Modified TOC Heuristics when Complete Shipment and Multiple Physical Resources are Constraints

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Abstract--- Over the last three decades, theory of constraints (TOC) has emerged as a complete management philosophy. Fox extended the application of (TOC) to the product mix situations and suggested the theory of constraints heuristics (TOCh). This algorithm was applicable to single constraint situations only. So, it was later on modified by Lea(RTOCh) for multiple constraint situations. Neither of these heuristics is applicable to the situations where complete shipment to different customers is an additional constraint over and above the physical constraints. In this paper, some modifications are suggested in RTOCh proposed by Lea. By applying these modifications, the heuristics can be used in the complete shipment constraint situations. The working of this modified heuristics has been explained by taking two illustrative examples. By comparing the results obtained from the proposed heuristics with those obtained from integer linear programming model, it has been proved that the heuristics gives satisfactory results.

Keywords---Theory of constraints, Theory of constraints heuristics (TOCh), integer linear programming

I. Introduction

A PRACTICING manager has to frequently deal with product mix problems. This problem arises when the capacity of one or more resources is less than total demand placed on it. Methods from Operations research like linear programming, genetic algorithm, tabu search-simulated annealing etc can be used to solve such problems. Theory of constraints has been emerging as a powerful management philosophy over the past 30 years. [1] tried to expand the application of Theory of constraints (TOC) to product mix problems and gave Theory of constraints heuristics (TOCh) which can be used to determine the optimal product mix. [12] have explained the steps in TOCh. Many researchers have compared the results obtained from TOCh with those obtained by using other management science techniques. [12], from his work, showed that TOCh heuristic gives the optimal solution in case of single constraint situations only. But it yields either sub-optimal solution or infeasible solution, if there are multiple constraints. [14] developed a hybrid Tabu Search- Simulated Annealing model to solve product mix problem and by taking illustrative example concluded that the hybrid Tabu- simulated annealing model yields better results than TOC heuristics and linear programming. Similarly, [10] concluded from their mathematical model that TOC model generates more profits, if management has no control over labour and overhead expenses, while ABC generates higher profits if the management has complete control over the labour and overhead expenses. [17] using numerical examples proved that throughput accounting outperforms ABC (Activity based costing) in product mix decisions while the findings of [9] are contrary to it. To overcome the limitations of TOCh proposed by [1], [2] proposed a Revised Theory of constraints heuristics (RTOCh) for multi-constraint situations and have compared the outcome of their proposed heuristics with the solution generated by integer linear programming. [13] concluded from their analysis that TOC and linear programming yield similar results.

For determining the product mix, the demand of various products from different customers/distributors is aggregated and this aggregated demand is used in product mix problems. While solving these problems at aggregate level, it is assumed that a customer can be supplied with partial shipments, but this is not the case in many situations. A visit to a number of companies in the local region confirmed this fact. Jalandhar is a hub of hand tool industries. Companies of this region supply hand tools to the customers in various parts of the world. To find their method of production planning, we visited many hand tool- manufacturing units of the region. From the survey, it was found that these companies have negligible set up change over time particularly for the operations after forging. So they manufacture the products order by order, particularly on the operations like broaching, grinding, polishing, shot blasting, electroplating and packing etc. Also the international customers do not accept partial shipments. None of the researchers have worked on modifications needed in TOCh to make it applicable under such situations. So, the objective of the present study is to propose a modified TOCh for the situations where partial shipments are not permitted and to compare the results obtained from the proposed heuristic with the results obtained from integer linear programming model. To start with, the RTOCh, as proposed by Fredendall and Lea is presented below

A. Multi Constraint Scenario

If more than one resource is short of capacity required to manufacture the products demanded, it is a case of multi-constraint situation. It was proved by Planert and many other researchers that TOCh heuristics (given by Fox) gives either sub-optimal solution or infeasible solution in case of multi-constraint scenarios. To over-come this limitation, revised

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TOCh was developed by Fredendall and Lea to make TOC heuristics applicable to multi constraint scenarios. The steps of this heuristics are given in the following section. But before that, some notations used are explained below

