# An Efficient Robust Aerodynamic Design Optimization under Aleatory Uncertainty

Muhammad Shahbaz, Zhong-Hua Han, Jun Liu, and Wen-Ping Song

***Abstract*****—**This paper presents an efficient uncertainty quantification (UQ) method based on in-expensive Monte Carlo (IMC) method to reduce the statistical parameters of objective function (e.g. $C_d$) subject to aerodynamic and geometric constraints. The robustness of a candidate design is evaluated by IMC approach with kriging surrogate model for estimating statistics and probability density functions of output response quantities. An in-house code, PMNS2D, is used for CFD calculations of transonic flows over RAE2822 airfoil. Input parameters such as free stream Mach number and angle of attack are varied and the propagation of their corresponding aleatory uncertainties on output properties of interest is studied. The drag coefficient of the robust airfoil is insensitive to variations of uncertain parameter. and reduction in objective function is 35.8% and 35.3% for Mach and alpha uncertainty respectively. The small fluctuations in $C_d$ over uncertainty range means that the performance of robust design is insensitive to uncertain operating conditions.

***Keywords*****—**Robust aerodynamic design, Inexpensive Monte Carlo, Uncertainty Quantification, Surrogate Model, Airfoil design.

## I. Introduction

TRADITIONALLY aerodynamic design optimization (ADO) is formulated as a deterministic optimization problem at the given operating conditions. Practically, aircraft cannot operate exactly at the certain conditions. The operating conditions, such as angle of attack, Mach number, and Reynolds number may fluctuate during flight. These parameters are treated as constant value in conventional design. Such type of uncertain design parameters are inevitable in ADO and must be considered to produce reliable optimal solution (high performance and low sensitivity). In last two decades, various non-deterministic methods have been developed to cater uncertainty i.e., reliability based method and robust design method. Taguchi [1] developed the foundation of robust design in 1950's and applied his methods in electronics & automotive products. The Taguchi robust design method aims to look for a solution with better mean performance and less variance. Reference [2] provides the detailed review of robust optimal design for Taguchi method. In aerodynamic design, the research on robust design optimization (RDO) can find its origin from the multi-point aerodynamic optimization. A simplified version of airfoil problem was studied in 1977 [3] where it was observed that single point optimization (SPO) perform well at one design point but have poor off-design characteristics. To overcome this issue, Drela proposed multi-point optimization method [4] for drag minimization of an airfoil. The shortcomings of multi-point optimization were discussed in [5] and robust optimization method was proposed for airfoil optimization [6]. A maximum expected value (MEV) criteria based on Von Neumann decision theory [7] was utilized to formulate the robust design [8] but MEV only considered the minimization of the mean value of objective function and didn't address the variability. A second criterion of minimizing the variance was coupled with MEV to formulate the robust design. Now, robust design is extensively being used in aerodynamics. The ADO under uncertainty can be considered as a robust design optimization in which solution is sought that is relatively insensitive to small changes in uncertain quantities. The main objective of RDO is to optimize the mean performance while minimizing the variation caused by various uncertainties.

Uncertainty is ubiquitous in aerodynamic design. The needs and opportunities for uncertainty based design for aerospace vehicles were given in [9]. Oberkampf [10] defined Uncertainty as "potential deficiency in any phase or activity of modeling process that is due to lack of knowledge". Two types of input uncertainty can be considered in RDO studies i.e., inherent (aleatory) uncertainty and model-form (epistemic) uncertainty. Aleatory uncertainty, being probabilistic and irreducible, describes the inherent variability of the physical system and arises due to natural and unpredictable variations in operating conditions). Epistemic uncertainty is reducible and described as unavailability or lack of information in any phase of design process. In uncertainty quantification (UQ), probability theory is mostly used to cater input aleatory uncertainty, which often can be well represented by probability density function with sufficient information on the type of distribution such as normal and uniform. UQ analysis has got keen interest in these days [11][12]. Different methods are available for uncertainty propagation. The most straight forward and accurate method is a full nonlinear Monte Carlo (MC) simulation, but it is prohibitively expensive for high-fidelity CFD computations. Another method to assess

Muhammad Shahbaz is Graduate Student in School of Aeronautics, Northwestern Polytechnical University, Xi'an,710072, P. R. China. (0086-15686491401; e-mail:engr_ch_shahbaz@yahoo.com).

