAR model [23]

D. The Nedbør-Afrstrømnings Model (NAM)

NAM is the abbreviation of the Danish "Nedbør-Afstrømnings-Model", meaning precipitation-runoff-model [8]. Nam is conceptual rainfall-runoff model, it was originally developed by the Department of Hydrodynamics and Water Resources at the Technical University of Denmark. The NAM structure is based on physical structures and equations used together with semi-empirical ones. The NAM simulates the rainfall-runoff process by continuously accounting for the water content in four different and mutually interrelated storages that represent different physical elements of the catchment. These storages are snow storage, surface storage, lower or root zone storage and groundwater storage (see Fig. 5). In this study, the snow component is not relevant and so the parameter corresponding to this process is set to zero. The model structure is presented in Fig 5.

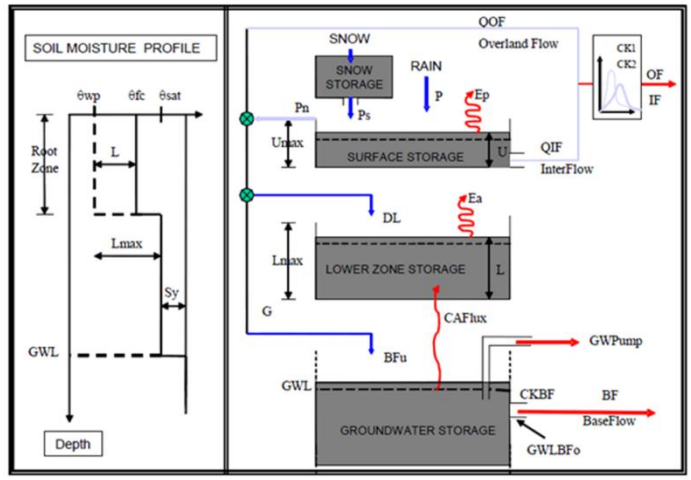


Fig. 5 Structure of the NAM

IV. Combination Methods

The first combining method has addressed in economic forecasting by Bates and Granger [5]. Since the combination methods showed the ability method for improving the performance of forecasts to combine the single model. The results of combination methods in the literature reviews indicate that in many cases the combined forecasts outperform the single model [6][16][23][24[3]. In this study, the results from four selected rainfall-runoff models were combined using three combination methods for improving the accuracy and reliability of simulated river flows of Ohinemuri catchment, New Zealand. The description of the combination methods is given in the following section below.

A. Simple average method (SAM)

SAM is the most popular combined methods on forecasting, which is the simple method provides forecasts that are more accurate than the complex method [4]. The SAM involves computing the average of the combining the outputs of different individual models. Many of published studies have shown that the simple average provides an alternative can perform better than the individual forecasts [17][7][23][25]. The SAM combination method can be expressed by the following equation;

$$Q_{ci} = \frac{1}{N}\sum_{j=1}^{N} Q_{j,i} \qquad (2)$$

where Q_{ci} is a combined estimate of discharge of the *jth* rainfall-runoff models for the *ith* time period, $Q_{j,i}$ is the simulated discharge of the *jth* rainfall-runoff models for the *ith* time period.

B. Weighted average method

Bates and Granger [5] were the first to apply the WAM for combining forecasts. After, Makridakis and Winkler, 1983 [17] advised the weighted average method was used to combine the forecasts perform better that the individual forecasts. The WAM utilizes the multiple linear regression technique to combine the simulated outputs. The WAM

combination method can be expressed as;

$$Q_{ci} = \sum_{j=1}^{N} a_j Q_{j,i} + e_i \quad (3)$$

Where $Q_{j,i}$ is the observed discharge of the *ith* time period, a_j is the weight assigned to the *jth* model estimated discharge, Q_{ci} and e_i is the combination error term. The sum of the model weights are constrained to be always positive and to be equal to unity. The weights are calculated by the ordinary least square technique.

