Gyroscope Mystery is Solved

Ryspek Usubamatov, Azmi B. Harun, and Mohd Fidzwan B. Md. Amin Hamzas

***Abstract*—** Numerous publications represent the gyroscope theory using mathematical models based on the law of kinetic energy conservation and the rate of change in angular momentum of a spinning rotor. In reality, gyroscope effects are more complex and known mathematical models do not reflect the actual motions. Analysis of forces acting on a gyroscope shows that three dynamic components act simultaneously: the centrifugal force, inertial force and the rate of change in angular momentum of the spinning rotor. The spinning rotor creates a rotating plane of centrifugal forces that resists the twisting of the rotor with external torque applied. The forced inclination of the spinning rotor creates inertial forces, resulting in precession torque of a gyroscope. The rate of change of angular momentum creates resisting and precession torques. A new mathematical model for gyroscope motions was tested and the results were validated. This model can be used as a base for new gyroscope theory.

***Keywords*—**Gyroscope Theory

I. INTRODUCTION

IN 1765, Leonhard Euler first laid out the mathematical foundations for the gyroscope theory in his work on the dynamics of rigid bodies. Later, Sir Isaac Newton and many other scientists developed and added new interpretations for the gyroscope phenomenon. The primary attribute of a gyroscope is a rotor that persists in maintaining its plane of rotation, creating a gyroscope effect. Gyroscope effects are used in many engineering calculations of rotating parts, and a gyroscope is the basic unit of numerous devices and instruments used in aviation, space, marine and other industries. Many publications discuss the gyroscope theory and the many approaches and mathematical solutions describing some of the new properties of gyroscopes. The most fundamental textbooks of classical mechanics have chapters representing gyroscope theory [1 - 4]. There are also many publications dedicated to theory and applications of gyroscope in engineering [5 - 8]. A simple explanation of the gyroscope effect is the rate of change in angular momentum vector that creates the precession torque. However, practice demonstrates this approach does not give the full picture of gyroscope motions. Analyzing gyroscope properties under the action of an applied force, occasionally leads to problems that need to be solved using a clear and understandable presentation of gyroscope motions. Gyroscope theory still attracts many researchers who continue to discover new properties of gyroscopic devices [9 - 11]. There are many publications that detail results that not well-matched with analytical calculations and practice [12 - 14].

The nature of gyroscopic effects is more complex. Analyses of motions in gyroscope devices shows pseudo centrifugal and inertial forces of the spinning rotor are fundamental forces that lead to gyroscope effects [15 – 16]. The known rate of change in angular momentum attributes to only 30% of gyroscopic effects. External torque applied to the gyroscope creates the centrifugal and inertial forces and the rate of change in angular momentum of the spinning rotor. The centrifugal force creates resisting torque of change at the rotor's location; the inertial force creates the net of procession torques and the rate of change in the angular momentum involves both resisting and precession torques, but is not the primary force in gyroscope effects. The simultaneous action of these components has not been described in the physics of gyroscope effects. Based on new fundamental approaches, we derived a new gyroscope theory. The new mathematical model matched with practice and was confirmed by preliminary laboratory tests of the Super Precision Gyroscope, "Brightfusion Ltd". The gyroscope mystery was solved and a new approach to the gyroscope theory now needs new extended tests and research. This paper represents a new mathematical model of gyroscope effects and describes motions of the spinning rotor based on actions of the centrifugal and inertial forces and the rate of change in the angular momentum, which are results of an external torque applied to a gyroscope.

II. NOMENCLATURE

J_i - mass moment of inertia for the rotor's disc around the i axis
M - mass of rotor's disc
R - external radius of rotor
r_m - radius location of the mass element
T - torque applied
T_{am} - torque created by the rate of change in the angular momentum
T_{ctr}, T_{inr} - torque created by the centrifugal and inertial forces
T_i - torque around axis i
W – weight of gyroscope
ω - angular velocity of rotor
ω_p - angular velocity of precession
$\omega_{p.i}$ - angular velocity of precession around axis i

Ryspek Usubamatov is with the, University Malaysia Perlis, 02600 Arau, Perlis, Malaysia, (Phone: 604-9885035, e-mail: ryspek@unimap.edu.my).

Azmi B. Harun, is with the University Malaysia Perlis, 02600 Arau, Perlis, Malaysia, (e-mail: azmiharun@unimap.edu.my).

