Optimal Sizing and Placement of Distributed Generators for Profit Maximization Using Firefly Algorithm

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Abstract—This paper presents a firefly based algorithm for optimal sizing and allocation of distributed generators for profit maximization. Distributed generators in the proposed algorithm are of photovoltaic (PV) and combined heat and power (CHP) technologies. Combined heat and power distributed generators are modeled as voltage controlled nodes while photovoltaic distributed generators are modeled as constant power nodes. The proposed algorithm is implemented in MATLAB environment and tested the unbalanced IEEE 37-node feeder. The results show the effectiveness of the proposed algorithm in optimal selection of distributed generators size and site in order to maximize the total system profit.

Keywords—Distributed generators, firefly algorithm, IEEE 37-node feeder, profit maximization

I. INTRODUCTION

Distributed generation incorporation to the distribution networks attracted the researchers’ attention in the last few decades. Distributed generators (DGs) improve the voltage profile, reduce the power losses, enhance the system reliability and delay the transmission and distribution networks investments. Hence, it reduces the system cost and increases its profit.

Optimal location and capacity is the main issue of studies done on DGs as the proper sizing and placement of DGs allow the system operator to gain the maximum benefits from the DGs, on the other side improper placement or sizing of DGs may cause undesirable effects. The search space of optimal location and capacity of DGs is wide, different optimization methods are used in this field for the sake of power loss minimization, cost reduction, profit maximization and environmental emission reduction. The optimization methods could be analytical, numerical and heuristic. Reference [1] presents a fuzzy genetic algorithm (GA) is used to find the optimal location and size of single DG in order to minimize the system cost and active power losses. A heuristic cost-benefit approach for optimally select the DGs size and site to serve peak demands optimally in a competitive electricity market is introduced in [2]. An integrated distribution network planning model, implementing optimal DG power and location as an alternative option is presented in [3]. The distribution network capacity for the connection of DG is computed by an optimal power flow formulation that is solved by an interior point method [4]. Sequential Quadratic Programming is applied to solve optimization problem with fault level constraints in [5]. Gradient search for the optimal sizing of DGs in meshed networks considering fault level constraints is proposed in [6]. Linear Programming is used to solve the optimization problem for achieving maximum DG energy harvesting [7]. Ordinal Optimization method is developed in [8] for specifying the locations and sizes of multiple DGs such that a tradeoff between loss minimization and DG capacity maximization is achieved. GA is used to solve an optimization problem that considers constant power concentrated loads [9]. A GA is employed to solve optimization problem that maximizes the profit of the system by the optimal placement of DGs [10].

Firefly Algorithm (FA) was first introduced by Xin-She Yang [11] for solving nonlinear multidimensional optimization problems. Few publications utilize the firefly method for optimizing the power flow problem [12-15]. In [12] the authors applied the FA algorithm to solve economic dispatch problem. The results were compared with continuous genetic algorithm to show the effectiveness of FA. Authors in [13], [14] use FA to evaluate the optimal location and size of one or two distributed generators on a balanced radial feeder for power loss minimization. In [15] an application of FA on optimal allocation of DG based on real and reactive power losses and voltage profile optimization for different load models.

This paper presents a firefly based method for finding the optimal location and capacity of combined heat and power (CHP) DG as well as photovoltaic DG connected to unbalanced distribution feeders for profit maximization. The CHP DG in the proposed algorithm is modeled as voltage controlled (PV) node with the flexibility to be converted to constant power (PQ) node in case of reactive power limit violation while the photovoltaic DG is modeled as PQ node as...
II. PROBLEM FORMULATION

In this paper, the objective is carrying out an optimal dispatch that the operator of a distribution network is able to determine its economical optimum in consideration of the relevant technical constraints. The objective function is to maximize the total profit of the distribution company for participating in day-ahead market for 24 hours period as formulated in the following equations.

