Simultaneous Optimal Selection of Design and Manufacturing Tolerances with Different Stack-up Conditions using TLBO Algorithm

Ravipudi Venkata Rao, and Kiran Chunilal More

***Abstract*—** Tolerance is one of the most important parameters in design and manufacturing. The allocation of design and machining tolerances has a significant impact on manufacturing cost and product quality. This article presents an analytical model for simultaneously allocating design and machining tolerances based on the least-manufacturing-cost criterion. Traditionally, it is carried out in two phases; CAD and CAPP, in a sequential manner. This approach has the drawbacks of more lead-time and sub optimality. In this study, tolerance allocation is formulated as a non-linear optimization model based on the cost-tolerance relationship. This paper reports on an integrated approach for simultaneous selection of design and manufacturing tolerances based on the minimization of the total manufacturing cost. In this paper, a recently developed advanced optimization algorithm called Teaching-Learning-Based Optimization (TLBO) algorithm is used for optimal selection of design and manufacturing tolerances with an alternative manufacturing process to obtain the global optimal solution. The comparison of the proposed TLBO algorithm is made with the Genetic Algorithm (GA), Simulated Annealing Algorithm (SA) and Scatter search Algorithm (SS). It is found that the TLBO algorithm has produced better results as compared to those obtained by using GA, SA and SS algorithms.

***Keywords*—**Tolerance design, Teaching-Learning-Based Optimization (TLBO), Optimal Selection, RSS criterion.

I. Introduction

THE tolerance design problem becomes more complex in the presence of alternative processes (or machines) for manufacturing of each dimension. This is because the manufacturing cost–tolerance characteristics differ from process to process, and from machine to machine. All costs incurred during a product's life cycle can be divided in two main categories: manufacturing cost, which occurs before the product reaches the customer; and quality loss, which occurs after the product is sold [1]. A loose tolerance (low manufacturing cost) indicates that the variability of product quality characteristics shall be great (high quality loss). Cost of a manufactured dimension in general depends on the tolerance associated with the dimension. Manufacturing cost normally decreases with increase in the tolerance. Several authors have presented/proposed mathematical models to represent the manufacturing cost–tolerance relationship for tolerance design [2-4].

Traditionally tolerance design practice is based on design/manufacturing standards, handbooks and designers' experience without considering the impact of tolerances on manufacturing cost. The resulting set of tolerances usually does not ensure the minimum manufacturing cost. Zhang et al. [5] proposed a mathematical model for the selection of design and manufacturing tolerances. The tolerance optimization model was formulated as a nonlinear multivariable problem. Simulated annealing was applied to solve the model to get the global solution. Later Al-Ansary and Deiab [6] applied genetic algorithm to the same model proposed by Zhang et al. to get more accurate results. However they have considered only the worst-case stack-up criterion for simultaneous allocation of the design and manufacturing tolerances. This criterion results in very tight tolerances and hence leads to the high manufacturing cost.

Singh et al. [7] extended the same model to include other stack-up criteria like root sum square, spotts, and estimated mean shift criteria and solved by using genetic algorithm. Krishna and Rao [8] used the same model to include other stack-up criteria like root sum square, spotts, and estimated mean shift criteria and solved by using Scatter search algorithm. Zhang [9] attempted the problem in a totally different manner, introducing a new concept of interim tolerances that help determine appropriate manufacturing processes and solved the problem using a nonlinear programming technique.

In order to make manufacturing more efficient and economical, they should be implemented for both design and manufacturing. Two types of tolerances are often used: design tolerances and manufacturing tolerances.

The next section presents the details of the optimization modeling of design and manufacturing tolerances.

II. Optimization Modeling of Design and Manufacturing Tolerances

Although a mechanical assembly consists of several individual components/dimensions, only a few affect the functionality of the assembly. The tolerances of these individual dimensions determine the tolerance of the assembly.