Zhong-Hua Han, is professor in School of Aeronautics, Northwestern Polytechnical University, Xi'an, 710072, P. R. China. (e-mail: hanzh@nwpu.edu.cn).

Jun Liu, is PhD candidate in School of Aeronautics, Northwestern Polytechnical University, Xi'an, 710072, P. R. China. (e-mail: 470381562@qq.com)

Wen-Ping Song is professor in School of Aeronautics, Northwestern Polytechnical University, Xi'an, 710072, P. R. China. (e-mail: wpsong@nwpu.edu.cn).

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uncertainty is moment method [12], [13] based on Taylor series expansion. An alternate approach for uncertainty propagation is the use of surrogate with Monte Carlo simulation. The surrogate models are used to replace expensive high-fidelity CFD simulations with an analytical model which is constructed through selective sampling of the high-fidelity model. A Monte Carlo efficiently used with surrogate model, is often referred to as inexpensive Monte Carlo (IMC) simulation approach. Its estimated function values can be used for MC simulation to obtain not only mean and standard deviation, but also an approximate probability density function (PDF) of the output function. This approach is promising for the extension to RDO studies in aerodynamics [14]-[17] because of its low computational cost for uncertainty analysis. The construction of an accurate surrogate model is necessary for uncertainty propagation through IMC. The kriging model [18]-[26] performs well in aerospace engineering and predicts the function value by using stochastic processes, and has flexibility to represent multimodal/nonlinear functions. An efficient kriging based optimization is established [26] and compared with RSM-based optimization which shows that Kriging-based method outperforms RSM-based method both for efficiency and efficacy.

In this research, kriging-based inexpensive Monte Carlo method is applied to a RDO problem of an airfoil using kriging -based optimization method developed in [26]. The uncertain Mach number and angle of attack are supposed to follow normal distribution. A low cost IMC uncertainty analyses on the kriging models are utilized to evaluate the mean and standard deviation of drag coefficient. The RDO is carried out using Genetic Algorithm (GA) based optimizer, *SurroOpt*.

## II. Uncertainty Quantification Method

Uncertainty quantification is a major task in RDO which is used to characterize the uncertainty of the output, when the input uncertainties are given. The goal of UQ is to determine how random variation (aleatory) affects the sensitivity, performance, or reliability of the system that is being modeled. Among different uncertainty propagation methods, Monte Carlo simulation (MCS) and inexpensive Monte Carlo (IMC) methods are described briefly in this section.

### *a. Monte Carlo Simulation (MC)*

Monte Carlo simulation method [27] has been extensively used in uncertainty analysis since 1940s. The probability distribution of the output of a process induced by the probability distribution of stochastic inputs is obtained by performing $N$ repetitions of the process. For an uncertainty analysis, main goal is to obtain the mean and variance of the function $f$ with respect to the random variable $x$. The first two statistical moments are as under.

$$\mu_f = \int_{\Omega} f(x)\, p(x) d(x), \tag{1}$$

$$\sigma_f^2 = \int_{\Omega} (f(x) - \mu_f)^2\, p(x) d(x), \tag{2}$$

where $\Omega$ and $p$ are the range and PDF of the random variable $x$.Taking $N$ sample points for $x$ by following the distribution of $p$, the mean and standard deviation of $f$ can be estimated by MC simulation as follows.

$$\mu_f \approx \frac{1}{N} \sum_{i=1}^{N} f(x_i), \tag{3}$$

$$\sigma_f^2 \approx \frac{1}{N-1} \sum_{i=1}^{N} (f(x_i) - \mu_f)^2, \tag{4}$$

Equations (3) and (4) can be used for random samples, $N$ which follow any type of distribution i.e., uniform or normal. The PDF of $f$ can be predicted by $f(x_i)$. The standard deviation of the predicted mean from its exact value is $O(M^{-1/2})$. Although, it is the most popular method but it requires large number of CFD performance evaluations for obtaining accurate results. Due to which its computational cost is very large especially with the increase in the number of random variables.