Fidzwan B. Md. Amin Hamzas is with the is with the University Malaysia Perlis, 02600 Arau, Perlis, Malaysia, (e-mail: fidzwan@unimap.edu.my).

III. METHODOLOGY

Centrifugal forces naturally counteract the action of the forces directed to changing the location of the spinning rotor plane. The spinning rotor experiences a radial acceleration and a pseudo centrifugal force. This acceleration and rotation of mass elements create the centrifugal forces' pseudo plane, which acts strictly perpendicular to the axis of the spinning rotor. The plane of rotating centrifugal forces resists on declination, if an external torque is applied.

In uniform circular motions the magnitude of the body's velocity does not change. However, since velocity is a vector quantity its direction changes continuously, i.e. the body is under acceleration. Centrifugal forces create the rotating forces' pseudo plane, which acts strictly perpendicular to the axis of the spinning rotor. The external torque leads to change in the angular location of the spinning rotor plane and creates a pseudo contracting moment of the centrifugal forces' components. The plane of rotating centrifugal forces declines and resists action of the external torque. Wee derived the mathematical model of the resistance torque created by the centrifugal forces for a thin disc-type spinning rotor. The mathematical model used the following simplifications: the weight of the rotor shaft is neglected and bearing friction is negligible. The mathematical model of resistance torque created by the rotating centrifugal forces plane is represented by the following equation [15].

$$T_{ctr} = \frac{8MR^2\omega\omega_p}{9} = (4/3)^2 J\omega\omega_p \tag{1}$$

Where T_{ctr} is a resistance torque created by the centrifugal forces, J is the rotor's mass moment of inertia, ω is the angular velocity of the spinning rotor, and ω_{p} is the angular velocity of a forced precession of a spinning rotor.

Analysis of Eq. (1) shows the centrifugal forces' resistance on the external torque depends on the angular velocity, ω, of the spinning rotor, its radius, R, and mass, M, and on the angular velocity of the forced precession $\omega_{p.}$ The angular velocity of forced precession, $\omega_{p,}$ causes the action of the external torque applying T to the spinning rotor. Absence of external torque means the angular velocity of the forced precession, ω_p = 0. Then, the Eq. (1) gives the resistance torque's equation of the centrifugal forces, T_{ctr} = 0, which is a natural result.

In uniform circular motion, the tangential velocity direction of mass elements changes continuously. In the case of external torque applied to a gyroscope, the inclination of the disc of a spinning rotor changes the tangential velocity's direction of a mass element. The tangential velocity's change in direction of creates acceleration and the inertial force of a mass element. These inertial forces start act perpendicular to the plane of the spinning rotor, turning it and cause the angular torque and the angular velocity of the rotor precession. This torque acts in the plane perpendicular to the plane of the resistance torque action, which was created by centrifugal forces. The following equation represents the mathematical model of the precession torque created by the inertial forces [16]:

$$T_{inr} = \frac{8MR^2\omega\omega_p}{9} = (4/3)^2 J\omega\omega_p \tag{2}$$

Where T_{inr} is the precession torque created by inertial forces, other parameters are as specified above.

Analysis of Eq. (2) shows the precession torque created by the axial inertial forces of the spinning rotor is the same as Eq. (1), i.e., precession and resistance torques are created by the same rotating masses and accelerations are perpendicular to each other. The external torque applied to the spinning rotor causes angular velocity of precession around the axis that is perpendicular to the rotor's axis.

The rate of change in angular momentum of the spinning rotor creates the torque, which acts in two directions as precession and resistance torques and represented by the following equation [1-4]

$$T_{a.m} = (MR^2/2)\omega\omega_{p.i} = J\omega\omega_{p.i} \tag{3}$$

Where $T_{\text{a.m}}$ is the torque created by the rate of change in angular momentum, $\omega_{\text{p.i}}$ is the angular velocity of a precession of a spinning rotor in i direction and other parameters are as specified above.

The defined toques values are based on the centrifugal and inertial forces and the rate of change in angular momentum acting simultaneously on the spinning rotor. Calculation shows the torque of the rate of change in angular momentum represents 30% of the total acting torques. We can show the values and results of the forces and motions calculated by using known gyroscope theory.

IV. WORKING EXAMPLE

The propeller on an airplane has a mass of M = 15 kg and a radius of gyration r_{g} = 0.3 m about the axis of spin. The propeller is turning at ω = 350 rad/s. The airplane is traveling at V = 300 km/hr and enters a vertical curve having a radius of L = 80 m. We can determine gyroscopic bending moment, which the propeller exerts on the bearings of the engine (Fig. 1).

