The Application of the Markov Chain in Statistical Quality Monitoring

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***Abstract*— M**onitoring the quality of process is very common in a wide range of industries. The control charts are one of the most important tools used for this intention, in order to satisfy the market specifications. For the economical and statistical design of control charts a Markovian approach has been implemented in the scientific literature. The purpose of this paper is to study the literature regarding the applications of the Markov Chain in control chart based quality control. Journal articles were reviewed to illustrate the most important advances. A brief summary about the Markovian approach in different types of control charts and acceptance sampling is developed including the validation of the proposed methods. Statistical and economic designs were also found about the theme. To obtain better results in the research a classification method was developed beforehand. The analysis of the literature shown in this paper compares the different thesis proposed by the cited authors and resumes the advantages and disadvantages of their proposals.

***Keywords*—** Control charts, Economic design, Markov chain, Industrial statistics.

I. Introduction

NOWADAYS all industries afford monitoring quality processes in order to reduce costs associated with the nonconformity of products, wastes, and rework. All of this needs to be in-control to be able to compete in the guild. To achieve this, the continuous improvement of their methods is indispensable. Production companies try to stabilize their production lines, and try to encounter monitoring processes that provide a predictable distortion and variability.

The Markov chains have been implemented to predict the state of the system in a next sample. This process relies on the assumption that the actual state is the only thing that can be used to predict the next sample's state. The documentary research involves the Markov chain applications in control charts and acceptance sampling. For authors, it is important to find systematized Information that provides a new investigation theme or exposes the best practices.

In order to understand this investigation it is important to be clear in the concepts treated in the review. Brief definitions will be shared about control charts, the Markov chain application in this quality tools and statistic and economic design. Control charts are one of the statistic methods used to improve quality. They are applied in the analysis and monitoring of a process by a quality characteristic. In order to accomplish the charts function, samples are taken from a process and charts are drawn to control the mean and the variability of the process.

A control chart is said to be efficient if it meets the next three objectives: The shifts are detected rapidly when they occur, must present a low false alarm rate, and have a reasonable sample size. The first objective is measured by the Average Run Length (ARL) which is the average number of observations that must be graph before one of them presents an out of control condition [1]. Following this idea, Markov chain is applied by using the probabilities to find the ARL of a chart. In effect, the Markov chains represent a system that varies its state along time, having the next stages: (State1: In control, State 2: out of control) when a shift is presented, as shown in Table I. The Markov chain is made up of four sets of m transient states. The status of the process (in-control or out-of-control) when the (i+1)th sample is taken and the position of the ith sample point define the four sets of transient states [2].

TABLE I
THE STATES OF THE MARKOV CHAIN APPLIED TO CONTROL CHARTS

*i*th sample	(i^+1)st sample	
Sample point position (region)	Process mean status (on or off target)[a]	State of the Markov Chain
Warning	On	2
Warning	Off	4
Central	On	1
Central	Off	3

[a] On target means $\mu=\mu_o$ and off target means $\mu=\mu_o\pm\delta\sigma$

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II. RESEARCH METHODOLOGY

Following the research methodology proposed in Lage Junior and Godinho Filho [3], this paper implements the steps described below:

1) Perform a literature review concerning the application of the Markov Chain in quality processes.

2) Develop a classification method for the research.

3) Make use of the classification method to resume advances related to the theme.

4) Use the classification method to organize the literature for a better comprehension of the reader.

5) Summarize and present an analysis about the review.

The literature review included different applications in quality control charts and acceptance sampling involving the Markov Chain approaches. More than 20 articles were analyzed (step 1). A classification method was used to perform this paper (step 2) which will be explained in the next session. Making use of this organization, a table is presented to summarize the advances, until date, about the theme (step 3). In session 4 the reader can follow a sequence that has being selected from the classification method (step 4). The authors resume the papers founded in the research showing the contributions of the advances in the topic (step 5). The research was made using electronic database, such as Science Direct, IEEE and EBSCO.