\[
\text{Daily Profit} = \text{Daily revenue} - \text{Daily cost} \tag{1}
\]

\[
\text{Daily revenue} = \sum_{i=1}^{24} \sum_{h=1}^{24} E_{i,h} \lambda_h \tag{2}
\]

\[
\text{Daily cost} = \sum_{d=1}^{n_{DG}} E_{dg} C_{op} + \sum_{h=1}^{24} E_{loss} \lambda_h + C_{op,f} + E_{sub} C_{pur} + P_{inst} C_{inst} R / 365 \tag{3}
\]

Where, \( h \) is the number of hours, \( n \) is the number of system buses, \( n_{DG} \) is the total number of installed DGs, \( E_{i,h} \) is the \( i \)th Load hourly energy (kWh), \( \lambda_h \) is the energy price €/kWh, \( E_{dg} \) daily DG energy (kWh), \( C_{op} \) is the DG variable operating cost (€/kWh), \( E_{loss} \) is the hourly energy loss (kWh), \( C_{op,f} \) is the fixed operating cost (€), \( E_{sub} \) is the daily energy purchased from substation (kWh), \( C_{pur} \) is the Price of purchased energy (€/kWh), \( P_{inst} \) is the DG installed Capacity (kW), \( C_{inst} \) is the investment cost of DG (€) and \( R \) is the interest and depreciation rates (%). Subjected to the following constraints:

\[
0.9 \ p.u. \leq V \leq 1.1 \ p.u. \tag{4}
\]

- DG power limits:
  - Active, reactive and complex powers of the DG unit lie between min. and max. Power and this range should not be violated.

\[
0 \leq P_g \leq P_{gmax} \tag{5}
\]

\[
Q_{gmin} \leq Q_g \leq Q_{gmax} \tag{6}
\]

\[
0 \leq S_g \leq S_{load} \tag{7}
\]

In the proposed method DG maximum active power can be varied until its complex power reach its maximum limit which is the sum of loads complex power. The CHP DG power factor is bounded between 0.707 lead to 0.707 lag, hence, the reactive power varies as follow:

\[
-P_g \leq Q_g \leq P_g \tag{8}
\]

- Lines thermal limit (line Ampacity):

\[
I_{flow} \leq I_{thermal} \tag{9}
\]

- Substation limit:
  - This constraint represents the maximum apparent power that the substation can provide.

\[
S_{substation flow} \leq S_{substation max} \tag{10}
\]

- Power balance:
  - The sum of input power should be equal to the sum of output active power in addition to the active power loss. The input power may include the DG active power and the active power supplied by the utility. The active output power is the sum of loads active power.

\[
P_{substation} + \sum P_{DG} = \sum P_{loads} + P_{loss} \tag{11}
\]

III. METHODOLOGY

The proposed algorithm is composed of two major steps; the first step is developing an unbalanced load flow. The second step is applying the Firefly algorithm to determine the DG location and size in order to minimize the system operating cost.

The load flow algorithm is used to integrate multiple DG in the distribution network based on the backward forward sweep method. The detailed mathematical model of the distribution network can be found in [16] and [17]. Depending on the reactive power capability of DG it can operate in one of the following modes, to output power at specified power factor or to output power at a specified terminal voltage. Photovoltaic panels cannot supply reactive power to control the output voltage, this means that the generation node in this as is represented as constant power (PQ) node or constant negative load with current injection into the node. CHP DG can supply required reactive power, hence the generator node in this case must be modeled as voltage controlled (PV) node. When modeled as PV node DG behaves as voltage dependent current source as the amount of reactive current injection depends on the difference between the voltage magnitude of the PV node and the scheduled value, steps for modeling DG can be summarized in [18].

The firefly optimization algorithm is inspired from the natural behavior of the fireflies; a firefly of the maximum brightness has the largest ability to attract other fireflies regardless to their sex. The brightness of a firefly is affected or determined by the landscape of the objective function. For a maximization problem, the brightness can simply be proportional to the value of the objective function [11], whilst for a minimization problem; the brightness is inversely proportional to the value of the objective function.