C. Stochastic gradient boosting

The stochastic gradient boosting (SGB) proposed by Friedman [11], is a regression analysis for improving the accuracy of a predictive function by applying the function repeatedly in a series of trees. The final predicted value combines the output of each function with weighting, as a result that the total error of the prediction is minimized [21]. It is developed to improve the accuracy of decision trees models. SGB method is the functionally similar to decision trees because it creates a tree ensemble. The treeboost uses SGB to increase the predictive accuracy of decision tree model. The mathematically of a treeboost-SGB method can be described as:

$$\text{Predicted Target} = \mathrm{F}_0 + B_1 T_1(X) + B_2 T_2(X) + \cdots + B_M T_M(X) \quad (4)$$

where F_0 is the starting value for the series (the median target value for a regression model), X is a vector of "pseudo-residual" values remaining at this point in the series, $T_1(X)$, $T_2(X)$ are trees fitted to the pseudo-residuals and B_1, B_2, etc. are coefficients of the tree node predicted values that are computed by the treeboot algorithm.

V. Evaluation of Model Performance

To access the accuracy of the models, the most common evaluation criteria: root mean squared error (RMSE), coefficient of determination (R^2) and the Coefficient of Efficiency, CE [20] used in hydrology were applied in this study. Gupta et al., 2009 [14] proposed a recently method the Kling and Gupta Efficiency (KGF), which can help to reduce model calibration problem for river flow forecasting. The KGF was applied in this study. For further details of the KGE and its components see Gupta et al. 2009 [14]. The value of CE ranges from $-\infty$ to 1, in which the value closer to 1 represents a better fit of model. The RMSE provides different types of information about the predictive capabilities of the model. It is a non-negative metric that has no upper bond and for a perfect model the RMSE value is close to zero. For a perfect model, the KGE values are close to zero, which is similar to R^2 and CE. The graphical criteria, namely, the hydrograph plots were used in assessing the model performance. The equation used for calculating these statistics are shown below.

Coefficient of efficiency, *CE* is stated as:

$$CE = 1 - \frac{\sum_{i=1}^{n}(S_i - Q_i)^2}{\sum_{i=1}^{n}(Q_i - \bar{Q})^2} \quad (5)$$

Root mean square error, *RMSE* is stated as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(S_i - Q_i)^2}{n}} \quad (6)$$

Coefficient of Determination, R^2 is stated as:

$$R^2 = \left[\frac{\sum_{i=1}^{n}(Q_i - \bar{Q})(S_i - \bar{S})}{\sqrt{\sum_{i=1}^{n}(Q_i - \bar{Q})^2 \sum_{i=1}^{n}(S_i - \bar{S})^2}}\right]^2 \quad (7)$$

King-Gupta Efficiency, *KGE* is stated as:

$$ED = \sqrt{(r-1)^2 + (\alpha - 1)^2 + (\beta - 1)^2} \quad , \alpha = \frac{\sigma_{S_i}}{\sigma_{Q_i}}, \beta = \frac{\bar{S}}{\bar{Q}},$$
$$KGE = 1 - ED \quad (8)$$

Where Q_i is the observed runoff value at time i (m^3/s), S_i is the simulated runoff at time i (m^3/s), $\bar{Q}$ is the mean of the observed runoff data, $\bar{S}$ is the mean of the simulated runoff data and n is the number of data points.

VI. Results and Discussion

In this study, the four selected rainfall-runoff models: the LPM, LVGFM, the SMAR model and NAM were applied to the daily data of Ohinemuri catchment in New Zealand. They are used to simulate river flows at the outlet of an Ohinemuri river gauge (see Fig. 1). For model evaluation, the input data was split into two parts. The first part is 2/3 of the available data, which was used as the calibration period. The remaining 1/3 was used as the verification period (i.e. testing the consistency of the calibrated model on an independent set of data). All models were calibrated the catchment to determine the optimum set of the parameter values.