III. CLASSIFICATION METHOD

Articles have been classified in two main categories, as follows: *(a)* Application(s), and *(b)* Type of charts.

The objective of including the application (category A) is to understand the different usage that could contribute to quality processes, such as economic or statistic designs. The importance of this group is that the applications that are used to monitor processes, help in the decision making for improving quality and reduce involved costs.

Category B specifies the type of charts that are used to apply the Markov chain. The following types of charts have been taken into account by the authors: x, T^2, S^2 , r, CUSUM (or cumulative sum control chart), *EWMA* (or exponentially-weighted moving average chart), X^2, and *MEWMA*. The objective of involving these distinct tools is to analyze how different methodologies, types of data or applications react differently depending on the chart used.

IV. RESULTS

This section will show the most important papers related to the theme, especially those from the last decade (see table II). According to the classification method used in this review, the order of the topics to treat is as follows: Markov chain approaches; as mentioned in the past section, all of the papers are related to the Markov Chain implementation in the use of control chart to monitor quality processes.

Authors, such as Bai and Lee [5] used the Markov chain approach to derivate formulas as the average time to signal and average numbers of switches to signal.

According to Fu, et al. [6], various applications of the Markov chain approach rely on discretizing a continuous variable. They made a distinction between natural and artificial discretization and mentioned examples for both: "Examples where discretization arises naturally are charts with discrete monitoring statistics (such as counts) and charts with a continuous monitoring statistic which effectively divide the values of the statistic into two more regions" [6]. Examples where discretization is carried out artificially are Cusum and EWMA charts [7, 8] .

Fu, et al. [6] showed how the run length is bound to a finite Markov chain. They described rules related to natural and/or artificial discretization. In contrast Bai and Lee [5] used a Markov chain approach to derive the formulas to evaluate the Average Time to Signal and the Average Number of Switches to Signal.

TABLE II
THE STATES OF THE MARKOV CHAIN APPLIED TO CONTROL CHARTS

PUBLICATION YEAR	AUTHOR(S)	CATEGORY	
		A	*B*
1996	Antonio F.B. Costa		x
2001	James C. Fu, Fred A. Spiring, Hansheng Xie		x, Cusum, EWMA
2001	D.S. Bai, K.T. Lee		x
2003	James C. Fu, Galit Shmueli, Y.M. Chang		x, Cusum
2004	Su-Fen Yang	Economic	x
2005	Yu Chang Lin, Chao-Yu Chou	Statistic	x
2005	Maysa S. De Magalhaes, Antonio F.B. Costa, Francisco D. Moura Neto	Statistic	x, r
2005	Zachary G. Stoumbos, Marion R. Reynolds Jr.	Statistic, Economic	x, EWMA
2006	Alireza Faraz ~, Ahmad Parsian z	Statistic	t2, EWMA
2006	Yan-Kwang Chen, Kun-Lin Hsieh, Cheng-Chang Chang	Economic	x
2006	Dogan A. Serel, Herbert Moskowitz	Statistic, Economic	EWMA
2007	Su-Fen Yang, M.A. rahim	Statistic, Economic	x, s2
2008	Su-Fen Yang, Wan Yun Chen		x, s2
2009	Alireza Faraz, Erwin Saniga		x, t2
2009	Yung-Ming Chang, Tung-Lung Wu		x, Cusum, EWMA
2009	Alpaben K. Patel, Jyoti Divecha	Statistic	Cusum
2009	Seyed Taghi Akhavan Niaki, Mohammad Javad Ershadi, Mahdi Malaki	Statistic, Economic	8
2010	Su-Fen Yang, Wan Yun Chen		x
2010	Philippe Castagliola, PetrosE. Maravelakis		s2, Cusum, EWMA
2010	Pei-Hsi Lee	Statistic	x, r
2010	Antonio F.B. Costa, Marcela Aparecida Guerreiro Machado		x
2010	Antonio F.B. Costa, M.A. rahim	Statistic	x
2011	George Nenes	Economic	x, Cusum, EWMA
2011	Tzong-Ru Tsai, Jyun-You Chiang, Shing I. Chang	Statistic	X2
2011	Eugenio K. Epprecht, Marco A. De Luna, Francisco Aparisi	Economic	EWMA, 8
2012	Faraz, Kazemzadeh, Parsian, Moghadam	Economic	t2