The airplane flies by the curve, which creates the forced angular velocity of precession around the axis *oy*, that is:

$$\omega_y = \frac{V}{L} = \frac{300000(m)/3600(s)}{80(m)} = 1.04\frac{rad}{s}. \tag{4}$$

The centrifugal and inertial forces around the axes *oy* and *oz* respectively and the torque of the rate of change in the angular momentum around the axis *oy* (Fig. 1b) create the gyroscopic bending moments of the propeller. The flight of an airplane

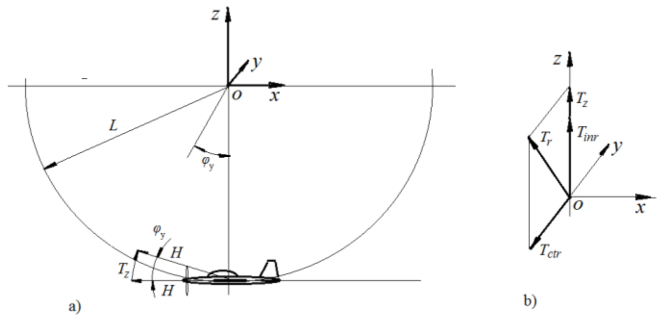

Fig. 1. Sketch for calculating the propeller's gyroscopic bending moment

Does not allow the determination of the angular velocity around the axes *oz* and *ox*. Hence, there is no other torques or bending moments around the axes. The bending moments acting on the propeller are calculated by Eqs. (1), (2) and (3). The total bending moment is represented by vector diagram (Fig. 1b) and by the following equation

$$T_r = \sqrt{T_{ctr}^2 + (T_{inr} + T_z)^2} = \sqrt{\left[\left(\frac{4}{3}\right)^2 J\omega\omega_y\right]^2 + \left[\left(\frac{4}{3}\right)^2 J\omega\omega_y + J\omega\omega_y\right]^2} = 3.2979 J\omega\omega_y \quad (5)$$

Where T_r is the resultant torque, T_{ctr} T_{inr} and T_z are torques created by the centrifugal and inertial forces and the rate of change in angular momentum for the spinning rotor, all other parameters are as specified above.

Equation (5) represents the equation of the propeller's gyration radius. The mass moment of the propeller's inertia is $J = Mr^2_g$ where M is mass and r_g is radius of gyration. The mass moment of inertia in Eq. (1) should be represented by the expression of the gyration's radius, i.e., $J = MR^2/2 = Mr^2_g$ hence $r^2_g = R^2/2$. Substituting defined parameters and transforming results in the equation of the total gyroscopic bending moment created by the propeller which has the following result:

$$T_r = 3.2979 J\omega\omega_y = 3.2979 M r_g^2 \omega\omega_y = 3.2979 * 15(kg) * (0.3m)^2 * 350(rad/s) * 1.04(rad/s) = 1620.5880 Nm$$

(6)

The equation of the gyroscopic bending moment, based on the principle of the rate of change in angular momentum created by the propeller, gives the following result:

$$T_{a.m} = J\omega\omega_{poz} = Mr^2\,\omega\omega_{poz} = 15(\text{kg})\times(0.3\text{m})^2\times 350(\text{rad/s})\times 1.04(\text{rad/s}) = 491.4\text{Nm}. \quad (7)$$

Analysis of the two approaches in calculating the bending moment acting on the propeller (Eqs. (6) and (7)) shows the actual bending moment calculated by the new equations is bigger than the bending moment calculated by the rate of change in angular momentum $T_r/T_{a.m}$ = 1620.5880/491.4 = 3.2979 times. The known gyroscope theory does not consider the torques generated by the centrifugal and inertial forces of the spinning propeller around the axes *ox* and *oz* created by the external torque around the axis *oy* (Fig 1a, b).

V. Practical Test

Defined equations of resistance, precession and the rate of change in angular momentum torques acting on the spinning rotor allows the creation of mathematical models of gyroscope effects. The mathematical models considered the most unaccountable motions of the simple gyroscope suspended on one pivot (Fig.2).