### *B. Inexpensive Monte Carlo Based on Surrogate Model*

The concept of the IMC is similar to that of MC simulation. However, the huge numbers of exact CFD evaluations are replaced by a cheaper surrogate model. The most appropriate choice among various surrogate model is kriging which is a statistical interpolation method proposed by Krige in 1951. Kriging was widely used in geo-statistical problems and it was extended by Sacks [18] for the design and analysis of deterministic computer experiments. In this work, kriging model(s) are built to quantify the uncertainly of each candidate airfoil, with uncertain flow condition parameters (such as Mach number and angle of attack) as variables. These surrogate models are further used in MC simulation method to estimate the mean and standard deviation of aerodynamic functions. The process of uncertainty quantification is shown in Fig. 1.

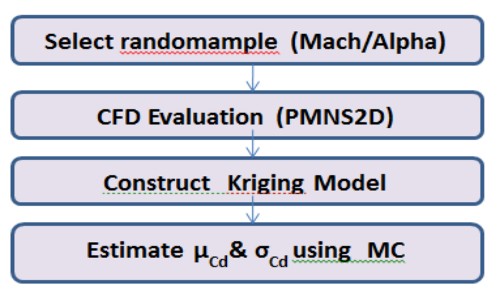


Fig. 1. Procedure of uncertainty quantification for a candidate airfoil using IMC

The reference [26] provides the detailed description of kriging model used in this research. Additional samples are used to verify the accuracy of kriging surrogate. Here two standard performance metrics are used to evaluate the

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performance 1) Root mean square error (RMSE), 2) Maximum absolute error (MAE). They are represented as follows.

$$RMSE = \sqrt{\frac{1}{n_t}\sum_{k=1}^{n_t}((f(x^k) - \bar{f}(x^k))^2} \tag{5}$$

$$MAE = \max_k |(f(x^k) - \bar{f}(x^k)| \tag{6}$$

where $f(x^k)$ is the exact function value for test point $x^k$, $\bar{f}(x^k)$ corresponding estimated function value and $n^t$ is the total number of test points. Seventy samples have been selected as test points and 3, 5, 10, 15, 20, and 25 are taken as training points. The surrogate model attained the desired accuracy by selecting 20 samples of uncertain parameter.

## III. Computational Fluid Dynamic Model

### a. *Airfoil Geometry*

The airfoil geometry for single point deterministic problem and robust design problem is selected as supercritical transonic airfoil RAE2822. The geometry of the RAE 2822 airfoil is described by the design airfoil coordinates [28] with a maximum thickness–to–chord ratio (t/c) of 0.1214.

### B. *Grid Generation*

Computational grid for an airfoil is generated using in-house code called "*Foilgrid*". The domain boundaries are placed at distance of 18 chord length around the airfoil. The computational meshes are of structured curvilinear body fitted C-topology with elements clustering around the airfoil. The RAE2822 airfoil is designed for a free stream Mach number, $M_\infty$ = 0.66 and lift coefficient, $C_l$ = 0.56 at angle of attack, α = 1.06º. The off–design nominal flow conditions considered in this study corresponds to free stream Mach number, $M_\infty$ = 0.73, angle of attack, α= 2.79º, and Reynolds number, Re = $6.5\times10^6$. A mesh for RAE2822 airfoil is shown in Fig. 2. A grid with 20,865 mesh cells is used for robust airfoil optimization.

### C. *Flow Solver*

The flow analyses are performed with in-house code, called "*PMNS2D*". It solves the Reynolds-Averaged Navier-Stokes (RANS) equations to simulate the flow around the airfoil. The equations are solved on structured meshes using the cell - centered finite-volume approach. The Spalart-Allmaras one-equation turbulence model is used for turbulence closure. The second-order Jameson's central scheme is used as spatial scheme. Implicit residual smoothing, local time-stepping and multi-grid techniques are used to accelerate the solution to converge to the steady state. The pressure coefficient ($C_p$) comparison with experimental data shown in Fig. 3 depicts that grid as well as flow solver is capable to capture flow phenomenon and can be used for aerodynamic optimization.

