Unbiased FIR Filter Digital Phase-Locked Loop

Sung Hyun Yoo, In Hwan Choi, Jun Ho Chung, Choon Ki Ahn, and Myo Taeg Lim

Abstract**—**In this paper, a new digital phase-locked loop (DPLL) is proposed based on unbiased finite impulse response (UFIR) filters. In, this paper, we compare the fixed gain DPLL, the DPLL based on adaptive Kalman filter, and the proposed DPLL Through simulation, we show the robustness of the proposed DPLL against incorrect noise information.

Keywords**—**Unbiased finite impulse response (UFIR) filter, adaptive Kalman filter (AKF), digital phase-locked loop (DPLL), synchronization.

I. Introduction

DIGITAL phase-locked loop (DPLL) was introduced in 1960s to overcome analog phase-locked loops drawbacks [1]. The DPLL has been a fundamental method to track the signals used in many areas such as carrier tracking, symbol synchronization, clock recovery and re-generation, and motor control. We can estimate the phase information using DPLL information by comparing the input and output signals [1-2].

The fixed gain DPLL was the typical method to estimate the sinusoidal signals. However the fixed gain method has shown unreliable performance when noise information is incorrect. To overcome the fixed gain method, the DPLL based on adaptive Kalman filter (AKF) is developed. The DPLL based on AKF obtain the time varying DPLL loop gains. However AKF is infinite impulse response and can diverge or show poor performance in incorrect noise information.

In signal processing area, finite impulse response (FIR) structure is one of the most preferable methods. FIR structure guarantee stability and robustness to incorrect noise information and modeling uncertainties. When signal models are represented by state-space models, various FIR structures have been applied to solve the filtering problems. The H_∞ FIR filters, the strictly passive FIR filters, the optimal FIR filters, the unbiased FIR filters, nonlinear FIR filter and the induced l_∞ FIR filers have been presented in [5-9]. Recently, FIR filters were applied to output feedback control problems in [10-11].

In this paper, we derive the DPLL filter loop gain using unbiased memory (UFIR) filters. UFIR filter has fast convergence speed in physical situations such as incorrect noise information, thermal noise, etc. with an unexpected condition [4-10]. Furthermore, FIR structure is a noncumulative structure that guarantees robustness in unexpected conditions where infinite impulse response (IIR) structure cannot solve this defect due to error accumulation. Therefore, we apply the UFIR filter to obtain the DPLL loop gain to improve the performance.

This paper is composed of as follows. In Section 2, a new DPLL loop filter gain using UFIR filter is represented. In Section 3, simulation results for an incorrect noise case are given. Finally, conclusion is stated in Section 4.

II. Unbiased FIR DPLL

In this section, we consider a second-order a zero crossing DPLL model as in [2]. The clock rates of the receiver are $f_0 = 1/T_0$ and transmitter $f_1 = 1/T_1$. The timing offset is defined by $\beta_k = T_1 - T_0$ and assumed to be a constant value and a positive zero-crossing point at $k-th$ sequence is represented as

$$\alpha_k = t_0 + k(T_1 - T_0)\,. \tag{1}$$

Consider the state-space model of DPLL

$$\begin{aligned} x_k &= Ax_{k-1} + w_k\,, \\ y_k &= Cx_k + v_k\,, \end{aligned} \tag{2}$$

where the state vector, the input signal, the matrices A and C, the process noise covariance and measurement noise covariance are given by

$$\begin{aligned} x_k &= \begin{bmatrix} \alpha_k & \beta_k \end{bmatrix}^T, \\ y_k &= \alpha_k + v_k\,, \\ A &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \\ C &= \begin{bmatrix} 1 & 0 \end{bmatrix}, \\ E\left[w_k w_k^{\,T}\right] &= \begin{bmatrix} q_1^{\,2} & 0 \\ 0 & q_2^{\,2} \end{bmatrix} := Q\,, \\ E\left[v_k^{\,2}\right] &= r^2 := R\,. \end{aligned}$$

We assume that the process noise and the measurement noise are zero-mean white noise.

From [1], a second-order DPLL estimated in frequency domain is represented by

$$\hat{\alpha}_{k+1} = \hat{\alpha}_k + T_0 + K_0 y_k + K_1 \sum_{i=0}^{k} y_i\,. \tag{3}$$

where K_0, and K_1 are the fixed gains. In equation (3), all measurements are used to estimate the zero-crossing time. Since the measurement contains noise term in (2), the incorrect noise information will cause divergence or poor performance.

Manuscript received November 20, 2013.