Euler's differential equations of gyroscope's motions based on torques summed about the point *O* (Fig. 2) are represented by the following [1-4]

$$J_z \frac{d\omega_z}{dt} = Wl - \left(\frac{4}{3}\right)^2 J_x\omega\omega_p - J_y\omega_x\omega_y$$
$$J_y \frac{d\omega_y}{dt} = \left(\frac{4}{3}\right)^2 J_x\omega\omega_p - \left(\frac{4}{3}\right)^2 J_x\omega\omega_p + J_z\omega_z\omega_x$$

(8)

where Wl is the torque created by the gyroscope weight around the pivot *o; l* is the distance from the pivot to center of mass of the gyroscope, $J_y = J_z = (MR^2/4) + Ml^2$ are the mass moments of gyroscope inertia around axis *oy* and *oz* and calculated by the parallel axis theorem, $J_y = J_z = (MR^2/4)$ is the rotor's mass moment of inertia around the center of mass and axis parallel to axis *oy* and *oz*,, where M is the mass of rotating parts, R is the external radius of the rotor, ω_p is angular velocity of precession around the referred axes *i*, $J_x = (MR^2/2)$ is the rotor's mass moment of inertia around the axis *ox*, ω is the angular velocity of the rotor's disc and other parameters are as specified above.

Equation (8) represents torques and motions in the planes *xoz* and *xoy* respectively. After the condition action of the gyroscope's weight creates the applied torque. The torque *T* generates the resistance torque of centrifugal forces around the axis *oz* and the procession torque of the inertial forces around the axis *oy*, (Eqs. (1) and (2) respectively). The angular motions of the gyroscope around the axis *oz* an *oy* generate torques of the rate of change in angular momentum (Eq. (3)) which act around axis *oy* and *oz* respectively. Eq. (8) is solved by the special method and enables the calculation of angular velocities of precession around the axes *oy* and *oz.* Full solution of Eq. (8) and its derived method are not presented in this paper.

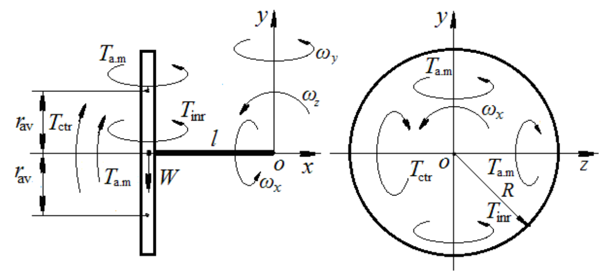

Fig. 2 Scheme of acting forces on the gyroscope with one pivot

The test on angular velocities of a gyroscope precession is conducted on the base of the precision gyroscope (Fig. 3) which technical data represented in Table 1. One side of the gyroscope is suspended on the cord and its weight creates the external torque, *T*, resulting in the gyroscope precession about point *o*, which is the center of coordinate system $\Sigma oxyz$ (Fig. 3). The precession angular velocities of the gyroscope are determined.

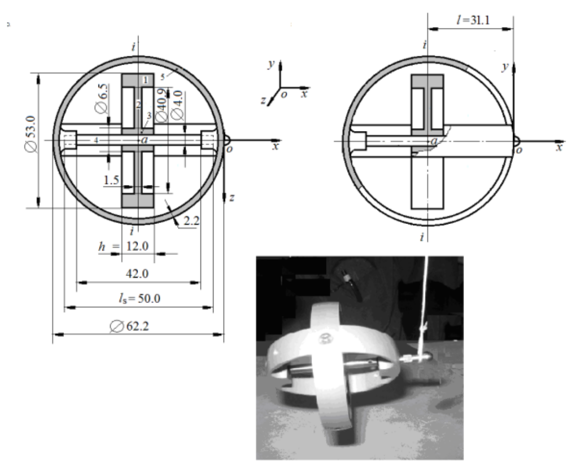

Fig. 3 Geometrical parameters of the gyroscope components

Differential Eq. (8) represents the motions of the gyroscope. For simplification, the rotor's disc with shaft is considered the thin disc , whith radius of location at the center of mass elements and the conventional external radius of the disc is calculated. The rotor disc and shaft of the gyroscope present the complex form and different radii of its components. The components *i* of the disc are the thick walled cylindrical tube with open ends of inner radius r_{in}, outer radius r_{out}, length *h* and mass $m_i = \rho\pi(r^2_{out} - r^2_{in})h$, where ρ is density of metal, and other parameters are as specified above.