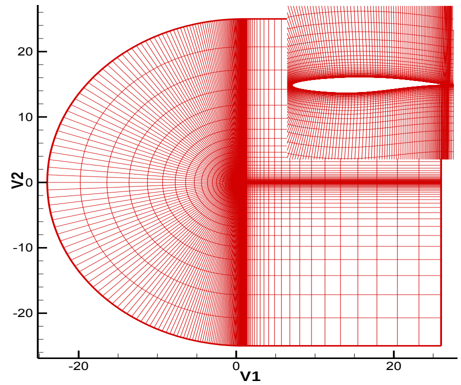



Fig. 2. Computational grid for RAE 2822( 20,865 cells)

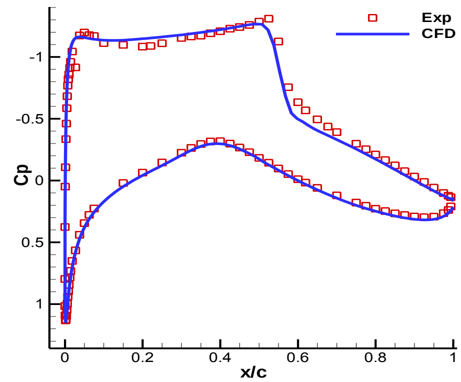



Fig. 3. Cp comparison at M∞ =0.73 and α=2.79º

## IV. Surrogate Based Optimization

An efficient surrogate-based optimization toolbox called "*SurroOpt*" which can solve any single objective and/or, multi-objective optimization problems is used in this research. The *SurroOpt* code includes several types of design of experimental (DoE) methods such as Latin hypercube sampling (LHS) [29], uniform design (UD) [30] etc; surrogate models such as quadratic response surface, kriging models (its variants such as gradient enhanced kriging, and radial-basis functions, etc; and traditional optimizers such as genetic algorithm (GA), BFGS, SQP, pattern search etc. In this research, the LHS, kriging model, and GA are used respectively. Two cases are addressed in this section 1) Deterministic single point optimization, 2) In-deterministic RDO under aleatory uncertainty of Mach number or angle of attack. The airfoil is parameterized by Class/Shape transformation function (CST) [31] for both cases. The RAE2822 is selected as the baseline airfoil. Design space is chosen within ±25% of baseline airfoil with 14 design variables on airfoil as shown in Fig. 4. Two cases are described as follows.

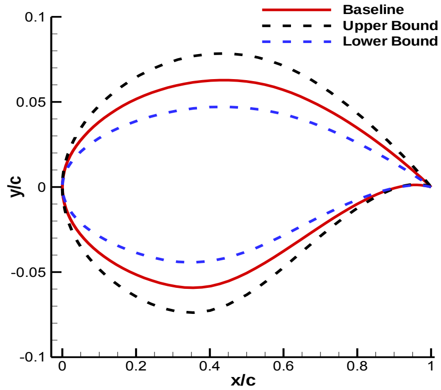



Fig. 4. Baseline airfoil with ±25% upper and lower bounds

*a. Case 1: Deterministic Single Point Optimization*

In this case, deterministic single point optimization is performed for drag minimization of RAE2822 airfoil subject to aerodynamic and geometric constraints. The optimization has been carried out at Mach number, $M_\infty$=0.73, angle of attack, α=2.79º and Reynolds number, Re = 6.5×$10^6$ using *SurroOpt*. The problem formulation is as follows.

$$\begin{aligned} &Objective\ function: Minimize\ C_d \\ &st: Thickness \geq Thickness_0 \\ &C_l \geq C \\ &|C_m| \leq |C_{m0}|, \end{aligned} \tag{6}$$

where $C_{l0}, C_{m0}, and\ Thickness_0$ corresponds to lift coefficient, pitching moment and maximum thickness of baseline airfoil, respectively.

Table 1 shows the results of an optimized airfoil which depicts that the drag reduction of an optimum airfoil is quite reasonable (31.67%), satisfying the required constraints. The comparison of optimum airfoil geometry with baseline is shown in Fig. 5. The $C_p$ distribution comparison in Fig. 6 shows that the strong shock of the baseline is dramatically weakened and a smooth $C_p$ distribution is obtained for optimized airfoil.