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The assembly tolerance obtained in such a manner needs to be in a certain range for proper functioning of the assembly. There exist several methods for estimating the accumulated tolerance, called stack-up conditions. The most commonly used stack-up criteria are as follows:

$$\sum_{i=1}^{n_k} \delta_{id} \leq \Delta X_k \tag{1}$$

$$\sqrt{\sum_{i=1}^{n_k} \delta^2{}_{id}} \leq \Delta X_K \tag{2}$$

$$\frac{1}{2}\left[\sum_{i=1}^{n_k} \delta_{id} + \sum_{i=1}^{n_k} \delta^2{}_{id}\right] \leq \Delta X_K \tag{3}$$

$$\sum_{i=1}^{n_k} m_i \delta_{id} + \frac{Z}{3}\sqrt{\sum_{i=1}^{n_k} \left(1-m_i\right)^2 \delta_{id}^2} \leq \Delta X_K \tag{4}$$

Where, i_d are the design tolerances on the constituent dimensions and n_k is the number of constituent dimensions associated with the k^{th} dimensional chain; mi are the mean shift factors of process distribution; and Y_k is the permissible variation in the assembly dimension, also known as assembly tolerance. Z=3:00 corresponds to 99.73% yield; this value is most commonly considered in analytical treatment.

The advanced optimization techniques like GA, SA, SS etc. need tuning of algorithm-specific parameters. For example, GA requires crossover probability; mutation probability SA requires controlling temperature and Boltzmann constant. SS requires no. of reference sets, reference sets size, Psize. The proper tuning of the algorithm-specific parameters is very essential factor, which affects the performance of those algorithms. The improper tuning of algorithm-specific parameters either increases the computational effort or yields the local optimal solution [13]. Hence to overcome the problem of tuning of algorithm-specific parameters, we have used a recently developed algorithm-specific parameter-less algorithm known as teaching-learning-based optimization (TLBO) algorithm [10, 11, and 12].

The next section presents the details of the TLBO algorithm.

III. Teaching-learning-based optimization algorithm.

TLBO is a teaching-learning process inspired algorithm proposed recently by Rao et al. [13, 44] based on the effect of influence of a teacher on the output of learners in a class. The algorithm mimics the teaching - learning ability of teachers and learners in a classroom. Teacher and learners are the two vital components of the algorithm and describe two basic modes of the learning, through teacher (known as teacher phase) and interacting with the other learners (known as learner phase) [11, 12, and 13].

The output in TLBO algorithm is considered in terms of results or grades of the learners which depend on the quality of teacher. So, teacher is usually considered as a highly learned person who trains learners so that they can have better results in terms of their marks or grades. Moreover, learners also learn from the interaction among themselves which also helps in improving their results. TLBO is a population based method and a group of learners is considered as population and different design variables are considered as different subjects offered to the learners and learners' result is analogous to the 'fitness' value of the optimization problem. In the entire population, the best solution is considered as the teacher. The working of TLBO is divided into two parts, 'teacher phase' and 'learner phase'. Working of both the phases is explained below.

A. *Teacher phase*

It is the first part of the algorithm where learners learn through the teacher. During this phase a teacher tries to increase the mean result of the class room from any value M1 to his or her level (i.e. TA). But practically it is not possible and a teacher can move the mean of the class room M1 to any other value M2which is better than M1depending on his or her capability. Consider Mj be the mean and Ti be the teacher at any iteration i. Now Ti will try to improve the existing mean Mj towards him/her so that the new mean will be designated as Mnew and the difference between the existing mean and new mean is given by,

$$Difference\_mean_i = r_i\left(M_{new} - T_f M_j\right) \tag{5}$$

Where T_f is the teaching factor which decides the value of mean to be changed, and r_i is the random number in the range [0, 1]. Value of T_f can be either 1 or 2 which is a heuristic step and it is decided randomly with equal probability as,

$$T_f = round[rand(0,1)\{2-1\}] \tag{6}$$

Based on this Difference_Mean, the existing solution is updated according to the following expression

$$X_{new,i} = X_{old,i} + Difference\_mean_i \tag{7}$$

II. Learner phase

It is the second part of the algorithm where learners increase their knowledge by interaction among themselves. A learner interacts randomly with other learners for enhancing his or her knowledge. A learner learns new things if the other learner has more knowledge than him or her. Mathematically the learning phenomenon of this phase is expressed below.

At any iteration i, considering two different learners Xi and Xj where i≠ j

If, $f(x_i) < f(x_j)$

$$X_{new,i} = X_{old,i} + r_i(X_{old,i} - X_{old,j}) \tag{8}$$

$$X_{new,i} = X_{old,i} + r_i(X_{old,j} - X_{old,i}) \tag{9}$$

If, $f(x_i) > f(x_j)$

$$X_{new,i} = X_{old,i} + r_i(X_{old,j} - X_{old,i}) \tag{10}$$

$$X_{new,i} = X_{old,i} + r_i(X_{old,i} - X_{old,j}) \tag{11}$$

Accept Xnew if it gives better function value. The equations (8) and (10) are applicable for minimization problems and the equations (9) and (11) are applicable for maximization problems.