The following steps describe the firefly algorithm:

Step (1): Generate a set of random fireflies bounded inside...
certain preset region. Two types of fireflies are used, the first represent the DG locations and the second represent the DG powers.

Step (2): Evaluate the brightness of each firefly based on the type of the objective function.

Step (3): Calculate the distances between all fireflies, the distance between two fireflies $i, j$ for a $d$ dimensional problem can be calculated as follow:

$$ r_{ij} = \sqrt{\sum_{k=1}^{d} (x_{i,k} - x_{j,k})^2} $$

(12)

Step (4): Move the less attraction (brightness) firefly $i$ towards the more attractive firefly $j$ by using the following equation:

$$ x_i = x_i + \beta_0 \exp(-\gamma r_{ij}^2) \times (x_j - x_i) + \alpha (\text{rand} - \frac{1}{2}) $$

(13)

Where the first term is the current firefly position, the second term is used to update the firefly position based on the brightness of the fireflies and the third term is used to randomize the movement of firefly. $\beta_0$ is the initial attractiveness, $\gamma$ is the absorption coefficient , the values of these parameters are determined according to the optimization problem. $\alpha$ is a randomization parameter that decrease at each iteration by equation (14) and rand is a random number generator uniformly distributed between $[0,1]$.

$$ \alpha^{k+1} = \alpha^k \left( \frac{1}{2K_{\text{max}}} \right)^{1/K_{\text{max}}} $$

(14)

Step (5): Evaluate the brightness of the fireflies at the updated positions by calculating the objective function from equations (1)-(3) and repeat steps (3), (4), (5) until reach the maximum number of iterations.

IV. SYSTEM DESCRIPTION

Fig.1 shows the general layout of an IEEE 37 node distribution feeder which is an actual feeder in the state of California [19]. This feeder is complex as it is characterized by:

- Spot loads, single phase and three phases balanced and unbalanced loads, delta connected loads, constant active and reactive power, constant impedance, and constant current type loads.
- Three phase overhead and underground lines with different phasing of spaces.

The regulator was removed in order to clearly evaluate the effects of the DG on the system voltage profile for the IEEE 37 node feeders. The substation node is numbered as (0) as it is the reference node which has a constant voltage of 1 per-unit, the numbering of the other nodes is done in ascending order. Whenever a lateral branches off of the main feeder the lateral is indexed before returning to the main feeder.

V. TEST CASES AND RESULTS

The proposed algorithm is implemented in MATLAB environment and tested on the IEEE 37-node feeder to maximize the distribution system operator daily profit. The costs and prices used in the proposed algorithm are presented in Table I [20]-[21].

<table>
<thead>
<tr>
<th><strong>TABLE I</strong></th>
<th>MAIN ECONOMICAL PARAMETERS ASSUMED TO EVALUATE THE ELECTRICITY GENERATION COST</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Investment cost of the CHP (€/kW)</strong></td>
<td>1,117</td>
</tr>
<tr>
<td><strong>Fixed maintenance and operation cost of the CHP (€/year)</strong></td>
<td>14,896</td>
</tr>
<tr>
<td><strong>Variable maintenance and operation cost of the CHP (€/kWh)</strong></td>
<td>0.006</td>
</tr>
<tr>
<td><strong>Investment cost of the PV (€/kW)</strong></td>
<td>2,500</td>
</tr>
<tr>
<td><strong>Yearly Maintenance and operational cost of the PV (€/kW)</strong></td>
<td>29.4</td>
</tr>
<tr>
<td><strong>Interest and depreciation rates</strong></td>
<td>6%</td>
</tr>
<tr>
<td><strong>Price of purchased energy from substation (€/kWh)</strong></td>
<td>0.11</td>
</tr>
<tr>
<td><strong>Price of energy sold to consumers (€/kWh)</strong></td>
<td>0.21</td>
</tr>
</tbody>
</table>

In the presented study it was assumed that the CHP unit is of 2000 KW capacity and the PV panel is of 1000 KW installed capacity. Five test cases were done on the system under study; one CHP DG was installed with constant load scaling factor (LSF) and with variable daily LSF, one Photovoltaic DG was installed with fixed and variable LSF. Three different types of loads residential, commercial and industrial are used as shown in Table II; the average hourly values of the load scaling factor curve is presented in Fig.2.