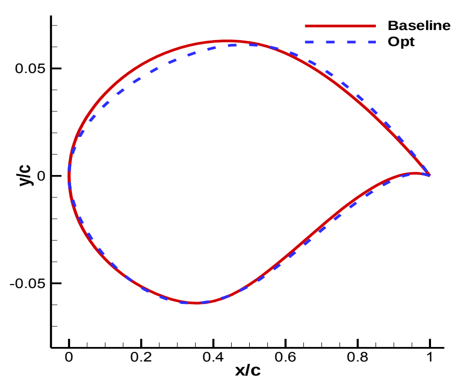



Fig. 5. Comparison of baseline airfoil and SPO airfoil

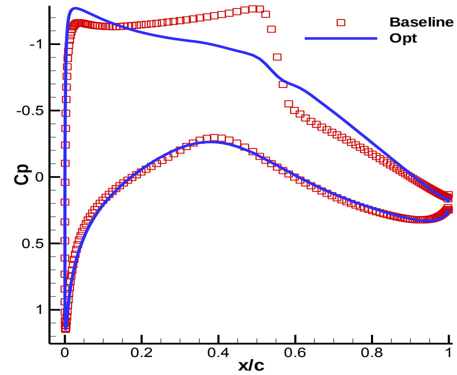



Fig. 6. Comparison of pressure coefficient between baseline and SPO airfoil at M∞=0.73 and α=2.79º

TABLE I
COMPARISON OF SINGLE POINT OPTIMIZATION RESULTS WITH RAE2822

| | $C_d$ | $C_l$ | $\lvert C_m \rvert$ | Thickness |
|---|---|---|---|---|
| RAE2822 | 0.01607 | 0.79562 | 0.09202 | 0.1214 |
| Optimized | 0.01098 | 0.796 | 0.09184 | 0.12265 |
| % diff | -31.67% | +0.047% | -0.195% | +1.02% |

*b. Case 2: Robust Design Optimization under Uncertainty*

The main objective of the robust design optimization under aleatory uncertainty is to reduce the mean and standard deviation of drag coefficient simultaneously to obtain the airfoil shape with minimum drag. Robustness can be achieved by minimizing the variance of $C_d$. In this research, Mach number or angle of attack are considered as uncertain parameters which represent normal distribution. For the robust airfoil optimization, two cases are presented. 1) RDO with Mach uncertainty; 2) RDO with an angle of attack uncertainty. Uncertainty is propagated through surrogate based inexpensive Monte Carlo simulation. Three constraints are imposed such as thickness, lift coefficient, and pitching moment coefficient. Robust design optimization was performed using *SurroOpt*. Following are the two cases which are discussed here.

i. RDO with Mach-Number Uncertainty

For Mach number uncertainty, the range of random Mach number sample for stochastic design space is set as -4 ≤ ξ ≤ +4 assuming normal uncertainty with mean, $\mu_M$ =0.73 and standard deviation, $\sigma_M$=0.005. A distribution of random Mach number is given by the following relation.

$$M_\infty = \mu_M + \sigma_M.\xi \tag{7}$$

The robust optimization for drag minimization of airfoil under Mach number uncertainty is formulated as follows.

$$\begin{aligned} &Objective\ Function: Minimize\ \mu_{Cd} + \sigma_{Cd} \\ &St: (1) thickness \geq thickness_0 \\ &(2)\ \mu_{C_l} \geq \mu_{C_0} \\ &(3)\ |\mu_{C_m}| \leq |\mu_{C_{m0}}| \end{aligned} \tag{8}$$

where $\mu_{C_{l0}}$ and $\mu_{C_{m0}}$ represent the mean values of target lift coefficient, and pitching moment of baseline airfoil. The flow chart for our robust optimization is shown in Fig. 7. Following procedure is adopted for robust airfoil optimization.

Step1:Select Design of experiment (DoE) for generating sample points (design variables) using LHS in design space for parameterization of airfoil geometry.

Step2:For each candidate shape considered during the airfoil shape optimization, twenty random samples are selected and evaluated by for CFD.; kriging model(s) are constructed and predicts the aerodynamic force coefficients on $2\times10^5$ samples selected by Monte Carlo simulation; then the mean and variance of drag, lift and moment coefficients are predicted and used to drive the shape optimization process ( Step 2 is referred to the process of UQ in Fig. 1).

Step3:Calculate the Objective function and set the constraints from predicted mean and variance of aerodynamic coefficients.

Step4:Construct the Kriging model and optimize the objective function by GA using infill criterion (e. g. EI).

Step5:Check and compare the objective function and constraints. If these are satisfied, optimum airfoil is achieved otherwise add new sample points, and repeat the UQ and surrogate-based optimization process.