Notations used

i denotes distributors (for various distributors i = 1,2,3….m)
j denotes products (For various products j = 1,2,3……n)
k denotes resources used in production (for various resources, k = 1,2,3……p)
Th_j denotes throughput per unit product j
D_{ij} denotes demand of distributor i for product j
x_i denotes a binary variable (x_i = 1 , if the demand of distributor i is met, 0 otherwise)
t_{kj} denotes time to process one unit of product j on the resource kQ_k Maximum time available on resource k
b denotes the bottleneck resource
t_{bj} denotes time to process one unit of product j on the bottleneck resource b.
R_i denotes the throughput to the constraint time ratio for customer i
dif_k denotes the difference between the capacity of a resource k and its load
MPS denotes set of customers in the master production schedule of the next planning period
NPS denotes set of customers not included in the master production schedule of the next planning period
CR is the set of bottleneck resources
Qt_j = Quantity of product j to be manufactured as per MPS
BN_k is the kth bottleneck resource
b is the dominant bottleneck (i.e. the resource which runs out of capacity before the other bottleneck resources)

THE REVISED PRODUCT MIX TOC HEURISTICS (RTOCh)

Step 1 Identify the systems constraints

a. Calculate the difference (dif_k) between the capacity of a resource and the demand placed on it.

$$dif_k = Q_k - \sum_{j=1}^{n} t_{kj} \, . D_j \qquad (k = 1,2,3 \ldots\ldots p)$$

Where t_{kj} is the processing time of one unit of product j on resource k

b. Determine CR= (BN_1, BN_2,………….BN_q), q ≤ p, where CR is the set of constraint resources with $Dif_k \le 0$ and $Dif_1 \le Dif_2 \le Dif_3 \ldots\ldots \le Dif_q$

Step 2 Decide how to exploit the systems constraint

a. Assume that BN_1 is the dominant bottleneck resource and calculate the ratio of Th_j to the product processing time as $R_j = Th_j/t_{j,BN1}$(where Th_j is the throughput/contribution margin per unit product j

b. Check if BN_1 is the dominant bottleneck

i If the number of resources (q) in CR equals to 1, then dominant bottleneck is BN_1, go to (c)

ii If q>1 and every product uses every resource, then again the BN_1 is the bottleneck constraint and the ranked constraint resources are BN_2, BN_3…….. BN_q , go to (c)

iii Otherwise determine the dominant bottleneck as follows

(1) Schedule the products in the non-ascending order of their R_j ratio until either one of the following two conditions are met

(a) Market demand is met (i.e. $Qt_j = D_j$)

(b) At least one of the constraint resources does not have sufficient capacity to schedule more of the product j

If the resource exhausted first is not BN_1, make the resource exhausted first as BN_1, increase the ranks of other resources by 1 unit i.e. earlier BN_1 becomes BN_2, BN_2 becomes BN_3 and so on. Recalculate R_j ratio for this new dominant bottleneck. Go to (c)

c. Schedule all the products requiring processing on dominant bottleneck resource in the non-ascending order of their R_j ratio. For each product P_j, schedule the largest quantity possible (i.e. the quantity for which market demand is not exceeded and there is adequate capacity on all the constraint resources). If a choice is to be made between two products, to be scheduled, having same R_j, then schedule the product with higher Th_j first.

d. When the dominant bottleneck capacity is exhausted or there is insufficient capacity left to produce another unit of product, then treat BN_2 as new bottleneck. Repeat steps (2c) and (2d) until every resource in CR is exhausted or there are no products remaining that need these constraint resources.

e. Schedule the free products (defined here as the products that do not use any of the constraint resources) until their market demand is met. The initial schedule is $P_1, P_2 \ldots\ldots\ldots\ldots P_n$.

f. Calculate the time left on the bottleneck resource BNq as follows

$$t_{left,q} = Q_{BN_q} - \sum_{j=1}^{n} t_{qj} \, . Qt_j$$

Where Qt_j is the quantity of product j scheduled.

g. Examine whether reducing a Qt_l and increasing some Qt_j (where j > l) increases throughput (i.e. examine whether trading off production of some products for the production of other products increases throughput). A neighbourhood search is conducted to determine which products will be candidates for this interchange. In this search, neighborhood of the product (i) is defined as the nearest downstream product (i.e. i + 1).