A. Test case (1): CHP DG, fixed load scaling factor

One CHP DG is connected to the feeder under test and the LSF is kept constant at 100%, the optimal location, capacity of the CHP unit for profit maximization and the maximum daily profit is presented in Table 3, Fig. 3 shows the convergence characteristics of the proposed method.
### TABLE II
**SYSTEM BUSES TYPES**

<table>
<thead>
<tr>
<th>System</th>
<th>Nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential</td>
<td>5, 9, 15, 21, 23, 30</td>
</tr>
<tr>
<td>Industrial</td>
<td>1, 4, 6, 10, 12, 17, 18, 19, 26, 29, 31, 33, 35</td>
</tr>
<tr>
<td>Commercial</td>
<td>8, 14, 20, 27, 32, 36</td>
</tr>
</tbody>
</table>

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**Fig. 2 Typical buses load curve**

### TABLE III
**CHP OPTIMAL LOCATION AND MAX. PROFIT**

<table>
<thead>
<tr>
<th>Optimal location</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHP active power (kW)</td>
<td>2000</td>
</tr>
<tr>
<td>CHP reactive power (kVar)</td>
<td>724.3</td>
</tr>
<tr>
<td>CHP capacity (kVA)</td>
<td>2130</td>
</tr>
<tr>
<td>Maximum profit (€)</td>
<td>10286</td>
</tr>
</tbody>
</table>

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**Optimal location**

- Residential nodes: 5, 9, 15, 21, 23, 30
- Industrial nodes: 1, 4, 6, 10, 12, 17, 18, 19, 26, 29, 31, 33, 35
- Commercial nodes: 8, 14, 20, 27, 32, 36

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**B. Test case (II): photovoltaic DG, fixed load scaling factor.**

One photovoltaic DG is connected to the feeder under test, the photovoltaic data presented in Fig.4 represent actual data of photovoltaic panel installed at the German University in Cairo (GUC) the LSF is kept constant at 100%, the optimal location, energy of the photovoltaic unit for profit maximization and the maximum daily profit is presented in Table IV. It is clear that the CHP unit is more effective in daily profit maximization because of its greater capacity and its capability to supply reactive power.

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**Fig. 3 Profit convergence**

### TABLE IV
**PHOTOVOLTAIC OPTIMAL LOCATION AND MAX PROFIT.**

<table>
<thead>
<tr>
<th>Optimal location</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photovoltaic daily energy (kWh)</td>
<td>8815</td>
</tr>
<tr>
<td>Maximum daily profit (€)</td>
<td>4776.69</td>
</tr>
</tbody>
</table>

---

**C. Test case (III): CHP DG, variable load scaling factor.**

A CHP DG is connected to the feeder under test and the load scaling factors varied as presented in Fig.2, the optimal location; daily DG energy and maximum profit are presented in Table V.

### TABLE V
**CHP OPTIMAL LOCATION AND MAX PROFIT.**

<table>
<thead>
<tr>
<th>Optimal location</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum daily profit (€)</td>
<td>9264</td>
</tr>
</tbody>
</table>

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**D. Test case (IV): One photovoltaic DG, variable load scaling factor.**

One photovoltaic DG is installed and the load scaling factors varied as presented in Fig.2, the daily profit versus the DG location is presented in Fig.5, the optimal location, daily DG energy and maximum profit are presented in Table VI.

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**Fig. 4 Photovoltaic power**

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A firefly based method that determines accurately the optimal location, capacity of a CHP technology DG or a photovoltaic DG with fixed or variable loads for profit maximization is proposed and examined. The method is applied on unbalanced distribution feeder and the results show the effectiveness, high speed of convergence and accuracy of the proposed method.

VI. CONCLUSION

REFERENCES


http://dx.doi.org/10.15242/IIE.E0614014