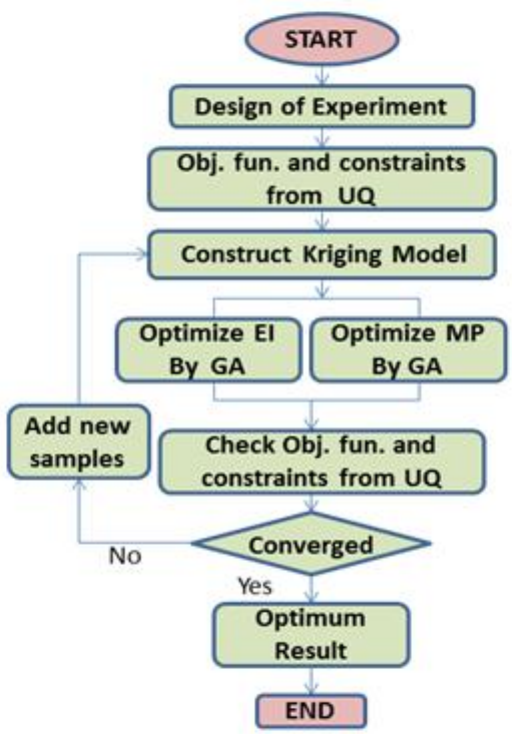



Fig. 7 Flow chart of surrogate-based robust optimization

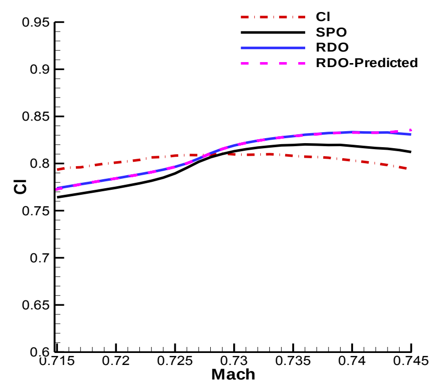
The results of optimized airfoil are tabulated in table II.

TABLE II
COMPARISON OF ROBUST OPTIMIZATION RESULTS WITH RAE2822

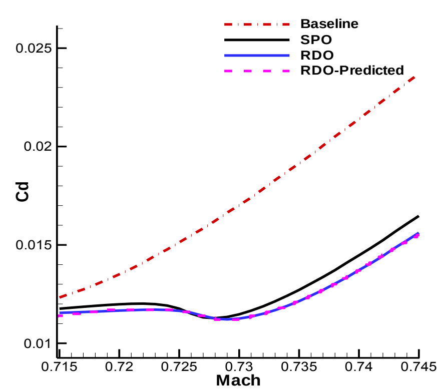
| | $\mu_{Cd}$ | $\sigma_{Cd}$ | Obj. Fun | $\mu_{Cl}$ | Thickness |
|---|---|---|---|---|---|
| RAE2822 | 0.01738 | 0.00227 | 0.01966 | 0.81152 | 0.1214 |
| Optimized | 0.01178 | 0.00084 | 0.01262 | 0.81465 | 0.12144 |
| % diff | -32.22% | -63.1% | -35.8% | +0.39% | +0.04% |

The mean and standard deviation values of the drag coefficient of robust optimized airfoil have been compared with the corresponding values of baseline airfoil which shows that there is significant reduction in mean and standard deviation of drag coefficient. Hence the objective function ($C_d$) of robust airfoil is reduced by significant amount of 35.8%. The aerodynamic lift and drag coefficients obtained by kriging-based IMC method are shown in Fig. , which indicates that the drag coefficient of optimized airfoil is smaller than baseline and SPO airfoil and it is relatively insensitive to the uncertain Mach number variations. The comparison between aerodynamic coefficients of robust airfoil and its surrogate predicted value indicates that the kriging surrogate has the capability to accurately predict the uncertainty of the aerodynamic force coefficients. The comparison of RDO airfoil geometry with baseline and SPO airfoil is shown in Fig. 9.

The probability densities of $C_l$ and $C_d$ gives more detailed insight into the effect of the random parameters on the aerodynamic force coefficients. Fig. presents the comparison of PDF of drag coefficient between robust and SPO airfoil which clearly indicates that variance of robust airfoil is reasonably reduced as compared to SPO airfoil, which shows the benefit of a RDO.