The mass moment of inertia around the axis *oy* for the gyroscope frame is represented as a thin sphere of radius r_f, mass m_f, and calculated by the equation $J_{y4} = (2/5)m_4r_4^2+m_4l^2$, where *l* is the distance between the axis *oy* and axis *i-i*. $T = Wl = mgl = 4.4316165\times10^{-2}$ kg.m^2/s^2 is the torque created by the weight of the gyroscope.

TABLE I

TECHNICAL DATA OF SUPER PRECISION GYROSCOPE, "BRIGHTFUSION LTD".

Weight, W	*kg*
Brass disk, ρ = 8,650 kg/m^3	0.1112
Steel shaft, ρ = 7,600 kg/m^3	0.0047
Total rotating components	0.1159
Aluminum frame	0.0294
Total gyroscope, *W*	**0.1453**

Mass moment of inertia	kg*m^2
Around axis *ox*	
Rotating components	5.7249×10^{-5}
Frame 5, J_5	1.0584×10^{-5}
Around axis *oy*	
Rotating components	14.0724×10^{-3}
Frame 5, J_5	3.9020×10^{-5}
Total	17.9744×10^{-5}

The mass moment of inertia of the gyroscope components around the axis *oy* is $J_y = J_{yf} + J_{yi} = 17.0783\times10^{-5}$ kg.m^2 (Table 1, Fig. 3). J_{yf} is the mass moment of inertia of the frame. J_{yi} is the mass moment of inertia of rotating parts.

The radius location of the mass element for the gyroscope rotating components is defined by theorem of location at the center of mass for the complex part:

$$r_m = \sum_{i=1}^{n} m_i r_i \Bigg/ \sum_{i=1}^{n} m_i \tag{9}$$

Where m_i is the mass of gyroscope elements, r_i is the radius of the location of the mass element.

The radius of the location of the mass element for the brass components is calculated by the following equation:

$$r_{mi} = \frac{m_{r.out}\times(2/3)r_{out} - m_{r.in}\times(2/3)r_{in}}{m_{r.out} - m_{r.in}} \tag{10}$$

Where $m_{r.i} = \rho\pi r_i^2 h/2\pi = \rho r_i^2 h/2$ is the mass element located on *i* radius.

Substituting defined expressions of the mass element $m_{r.i}$ and parameters presented in Table 1, into Eq. (10), and transforming it gives the following

$$r_{mi} = \frac{2}{3}\left(\frac{r^3_{i.out} - r^3_{i.in}}{r^2_{i.out} - r^2_{i.in}}\right) \tag{11}$$

Substituting parameters of the brass components and the shaft from Table 1 into Eq. (11) calculates the radii locations of the mass element for the following components:

- the brass disc component 1 (rim)

$$r_{m.r} = \frac{2}{3}\left(\frac{r_{r.out}^3 - r_{r.in}^3}{r_{r.out}^2 - r_{r.in}^2}\right) = 2.3604 \times 10^{-2}\, m$$

- the brass disc component (tie plate)

$$r_{m.p} = \frac{2}{3}\left(\frac{r_{p.out}^3 - r_{p.in}^3}{r_{p.out}^2 - r_{p.in}^2}\right) = 1.3930 \times 10^{-2}\, m$$

- the brass disc component (bush)

$$r_{m.b} = \frac{2}{3}\left(\frac{r_{b.out}^3 - r_{b.in}^3}{r_{b.out}^2 - r_{b.in}^2}\right) = 0.2674 \times 10^{-2}\, m$$

- the shaft 3

$$r_{m.3} = \frac{2}{3} r = 0.1333 \times 10^{-2}\, m$$

Substituting defined masses (Table 1) and radii by Eq. (11) into Eq. (9) calculates the radius location of the mass element of the gyroscope rotating components

$$r_m = \sum_{i=1}^{n} m_i r_i \Big/ \sum_{i=1}^{n} m_i = 0.020954 m$$

The defined radius location of the mass element for the gyroscope rotating components allow us to find its conventional external radius of uniform disc, which is $R_e = (3/2)r_m = 0.0314311$ m. Then, the mass moment of inertia of the rotating mass is calculated and represented by the following: $J_x = (MR_e^2/2)\omega_r = 5.724962\times10^{-5}\omega_r$. Hence, the rate of change in angular momentum of the rotating mass is calculated and represented by the following equation:

$$T_{a.m} = J_x \omega \omega_p = 5.724962 \times 10^{-5}\, \omega_r \omega_p \qquad (12)$$

The resistant and precession torques created by the centrifugal and inertial forces of the spinning rotor are calculated by the following equation:

$$T_{ctr} = T_{inr} = (4/3)^2 J_x \omega \omega_p = 10.177659 \times 10^{-5}\, \omega \omega_p \qquad (13)$$

Where all parameters are as specified above.