This study used three combination methods: SAM, WAM and SGB to produce the simulated river flows from four selected rainfall-runoff models of the Ohinemuri catchment, New Zealand. The SGB is a novel combination method in the context of rainfall-runoff modelling. On the other hand, the SAM and WAM are the most popular combining techniques and the methods were used to compare with other combination methods [16]. In this study, the SAM and WAM were used as a benchmark for comparing the results of the novel SGB combination method. The SAM is the simplest method, which assigns the same weight to each the component model output. The WAM assigns different weights to each of the component model outputs. The SGB was used to set up regression tree model for improving model outputs. In this study, DTREG software [21] was used to develop the SGB model. The SGB model was trained using 1334 data sets of the total results of individual rainfall-runoff model. The 80% of the total data sets were used for training the networks, and the remaining 20% data sets were used for testing the networks. This data set is used to avoid overfitting during the training process. The initial number of the tree is incremented by 1 up to a

maximum number of 1,000. The optimal number of the trees is determined when the value of the pseudo-residuals is minimal based on the influence trimming factor. Table I shows the ranking of the performance model of three combination methods and four selected rainfall-runoff models applied to the Ohinemuri river catchment, New Zealand. Analysis of the table indicates that, in the calibration period and verification period, the SGB method for combining four model outputs has better performance than that of the individual models and two combination methods. However, in the verification period, the LPM, in terms of KGE, is superior to all four individual rainfall-runoff modes and those of three combination methods. Figure 6 shows the observed and simulated flood hydrographs for the Ohinemuri river catchment produced the best individual model (LVGFM) and three combined models: SAM, WAM and SGB. The results in figure 6 show that the SGB fits the observed data value better than other models and the SGB is able to capture the flood peaks.

VII. CONCLUSION

This study is the first research has been addressed the use of stochastic gradient boosting combination method in the context of rainfall-runoff modelling. Overall results demonstrate that the proposed method has better performance than other individual simulation models and other two combination methods. The results obtained in this study also indicate that the use of combination technique can certainly improve the simulated river flows of four selected rainfall-runoff models for Ohinemuri catchment in New Zealand. Moreover, results indicate that the SGB is capable to develop a multi-model combination system of the Ohinemuri river catchment in New Zealand.

TABLE I
CALIBRATION AND VERIFICATION RESULTS FROM FOUR MODELS AND COMBINED MODELS FOR OHINEMURI CATCHMENT, NEW ZEALAND

Model	**Ohinemuri river, NZ (Area=285.39km^2)**							
	R^2	Rank	RMSE	Rank	CE	Rank	KGE	Rank
Calibration								
LPM	0.71	6	6.59	7	0.71	7	0.78	5
LVGFM	0.81	3	5.39	3	0.81	3	0.86	3
SMAR model	0.75	5	6.41	6	0.73	6	0.77	6
Nam	0.76	4	6.33	5	0.74	5	0.77	6
Combination models								
SAM	0.81	3	5.49	4	0.80	4	0.81	4
WAM	0.83	2	5.13	2	0.83	2	0.87	2
SGB	**0.91**	**1**	**3.69**	**1**	**0.91**	**1**	**0.88**	**1**
Verification								
LPM	0.64	4	6.14	3	0.61	3	**0.78**	**1**
LVGFM	0.55	7	6.71	5	0.54	4	0.64	5
SMAR model	0.57	6	6.80	6	0.53	5	0.58	7
Nam	0.65	3	6.21	4	0.61	3	0.60	6
Combination models								
SAM	0.66	2	5.91	2	0.64	2	0.66	4
WAM	0.62	5	6.14	3	0.61	3	0.68	3
SGB	**0.83**	**1**	**4.35**	**1**	**0.81**	**1**	0.72	2