Let X be the set of candidate products where reducing some Qt_l and increasing some Qt_j (j>l) may increase throughput. The set X is found as follows

i For k = 1 to q
For j = 1 to (n-1)
If $t_{j+1, BN_k} > 0$
Then if $R_{j+1} (t_{left,k} + t_{j, BN_k}) / Th_j \ge 1$then go to (ii)
Else j = j + 1
Else k = k + 1
ii if $Qt_j < D_j$ or $Qt_{j+1} < D_{l+1}$
then set l = j go to (iii)
else j = j +1 go to 1
iii X = (P_l, P_{l+1},………… P_n) if X = ϕ, the current product mix is optimal, so stop
otherwise go to step 2(h)

h. Reduce the current quantity Qt_l of the product P_l one unit at a time. This creates ($t_{l,BNq}$ + $t_{left,q}$) of available time on the dominant bottleneck resource. Schedule this available to process P_{l+1}, P_{l+2},………… P_n based on their R_j ratios (j = i+1, i+2……. n) in the non-ascending order using the procedure in step 2(c) . calculate the throughput gain of this step as

Gain = $\Delta Qt_l\ Th_l + \Delta Qt_{l+1}\ Th_{l+1} \ldots\ldots + \Delta Qt_n Th_n$

Where ΔQt_l is the change in units for P_l

Stop if the gain is less than 0 or stop if the solution becomes infeasible (i.e. $\Delta\ Qt_l\ > D_l$)

B. Modified Toch Heuristics (Mtoch2) For Multi Constraint Scenario

The above heuristics cannot be used in the situations where complete shipment to each customer is an additional constraint. Therefore, a new heuristics (MTOCh2) was developed and is explained below.

C. Steps In The Modified Toch Heuristics (Mtoch2)

1 Determine the capacity (Q_k) of each resource

2 Calculate the load on each resource by using the following equation

$$\text{Load on the resource k } = \sum_{j=1}^{n} t_{kj}\ (\sum_{i=1}^{m} D_{ij}\ )$$

3 Determine the CR (the set of bottleneck resources). , For this purpose, calculate the difference (dif_k) between capacity and load on each resource by using the following equation

$$dif_k \ = \ Q_k - \sum_{j=1}^{n} t_{kj}\ (\sum_{i=1}^{m} D_{ij}\ )$$

The resources for which Dif_k < 0 are the bottleneck resources.

CR = (BN_1, BN_2, ……….BN_w) where $w \leq k$ and BN_w are arranged in the descending order of their amount of overload. i.e. $Dif_1 < Dif_2 < \ldots\ldots\ldots\ldots\ Dif_w < 0$

4 Check if BN_1 is the dominant bottleneck. The steps for this are given below

a. If w = 1, then there is only one bottleneck resource i.e. BN_1

b. If all the customer orders require the use of all bottleneck the resources, even then BN_1 is the dominant bottleneck

c. If neither of the above two conditions are true, assume BN_1 to be the dominant bottleneck. Calculate the ratio (R_i) for BN_1. Schedule the customer orders in the descending order of this ratio. If BN_1 runs out of capacity before the other bottleneck resources then BN_1 is the dominant bottleneck. If some other resource runs out of capacity before BN_1, then make that resource as BN_1 and increase the rank of other resources by one unit.

5 Calculate the throughput for each customer order, by using the following formula

$$\text{Throughput for the customer i} = \sum_{j=1}^{n} d_{ij}\ .Th_j$$

6 Calculate the dominant bottleneck (b) time required to manufacture each customer order, by using the following formula

$$\text{Bottleneck time consumption for the customer i} = \sum_{j=1}^{n} d_{ij}\ .t_{bj}$$

7 Calculate the throughput to the constraint time ratio (R_i) for each customer, as given below

$$\text{Throughput per unit constraint minute for the customer i} = \frac{\sum_{j=1}^{n} d_{ij}\ .Th_j}{\sum_{j=1}^{n} d_{ij}\ .t_{bj}}$$

8 Arrange the customers in the descending order of this ratio

9 First allocate the capacity of the dominant bottleneck to manufacture the products needed to meet the requirement of the customer with highest ratio in step 7 then to next higher ratio and so on till the capacity of dominant. bottleneck is fully exhausted or the time left on constraint is not sufficient to meet the requirement of next customer in the list. Now take BN2 as the dominant bottleneck and schedule the products that do not require processing on BN1 in the descending order of their Ri ratio on BN2 (within the constraint of capacity of these resources and market demand). Repeat this process for all the resources in the set CR. Now schedule the customer orders that do not use any resource in the CR set according to the customer order. This determines the initial MPS.