(a) Lift Coefficient versus Mach variation




(b) Drag Coefficient versus Mach variation

Fig. 8 Comparison of Aerodynamic coefficients between RDO, Baseline and SPO airfoils

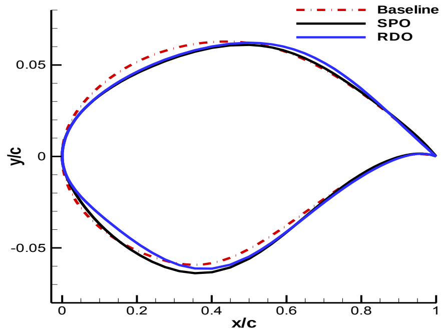



Fig. 9 Comparison of RDO airfoil geometry with Baseline and SPO airfoil (Mach number uncertainty)

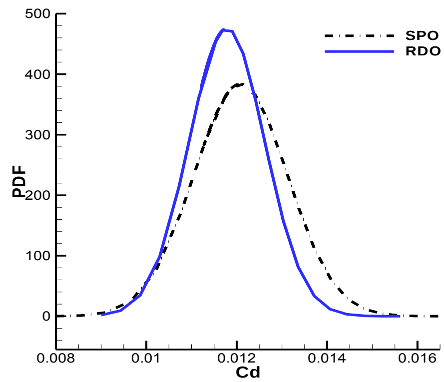



Fig. 10 Comparison of PDF of drag coefficient between robust airfoil and SPO airfoil (Mach number uncertainty)

2) RDO with Angle of Attack Uncertainty

In this case, the range of random alpha samples for stochastic design space is set as -4 ≤ ξ ≤ +4 assuming normal distribution with mean, $\mu_\alpha$ =2.79º and standard deviation, $\sigma_\alpha$=0.1. The RDO problem for drag minimization of airfoil under angle of attack uncertainty is formulated as follows.

$$\begin{aligned} &Objective\ Function: Minimize\ \mu_{Cd} + \sigma_{Cd} \\ &St: (1) thickness \geq thickness_0 \\ &(2)\ \mu_{C_l} \geq \mu_{C_0} \qquad (9) \\ &(3)\ |\mu_{C_m}| \leq |\mu_{C_{m0}}| \end{aligned}$$

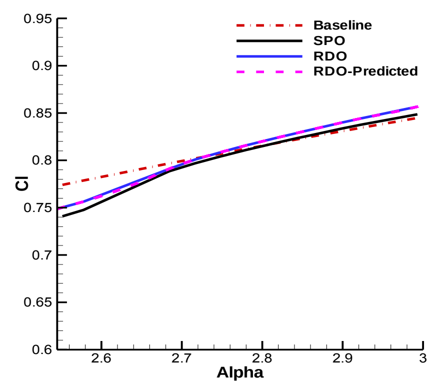
The optimization process is similar to RDO with Mach number uncertainty. The optimization results under angle of attack uncertainty shown in Table III. It can be found that the mean and standard deviation value of the drag coefficient of optimized airfoil is reduced up to 32.95% and 71.18% respectively. Hence objective function of robust airfoil is reduced by around 35.3%, satisfying the required constraints.

TABLE III
COMPARISON OF ROBUST AIRFOIL RESULTS WITH RAE2822

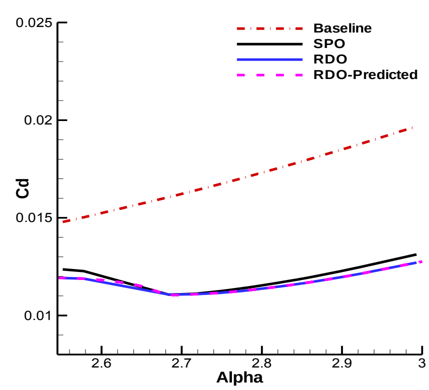
| | $\mu_{Cd}$ | $\sigma_{Cd}$ | Obj. Fun | $\mu_{Cl}$ | Thickness |
|---|---|---|---|---|---|
| RAE2822 | 0.01720 | 0.00113 | 0.01833 | 0.81284 | 0.12140 |
| Optimized | 0.01153 | 0.00032 | 0.01186 | 0.81568 | 0.12157 |
| % diff | -32.95% | -71.18% | -35.3% | +0.35% | +0.14% |

It is quite evident from the plot of drag coefficient in Fig. that for a given lift coefficient, the drag coefficient fluctuations of robust optimized airfoil are smaller than those of baseline and single-point optimized airfoil. The aerodynamic coefficients for robust airfoil have also been compared with its surrogate predicted value which shows that the surrogate has predicted the aerodynamic data efficiently. Fig. 12 depicts the comparison of RDO airfoil geometry with baseline and SPO airfoil. The PDF of drag coefficient of RDO airfoil shown in Fig. has also been compared with SPO airfoil. It is clearly indicated that the variance of drag coefficient of robust airfoil is reduced significantly. Fig. 14 shows the comparison of convergence history of SPO and robust airfoil.