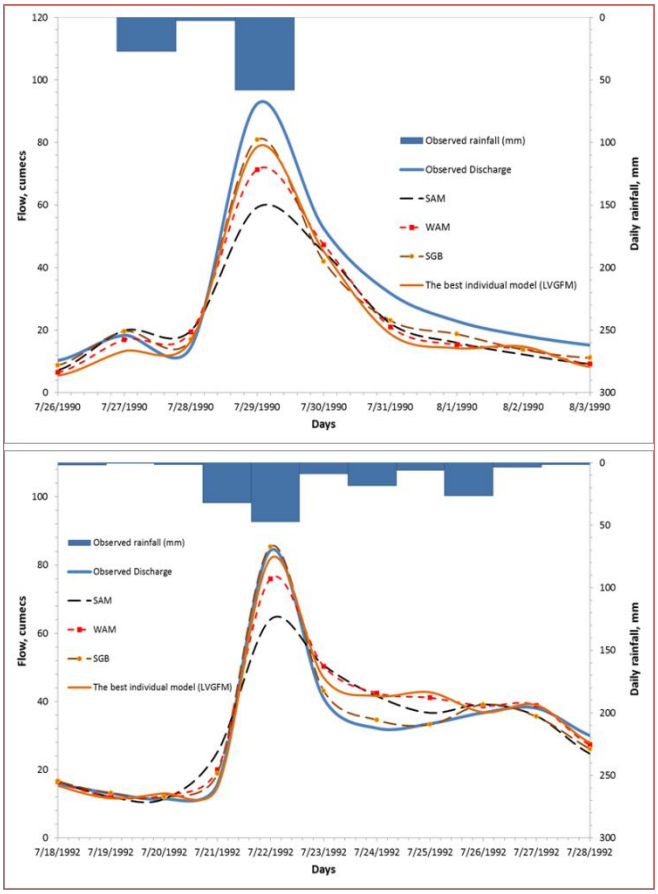


Fig. 6 Comparison of the observed and simulated flood hydrographs of the best model and three combination methods

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REFERENCES

[1] Abrahart, R. J. and L. See (2002). "Multi-model data fusion for river flow forecasting: an evaluation of six alternative methods based on two contrasting catchments". Hydrol. Earth Syst. Sci, 6(4): 655-670. http://dx.doi.org/10.5194/hess-6-655-2002

[2] Ahsan, M. and K. M. O'Connor (1994). "A simple non-linear rainfall-runoff model with a variable gain factor". Journal of Hydrology, 155(1-2): 151-183. http://dx.doi.org/10.1016/0022-1694(94)90163-5

[3] Ajami, N. K., et al. (2006). "Multi-model combination techniques for analysis of hydrological simulations: Application to distributed model intercomparison project results". Journal of Hydrometeorology, 7(4): 755-768. http://dx.doi.org/10.1175/JHM519.1

[4] Armstrong, J. S. (1999). "Forecasting for Environmental Decision Making". V.H. Dale and M.E. English.

[5] Bates, J. M. and C. W. J. Granger (1969). "The combination of forecasts". Journal of the Operational Research Society, (20): 451-468. http://dx.doi.org/10.1057/jors.1969.103

[6] Cavadias, G. and G. Morin (1986). "The Combination of Simulated Discharges of Hydrological Models". Nordic Hydrology, 17: 21-32.