It is important to mention that in the basic TLBO algorithm, the solution is updated in the teacher phase as well as in the learner phase [14]. For more details on TLBO algorithm, one may refer to: https://sites.google.com/site/tlborao.

The next section presents the details of a case study.

IV. A CASE STUDY

A case study of a linear mechanical assembly shown in Fig.1 is considered to explain the proposed methodology.

It is required to determine simultaneously the optimum design of manufacturing tolerances in such a way that the total manufacturing cost of the assembly is minimum. The details of the piston-cylinder assembly [8] are as follows:

Piston diameter: 50.8 mm

Cylinder bore diameter: 50.856 mm

Clearance (assembly dimension):0.056±0.0005 mm

Machining process plan for the piston: rough turning, finish turning, rough grinding and finish grinding.

Machining process plan for the cylinder bore: drilling, boring, finish boring and grinding.

The feasible ranges (mm) of the machining tolerances for the piston and cylinder bore are taken from the a fore mentioned reference and are shown in Table I. In the present problem, there is one assembly dimension (i.e.,K=1) and two component dimensions (i.e.,n_K= 2); one is for the piston and another one is for the cylinder. Hence there are two design tolerance parameters. In addition there are four machining tolerance parameters for machining the piston diameter and four more for the cylinder bore diameter. Totally the assembly consists of 10 tolerance parameters or design variables. The design tolerance for a given feature of a component is equal to the final machining tolerance for that feature. This gives $\delta_{1d} = \delta_{14}$ for the piston and $\delta_{2d} = \delta_{24}$ for the cylinder [8].

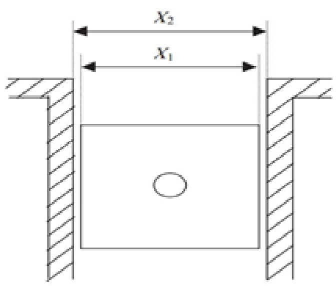

Fig. 1 A piston-cylinder bore assembly where, X1 is the piston diameter, X2 is the cylinder bore diameter and Yis the diametric clearance (assembly dimension)[8]

A. Formulation of the problem

1. Objective function

The objective function is to select design and manufacturing tolerances at which the cost of manufacturing the assembly is minimum. The cost of manufacturing the assembly is determined by summing up the cost of all processes involved in its manufacturing. In the proposed example, a modified form of the exponential cost function [8] is used to represent the relationship between the cost and tolerance of individual manufacturing processes. It is expressed as

$$C(\delta) = a_0 e^{-a1(\delta - a2)} + a_3 \tag{12}$$

Where, the constant parameters are determined from the test data. The parameters a_0, a2, a3 control the position, while a_1 governs the curvature of the cost function. Thus, the total manufacturing cost of the assembly can be expressed as:

$$C_{asm} = C_{11(\delta_{11})} + C_{12(\delta_{12})} + C_{13(\delta_{13})} + C_{14(\delta_{14})} + C_{21(\delta_{21})} + C_{22(\delta_{22})} + C_{23(\delta_{23})} + C_{24(\delta_{24})} \tag{13}$$

C_{asm} is the total assembly manufacturing cost,

Here the first subscript 1 refers to the piston and 2 refers to the cylinder bore; the second subscript refers to the four machining processes as per the respective process plan. Values of the constant parameters [8] for the cost functions of different manufacturing processes are given in Table II.

B. Constraints

The mentioned optimization problem is subjected to the constraints related to both the design and manufacturing tolerances. The design tolerances on different dimensions are based on the functionality considerations of the assembly. The tolerance on the assembly dimensions usually depends on other related dimensions that constitute the dimensional chains.

A design tolerance constraint is formulated to ensure that the accumulated tolerance in the dimensional chain does not exceed the specified tolerance on the assembly dimension. Accumulated tolerance can be estimated by using different approaches, which are applicable under different conditions. Apart from the commonly used worst case and RSS stack up approaches, a few non-traditional tolerance accumulation formulae have also been proposed (equations 14-17). These formulae lead to a set of design tolerance constraints for different assembly dimensions [8].