Deterministic single point optimization as well as robust design optimization takes approximately 5 minutes for one CFD evaluation with Pentium Dual Core CPU, 2.5Ghz. For total number of 100 iterations, the time for complete single point optimization took about 8 hours. RDO consisted of 20 CFD computations of candidate airfoil in one iteration. So, total time for complete RDO under Mach number or alpha uncertainty took about 5 days. Although the computational cost of RDO under Mach number or alpha uncertainty is about 15 times than SPO, the reduction in drag coefficient and its variance is significant as compare to SPO airfoil. The CFD evaluations for RDO were 2000 in comparison with 100 CFD evaluations of SPO. The cost of constructing kriging model is almost negligible in both cases of RDO using IMC. Similarly, the computational cost of RDO using surrogate based IMC method is much less than that of using simple MC method which may takes several months to perform robust optimization.




(a) Lift Coefficient versus angle of attack




(b) Drag Coefficient versus angle of attack

Fig. 11 Comparison of Aerodynamic coefficients of RDO airfoil with baseline and SPO airfoil

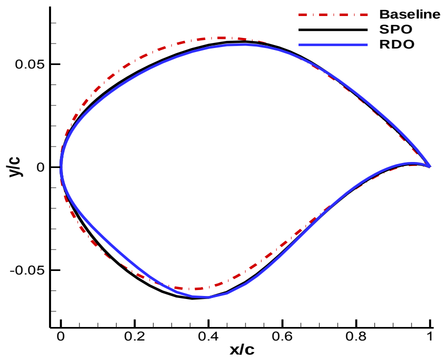



Fig. 12 Comparison of RDO airfoil geometry with baseline and SPO airfoil

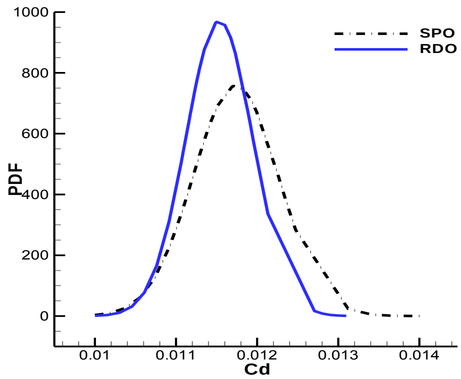



Fig. 13 Comparison of PDF of drag coefficient between robust airfoil and SPO airfoil (angle of attack uncertainty)

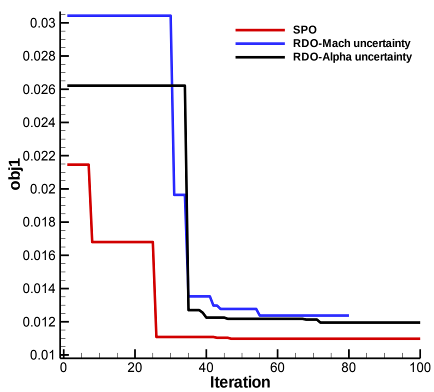



Fig. 14 Convergence history of RDO airfoil (angle of attack uncertainty)

## V. Conclusion

The aerodynamic design optimization under the uncertainty of free stream Mach number or angle of attack has been successfully applied to optimize the statistical moments of an RAE2822 airfoil using efficient surrogate-based uncertainty quantification IMC method. In both cases of Mach number and angle of attack uncertainty, the mean and variance of drag coefficient of robust airfoils are significantly reduced as compared to deterministic SPO method. The drag coefficient of optimized airfoil is relatively insensitive to the variations of the input uncertainty. In present work, the input uncertainty of Mach number and angle of attack were considered separately and the uncertain CFD output was assumed to follow the Gaussian (normal) distribution. In future, the combined effect of both uncertainties as well as geometric uncertainties will be incorporated. The real probability density function of the output of CFD code under uncertainties will be calculated without assuming Gaussian normal distribution. The method will be extended to 3D aerodynamic configuration towards industrial applications.

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