[7] Clemen, R. T. (1989). "Combining forecasts: A review and annotated bibliography". International Journal Forecasting, 5: 559-583. http://dx.doi.org/10.1016/0169-2070(89)90012-5
[8] DHI (2007). MIKE 11 - A modelling system for rivers and channels - Reference Manual. DHI software, 2007.
[9] Exbrayat, J.F., Viney, N.R., Seibert, J., Frede, H.G. and Breuer, L., 2011. "Multi-model data fusion as a tool for PUB: example in a Swedish mesoscale catchment". Adv. Geosci., 29: 43-50. http://dx.doi.org/10.5194/adgeo-29-43-2011
[10] Fernando, A., A. Shamseldin, et al. (2011). "Use of Gene Expression Programing for Multi-Model Combination of Rainfall-Runoff Models." J. Hydro. Eng., 17(9), 975-985. http://dx.doi.org/10.1061/(ASCE)HE.1943-5584.0000533
[11] Friedman, J. H. (1999). "Stochastic Gradient Boosting". Computational Statistics and Data Analysis, 38: 367-378. http://dx.doi.org/10.1061/(ASCE)HE.1943-5584.0000533
[12] Goswami, M. and O'Connor, K.M., 2007. Real-time flow forecasting in the absence of quantitative precipitation forecasts: A multi-model approach. Journal of Hydrology, 334(1-2): 125-140. http://dx.doi.org/10.1016/j.jhydrol.2006.10.002
[13] Granger, C. W. J. and P. Newbold (1977). "Forecasting economic time series". New York, Academic Press.
[14] Gupta, H. V., et al. (2009). "Decomposition of the mean squared error and NSE performance criteria: Implications for improving hydrological modeling". Journal of Hydrology, 377(1–2): 80-91. http://dx.doi.org/10.1016/j.jhydrol.2009.08.003
[15] Kachroo, R.K., Liang, G.C. and O'Connor, K.M., 1988. "Application of the linear perturbation model (LPM) to flood routing on the Mekong River". Hydrological Sciences Journal/Journal des Sciences Hydrologiques, 33(2): 193-214.
[16] Kim, Y. O., D. Jeong, et al. (2006). "Combining rainfall-runoff model outputs for improving ensemble streamflow prediction." J. Hydro. Eng., 11(6): 578-588. http://dx.doi.org/10.1061/(ASCE)1084-0699(2006)11:6(578)
[17] Makridakis, S. and R. L. Winkler (1983). "Averages of forecasts: some empirical results". Management Science, 29(7): 987-996. http://dx.doi.org/10.1287/mnsc.29.9.987
[18] Mandeville, A. N., et al. (1970). "River flow forecasting through conceptual models part III - The Ray catchment at Grendon Underwood". Journal of Hydrology, 11(2):109-128. http://dx.doi.org/10.1016/0022-1694(70)90098-3
[19] Nash, J. E. and B. I. Barsi (1983). "A hybrid model for flow forecasting on large catchments". Journal of Hydrology, 65(1–3): 125-137. http://dx.doi.org/10.1016/0022-1694(83)90213-5
[20] Nash, J.E. and Sutcliffe, J.V., 1970. River flow forecasting through conceptual models:Part 1 - A discussion of principles. Hydrology, 10: 282-290. http://dx.doi.org/10.1016/0022-1694(70)90255-6
[21] Phillip H. Sherrod (2003). DTREG Predictive Modeling Software. www.dtreg.com
[22] Refsgaard, J. C. (1997). "Validation and Intercomparison of Different Updating Procedures for Real-Time Forecasting." Nordic Hydrology 28: 65-84.
[23] Shamseldin, A. Y., et al. (1997). "Methods for combining the outputs of different rainfall-runoff models". Journal of Hydrology, 197(1-4): 203-229. http://dx.doi.org/10.1016/S0022-1694(96)03259-3
[24] Shamseldin, A. Y., et al. (2007). "A comparative study of three neural network forecast combination methods for simulated river flows of different rainfall–runoff models". Hydrological Sciences, 52(5). http://dx.doi.org/10.1623/hysj.52.5.896
[25] Timmermann, A. G. (2005). "Forecast combinations". Centre for Economic Policy Research, 90–98. Goswell Rd, London EC1V 7RR, UK.
[26] Zhang, L., et al. (2009). "Application of the Takagi–Sugeno fuzzy system forcombination forecasting of river flow in semiarid mountain regions". Hydrological Processes, 23(10): 1430-1436. http://dx.doi.org/10.1002/hyp.7265