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The UFIR filter equation is given by

$$\hat{x}_k = HY_{k-1} \ , \quad (4)$$

where horizon size is defined as N. The measurement vector and noise matrices are represented by

$$Y_{k-1} = \begin{bmatrix} y_{k-N} & y_{k-N+1} & \cdots & y_{k-1} \end{bmatrix}^T ,$$
$$W_{k-1} = \begin{bmatrix} w_{k-N}^T & w_{k-N+1}^T & \cdots & w_{k-1}^T \end{bmatrix}^T ,$$
$$V_{k-1} = \begin{bmatrix} v_{k-N}^T & v_{k-N+1}^T & \cdots & v_{k-1}^T \end{bmatrix}^T .$$

We can represent the state space model using FIR structure as follows :

$$Y_{k-1} = \bar{C}_N x_k + \bar{G}_N W_{k-1} + V_{k-1} \ , \quad (5)$$

where $\bar{C}_N$ and $\bar{G}_N$ are represented by

$$\bar{C}_N = \begin{bmatrix} CA^{-N} \\ CA^{1-N} \\ \vdots \\ CA^{-1} \end{bmatrix}, \quad \bar{G}_N = -\begin{bmatrix} CA^{-1} & CA^{-2} & \cdots & CA^{-N} \\ 0 & CA^{-1} & \cdots & CA^{1-N} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & CA^{-1} \end{bmatrix}.$$

The (4) can be represented by

$$\hat{x}_k = H\left(\bar{C}_N x_k + \bar{G}_N W_{k-1} + V_{k-1}\right).$$

Using an unbiased condition, we can obtain following constraint

$$H\bar{C}_N = I \ . \quad (6)$$

In (6), we can obtain the gain using pseudo inverse as follows :

$$H = \bar{C}_N^{\ \dagger},$$

Where $\dagger$ represents pseudo inverse.

In this paper, the second-order DPLL filter gain can be obtained by UFIR filter, which can estimate the zero-crossing time and the unknown phase only using the recent N measurement information and independent of noise information

III. Numerical Example

In this section, we perform simulation to compare the proposed DPLL with the fixed gain DPLL [1]. We define phase error $e_k = y_k - \hat{y}_k$ to confirm the filter performance, where $\hat{y}_k = C\hat{x}_k$ is the estimate of measurement. We set the horizon size $N = 2$ and SNR= 15dB, when comparing with the fixed gain DPLL. Measurement noise has a noise covariance of 3.16×10^{-10}. We use a measurement noise covariance of 3.16×10^{-8} and the initial state is unknown parameter given by $\hat{x}(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ when we design the filters. Fig. 1 shows the absolute value of errors of the proposed DPLL and the fixed gain DPLL. The results of another SNR are similar to this performance. Since the fixed gain DPLL has the IIR structure, the estimate error will diverge due to incorrect noise. On the other hand, the estimation error of the proposed DPLL converges with fast convergence speed in spite of incorrect noise information. According to the simulation result, we can conclude that the proposed DPLL is more robust than fixed gain DPLL in case of incorrect noise information.

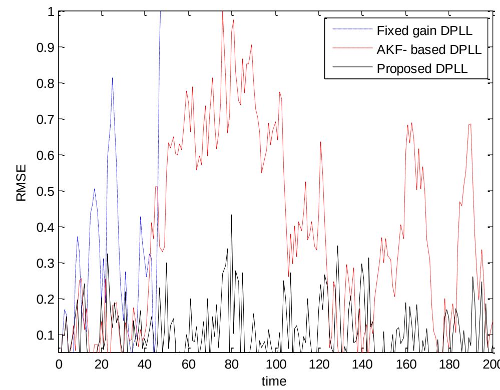

Fig. 1 Estimation errors of the proposed DPLL, fixed gain DPLL, and the DPLL based on adaptive Kalman filter

IV. Conclusion

This paper proposes a new second-order DPLL based on UFIR filter which allows the estimation of unknown phase. The proposed DPLL has a better performance against incorrect noise information than the fixed gain DPLL. Via numerical example, that proposed DPLL guarantees robust performance in poor condition such as incorrect noise information, whereas the fixed gain DPLL and the DPLL based on AKF show poor performance or divergence phenomenon in such condition. Therefore, we expect that the proposed DPLL can be applied to various fields of communication problems.

Acknowledgment

This work was supported by the Energy Efficiency & Resources Core Technology Program of the Korea Institute of Energy Technology Evaluation and Planning (KETEP), granted financial resource from the Ministry of Trade, Industry & Energy, Republic of Korea. (No. 20142010102390)

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