10 The above steps will divide the customers in two sets. First set contains the customers included in the master production schedule (MPS) and the second one contains those customers who are not included in the master production schedule (NPS)

11. Steps to check/make the solution optimal (optimality test)

a. Arrange the customers in the NPS set in the descending order of their throughput and let us call them NPS1, NPS2……….

b. Take the first customer (NPS1) from the set NPS

c. Determine the candidates from the set MPS that can be replaced by NPS1.the candidates to be replaced must meet the following conditions

i Their throughput should be less than the throughput of NPS1

ii Summation of their processing time on each of the constraint and the unutilized time on each constraint should be more than the processing time of NPS1 on each constraint.

d. From the list of such candidates, take the candidate with minimum throughput. Move this candidate to the end of NPS set and NPS1 to end of MPS set.

e. Repeat the steps c and d for all the candidates in the NPS set

This determines the optimal order mix for the MPS.

Note If at any stage, there is no candidate in the MPS set that meets the conditions i and ii, then the solution is optimal and the further iterations stop.

The working of this proposed heuristic has been tested by taking the following scenarios:

Resource	Product					Load	Capacity	Percentage Utilization	Over load
	A	B	C	D	E				
10	2.5	5.5	3.5	2	10	1015	2400	42.3	
20	9.5	3.5	8.5	8	20	2075	1825	113.7	250
30	6.5	1.5	9.5	10	15	1755	2400	73.1	
40	12	16	25	30	0	2620	2400	109.2	220
50	4	1	2	1	10	820	2400	34.2	
60	30	10	9	10	2	1680	2400	70.0	
Demand	20	30	40	30	60				
Sales Price	30	50	50	40	20				
Material Price	10	42	25	25	15				
Throughput	20	8	25	15	5				

II. Testing Of Proposed Heuristic (MTOCH2)

The heuristics was tested by taking two situations from the literature. In both the situations, the heuristics worked satisfactorily. This shows the quality of the heuristics proposed. .

III. Discussion Of Results

A product mix problem arises only when the total demand placed on a system is more than the ability of some or all of it's resources to meet it. Under these conditions, managers have to decide, which products to manufacture and which not to manufacture in the next planning period. Conventionally, linear programming has been used for taking such decisions. Fox [1987] extended the application of theory of constraints to such situations and proposed TOCh heuristics. TOCh was later enhanced to RTOCh by Fredendall and Lea [1997] to make it applicable to multiconstraint scenarios. TOCh or RTOCh are not applicable to situations where partial shipments are not allowed. So, in the present work, an attempt has been made to enhance the application of TOCh to such situations and a modified version of TOCh i.e. MTOCh2 has been proposed. The working of MTOCh2 has been explained by taking two illustrative problems from literature and the results obtained from this heuristics has been compared with the results obtained by using integer linear programming. In both the cases, the results have been found to be same. This confirms the quality of the heuristics proposed. But the question arises that if MTOCh2 gives similar results as are obtainable from linear programming, then why we should use them at all. The first reason is simplicity of construction and solution of MTOCh2 model as compared to integer linear programming model. A model that can be constructed and solved quickly is more likely to be used. Secondly, determining the product mix is only one part of the problem. The real issue is the ability to manufacture it in the uncertainties of the real world and to minimize the gap between the theoretically optimal results and the actual results obtained. Wrong sequencing can increase this gap. Integer linear programming does not provide any rule to prioritize the manufacturing of the products in MPS. While MTOCh2 does provide this sequencing. It states that the customer orders are to be manufactured in the order of decreasing value of R_i ratio. By doing so, even if we fail to manufacture all the orders in the MPS, we will miss the least profitable customer order only. On the other hand, sequencing the customer orders randomly, we may miss a more profitable customer. So, the gap between theoretically optimal and actual result obtained may be wider. TOC suggests the use of techniques like Drum-Buffer-Rope and Buffer Management to quickly highlight any negative fluctuation. So, timely action can be taken to minimize the harmful effects of these fluctuations. Furthermore, TOC mitigates the stochastic effects of production activities by establishing buffer stocks in front of production bottlenecks and providing protective capacity for non-constraint stochastic activities. So, the earlier TOC based product mix heuristics (TOCh and RTOCh) and the heuristics developed in the current work should be considered as important tools of TOC philosophy only. Their usage along with other techniques of TOC will result in near optimal results in the uncertainties of the real world. Comparing these techniques in isolation with the corresponding techniques from operations research can be mis-leading. A limitation of the present work is that algorithm has been tested by taking only two problems. It needs to be tested on more situations. This will further establish the validity of model. Similarly, testing it's applicability in real life situations is also needed. Developing computer programs of the proposed algorithms will further enhance the usage of the proposed heuristics.

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