Substituting defined parameters into Eq. (8), transforming and solving by the special method give the following results:

$$\omega_y = 680.3504/\omega_x, \quad \omega_z = 21.6963/\omega_x \qquad (14)$$

Equation (14) of angular velocities of the gyroscope precession around the axes *oy* and *oz* for the rotor's speed n = 10000 rpm gives the following:

$$\omega_z = \frac{21.6963}{\omega_x} = 0.0207 rad/s = 1.18^0/s \qquad (15)$$

$$\omega_y = \frac{735.5154}{\omega_x} = 0.707 rad/s = 40.206^0/s$$

Practical tests were conducted on the Precision Gyroscope, model "Brightfusion Ltd", (Fig. 3). The velocity of the spinning rotor was measured by the Optical Multimeter Tachoprobe Model 2108/LSR with range of measurement 0–60,000 rpm, Compact Instrument Ltd. The angular velocities for gyroscope precessions around the cord (axis *oy*) and around the axis *oz* for the suspended gyroscope were measured several times by stopwatch. The drop of the revolution velocity for the spinning rotor per time is presented in Fig. 4. The results are presented in Figs. 5 and 6.

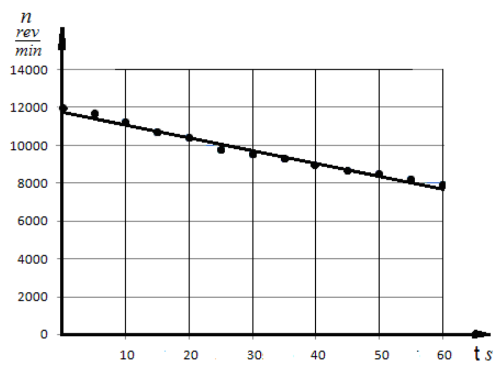

Fig. 4 Revolution velocity for the spinning rotor versus time.

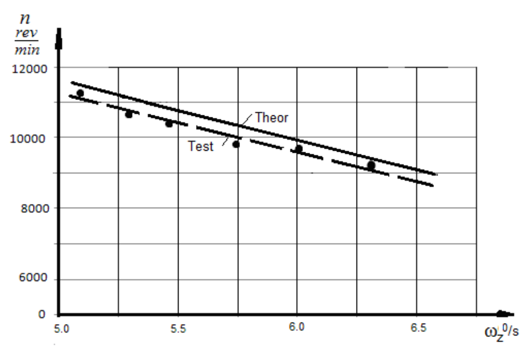

Fig. 5 Change of the rotor's angular precession around the axis *oz* versus the rotor's speed of revolution.

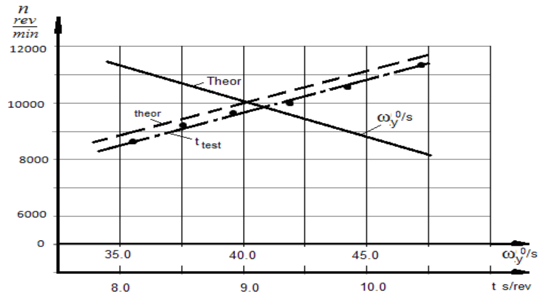

Fig. 6 Change of the rotor's angular precession ω_y and the time of one revolution for the gyroscope around the axis *oy* versus the rotor speed of revolution.

The recorded results of practical tests and theoretical calculations of gyroscope velocities of precessions are well matched. Differences in results are explained by simplifications in geometrical parameters and mechanical properties accepted for the gyroscope: the level of accuracy calculations of the gyroscope technical data, drop of the spinning rotor angular velocity and the level of accuracy measurement.

IV. DISCUSSION

The external torque applied to the gyroscope leads to an angular velocity of forced precession and activates the centrifugal and inertial forces and the rate of changes in angular momentum forces. The derived equations for the resistant and precession torques created by the external torque being applied to the spinning rotor, demonstrate that the torques depend on the mass, radius and angular velocity of the spinning rotor, as well as on angular velocity of its forced precession. The new analytical approach to gyroscopic problems shows that centrifugal and inertial forces of mass elements for the spinning rotor are as much real, active physical components as the rate of change in angular momentum. These forces act simultaneously and result in the resistance and precession torques. Experimental tests and results of the gyroscope effects are well-matched with the new mathematical model.