The constraints of the design tolerances are based on the stack-up conditions and are formulated as:

$$\delta_{14} + \delta_{24} \le 0.001 \qquad \textit{Worst case criterion} \tag{14}$$

$$\delta_{14}^2 + \delta_{24}^2 \le (0.001)^2 \qquad \textit{RSS criterion} \tag{15}$$

$$\frac{1}{2}\left[(\delta_{14} + \delta_{24}) + \sqrt{\left[\delta_{14}^2 + \delta_{24}^2\right]}\right] \le 0.001 \qquad \textit{Spotts criterion} \tag{16}$$

TABLE I
RANGES OF THE PRINCIPAL MACHINING TOLERANCES FOR THE PISTON AND CYLINDER BORE DIAMETER

Piston diameter (mm)			Cylinder bore diameter(mm)		
Notation	Lower limit	Upper limit	Notation	Lower limit	Upper limit
δ_{11}	0.005	0.02	δ_{21}	0.007	0.02
δ_{12}	0.002	0.012	δ_{22}	0.003	0.012
δ_{13}	0.0005	0.003	δ_{23}	0.0006	0.005
δ_{14}	0.0002	0.001	δ_{24}	0.0003	0.002

TABLE II
VALUES OF THE CONSTANT PARAMETERS FOR THE COST FUNCTIONS OF DIFFERENT MANUFACTURING PROCESSES

	a_0	a_1	a_2	a_3
$C_{11}(\delta_{11})$	5	309	5×10^{-3}	1.51
$C_{12}(\delta_{21})$	9	790	2.04×10^{-3}	4.36
$C_{13}(\delta_{13})$	13	3196	5.3×10^{-4}	7.48
$C_{14}(\delta_{14})$	18	8353	2.19×10^{-4}	11.99
$C_{21}(\delta_{21})$	4	299	7.02×10^{-3}	2.35
$C_{22}(\delta_{22})$	8	986	2.97×10^{-3}	5.29
$C_{23}(\delta_{23})$	10	3206	6×10^{-4}	9.67
$C_{24}(\delta_{24})$	2	9428	3.6×10^{-4}	13.12

The manufacturing tolerances are formulated based on Eq.5. The sum of the manufacturing tolerances for a process and its proceeding should be smaller than or equal to the difference of the nominal and the minimum machining allowances. The nominal and minimum machining allowances can be obtained from machining manuals or handbooks. The constraints are formulated [7] as

For piston:

$$\delta_{11} + \delta_{12} \leq 0.02 \quad (18)$$

$$\delta_{12} + \delta_{13} \leq 0.005 \quad (19)$$

$$\delta_{13} + \delta_{14} \leq 0.0018 \quad (20)$$

For cylinder bore:

$$\delta_{21} + \delta_{22} \leq 0.02 \quad (21)$$

$$\delta_{22} + \delta_{23} \leq 0.005 \quad (22)$$

$$\delta_{23} + \delta_{24} \leq 0.0018 \quad (23)$$

V. OPTIMIZATION

Finally the total manufacturing cost of the assembly as represented by equation (13) is optimized subject to the constraints and ranges of the tolerances (process limits). The optimization model for the tolerance design problem is formulated as a multi-variable, non-linear problem subjected to multiple constraints. The total manufacturing cost of the assembly is minimized subjected to the constraints and feasible ranges of the tolerances. The solution vector (X) for the optimization model constitutes eight design variables; δ_{11}, δ_{12}, δ_{13}, δ_{14}, δ_{21}, δ_{22}, δ_{23}, and δ_{24}. The scheme of simple TLBOs as explained above is used as the optimization strategy.

The next section presents the details of the result and discussion.

VI. RESULT AND DISCUSSION

The tolerance allocation result obtained by the TLBO algorithm is shown in Table III-V. For comparison purposes, another optimization algorithm is also employed to solve the example problem. Table VI shows the comparison of the optimum total machining cost obtained by TLBO with other methods like SS and GA for the worst-case criterion, RSS criterion and Spotts criterion.