V.CONCLUSION

The gyroscope theory in classical mechanics is one of the most complex and intricate in terms of analytical solutions. The known mathematical models in gyroscope theory are based on the actions of external torque applied and the rate of change in angular momentum of the spinning rotor. In spite of numerous publications, the gyroscope still attracts researches because of unsolved problems. However, the known mathematical models for gyroscope effects do not consider the action of the pseudo centrifugal and inertial forces of the rotating mass elements in the gyroscope's spinning rotor. The new analytical approach demonstrates that the centrifugal forces of the spinning rotor resist any inclination of the rotor's axis and create resistance torque. The axial inertial forces of the spinning rotor create the precession torque. The torque created by the rate of change in angular momentum vector acts in resistance and precession directions and only presents 30% of all acting forces in the gyroscope.

New mathematical models for gyroscope effects led to new properties and will be useful for modeling the behavior of gyroscopic devices. Derived analytical equations of the gyroscopic motions, based on principles of centrifugal and inertial forces' and the rate of change in angular momentum of the spinning rotor act simultaneously. The experimental tests and theoretical equations of the gyroscope effects are well matched. The new mathematical model for gyroscope motions is correct. The new analytical approach for the gyroscopic effects presents not only new properties with respect to a gyroscope, but also new challenges for future studies of the forces and motions in gyroscopic devices. The gyroscope mystery is solved. After expanded tests of the new mathematical model for gyroscope effects, there will be a new gyroscope theory that is different from what is known in engineering.

REFERENCES

[1] R. C. Hibbeler, *Engineering Mechanics - Statics and Dynamics*, 12th ed. Prentice Hall, Pearson, Singapore, 2010.

[2] D. R. Gregory, *Classical Mechanics*, Cambridge University Press, New York, 2006. http://dx.doi.org/10.1017/CBO9780511803789

[3] C. Ciminelli, et al, *Advances in Gyroscope Technologies*, Springer-erlag Berlin and Heidelberg GmbH & Co. KG, Berlin, 2010.

[4] R.F. Deimel, *Mechanics of the Gyroscope*, Dover Publications Inc. New York, 2003.

[5] G. Greenhill, *Report on Gyroscopic Theory*, General Books LLC, London, 2010.

[6] M.D. Ardema, *Analytical Dynamics. Theory and Application*. Academic/Plenum Publishers, New York, 2005.

[7] E. Butikov, Inertial rotation of a rigid body, *Eur. J. Phys*. 27, 2006, pp. 913 – 922. http://dx.doi.org/10.1088/0143-0807/27/4/022

[8] M.N. Aremenise, et al, *Advance in Gyroscope Technologies*, 1 st ed. Springer-Verlag, New York, LLC, 2010.

[9] T.Y. Quinn, & A. Picard, The mass of spinning rotors: no dependence on speed of sense or rotation, *Nature 343*, France, 1990, pp. 732-735.

[10] H. Weinberg, Gyro Mechanical Performance: the most important parameter, Technical Article MS-2158, *Analogue Devices*, Norwood, MA, 2011, pp. 1-5.

[11] Y. Ohshima, *Analysis and testing of gyroscope performance for the gravity probe B relativity mission*, Stanford University, CA, 2000.

[12] L. Zyga, Gyroscope's unexplained acceleration may be due to modified inertia, *PhysOrg.com*, 2011, July 26.

[13] J.E. Faller, et al., Gyroscope-weighing experiment with a null result. Physical Review Letters 64, *APS Journal*, Vol. 64. No. 8, 1990, pp. 825–826.

[14] Z. Zhang, J. Sun, and K. Wu, Error analysis and test study of fiber optic gyroscope north-finder. Advanced Sensor Systems and Applications II, *Proceedings of SPIE*, Vol. 5634, Cardiff, 2005, pp. 611-618.

[15] R. Usubamatov, Z.M. Zain, Y.N. Zaiazmin, and M.S. Hussin, Centrifugal forces are the cause of gyroscope stabilization, *Research Bulletin of the Australian Institute of High Energetic Materials*, April, 2011, pp. 226-233.

[16] R. Usubamatov, Analysis of inertial forces acting on gyroscope precession, *International Review of Aerospace Engineering*, Vol. 4, N2, 2011, pp. 103-108.