TABLE III
OPTIMAL TOLERANCES ALLOCATED USING TLBO
(BASED ON THE WORST CASE CRITERIA)

Piston tolerance (mm)		Cylinder bore tolerances (mm)		Minimum manufact-uring cost ($)	CPU time (s)
δ_{11}	0.016255	δ_{21}	0.016274	66.713	37(TLBO) and 350 (GA)
δ_{12}	0.003744	δ_{22}	0.003735		
δ_{13}	0.001255	δ_{23}	0.001274		
δ_{14}	0.000545	δ_{24}	0.000455		

TABLE IV
OPTIMAL TOLERANCES ALLOCATED USING TLBO (BASED ON RSS CRITERIA)

Piston tolerances(mm)		Cylinder bore tolerances(mm)		Minimum manufacturin g cost ($)	CPU time (s)
δ_{11}	0.016156	δ_{21}	0.01616	65.8084	33 (TLBO) and 330 (GA)
δ_{12}	0.003854	δ_{22}	0.00381		
δ_{13}	0.001149	δ_{23}	0.00117		
δ_{14}	0.00066	δ_{24}	0.00060		

TABLE V
OPTIMAL TOLERANCES ALLOCATED USING TLBO
(BASED ON RSS CRITERIA)

Piston tolerances(mm)		Cylinder bore tolerances(mm)		Minimum manufacturing cost($)	CPU time (s)
δ_{11}	0.01606	δ_{21}	0.01585	66.0197	32 (TLBO) and 300 (GA)
δ_{12}	0.00380	δ_{22}	0.00375		
δ_{13}	0.0012	δ_{23}	0.00125		
δ_{14}	0.00060	δ_{24}	0.00055		

It is observed that the solutions obtained by TLBO are superior to those of using genetic algorithms [7] and scatter search algorithm [8] in that TLBO always yields a lower total machining cost. The minimum costs obtained with different criteria are 66.713 (worst-case criterion), 65.80 (RSS criterion) and 66.0197 (spotts criterion) units. In worst case criterion SA [5] having cost 67.21, which is greater result than TLBO. The results indicate that the TLBO technique gives better results than GA, SA and SS algorithms.

TABLE VI
COMPARISON OF OPTIMAL MACHINING COSTS OBTAINED BY TLBO, SS AND GA FOR STACK-UP CONDITIONS.

Sr. No.	Stack-up condition	Optimal machining cost($)		
		Present work (TLBO)	SS Algorithm [8]	Genetic algorithm [7]
1	worst case criterion	**66.713**	66.77	66.85
2	RSS criterion	**65.8084**	65.97	65.92
3	Spotts criterion	**66.0197**	66.08	66.23

* The number in bold indicate the better values.

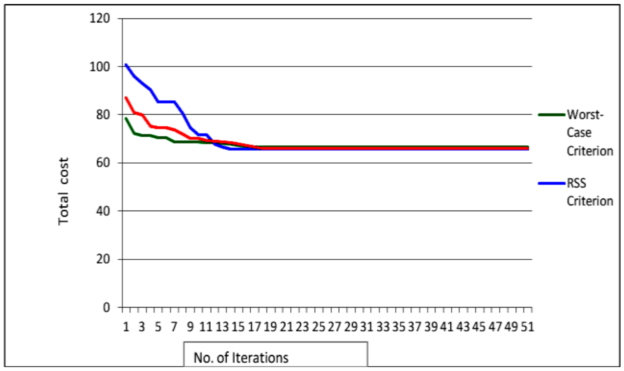

Fig. 2 Convergence for the worst-case criterion, RSS criterion and Spotts criterion obtained by TLBO algorithm for Engine assembly

Also, the computational time to find the optimum solutions in TLBO is taking only approximately $1/10^{th}$ of that of GA algorithms. Hence, the TLBO algorithm is proved better than the other optimization algorithms in terms of results and the convergence. Figure 2 shows the convergence rate of the TLBO algorithm for the worst-case criterion, RSS criterion and spotts criterion. For this problem we used 100 numbers of population and 50 numbers of generation. A personal computer using MATLAB2011b with 2 GB ram and 1.2 GHz processor.

VII. Conclusion

Tolerance design is a very important issue in product development. Conventionally tolerance design is carried out in two steps, CAD and CAPP, in a sequential manner. The approach suffers from several drawbacks such as more time consumption, suboptimality and an unhealthy working atmosphere. To overcome the drawbacks of this approach, an attempt was made at the simultaneous selection of optimal design and manufacturing tolerances. The approach of the TLBOs has been briefly explained and proposed for global optimization.

Although several conventional optimization techniques have been applied to solve tolerance design problems, their application is often limited because of the probability of getting stuck at local-optimal points and lack of robustness. The TLBO algorithm may be conveniently used for the optimal tolerance design of the other machine element Furthermore, as observed in the present study, TLBO can obtain superior solutions to other metaheuristics. TLBOs are a very powerful optimization tool and we are working to explore their application in complex tolerance optimization problems with different non-traditional cost functions.

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