Another approach is proposed to derive the economic - adjustment model of two dependent processes used to determine the design parameters of the *X* and cause-selecting control charts that together minimize the long-term cost resulting from processes over adjustment or under adjustment [9]. He said to be demonstrated that the expression for the economic - adjustment model is easier to obtain by the Markov chain approach.

The Markov chain approach is also used by De Magalhães, et al. [10] to measure the performance of the adaptive control charts. The properties of the T^2 chart with Double warning rules, used by Faraz and Saniga [11], where obtained using Markov chains too.

The average run length of the combined control scheme when the process is in control and out of control is computed by using the Markov chain approach [12]. A Markov chain approach is extended to derive the economic adjustment model used to determine the design parameters of the X and S^2 control charts, which together minimize the long term cost resulting from process over-adjustment or under adjustment [13].

Yang and Rahim [13], Yang and Chen [14] apply the Markov chain approach by deriving the adjusted average time to signal (AATS), used to measure the performance of the VSI control charts they proposed in their article. In the year 2009, an approach based on the use of discretization and finite Markov chain imbedding technique was done to investigate the run length properties for various control charts when the process observations are auto correlated [15].

The Markov chain model was implemented by Niaki, et al. [16] to estimate the Average Run Length in their MEWMA chart investigation. They also designed an experiment to obtain an optimal genetic algorithm applied to solve the model and estimate the optimal values of the control chart parameters.

Epprecht, et al. [17] developed a multidimensional Markov chain model to compute the ARLs of the set of EWMA control charts. In the same manner, they used an approximate algorithm to optimize the process.

A. *Statistic or Economic Design*

De Magalhães, et al. [10] contemplated the statistical model of an adaptive X and R chart. After evaluating their results with a numerical example, they conclude that using variable parameters, in their model, is statistically better than fix parameters.

An economico model was design by Stoumbos and Reynolds Jr [18], in the combination of EWMA and Shewhart X charts using variable sampling. Their economic model showed the long-run cost per time unit of using these combined charts as a funtion of their parameters. The statistic model was develop in this article in order to describe the behavior of the parameters associated with the operation of the combined charts. In the same manner, the cost of this model was quantify to analyse the proposal's reduction compared to the traditional control schemes with fix parameters.

An X chart with variable sample size and sampling interval VSSI has being shown superior to the traditional X chart with fixed sample size and sampling interval[19]. In the paper *Economic design of the VSSI X control charts for correlated data*, Chen, et al. [19] considered that the VSSI chart was still costly, for this reason they develop a cost model to know the cost per hour of using this chart in monitoring quality processes. These authors also found a difficulty in the implementation of economic design, which is related to finding the specific values of the input parameters. They propose to use sensitivity analysis to manage this difficulty.

An economic design for the EWMA chart scheme was presented by Serel and Herbert [12] in Joint economic design of EWMA control charts for mean and variance. They tried to minimize the cost of the model implementation by extending their purpose to the economic-statistical design using in-control and out of control average run lengths. In order to analyze the quality related production cost they used the Quadratic Loss Function published by Taguchi. In this manner they showed the optimal values of the variable parameters used. A statistic and economic design model was also presented by Yang and Rahim [13] applying X and S^2 control charts.

Another cost model was presented by Faraz, et al. [20] where they design an economical model for double warning lines in T^2 control charts for monitoring the process mean vector. They studied the effects of the model parameters on the chart parameters and resulting operating loss. These authors also compared possible variable ratio sampling (VRS) schemes and choose the one that yields, economically, the best proposal.

An economic and statistical – economic design was presented by Niaki, et al. [16] based on the implementation of Taguchi function approach in the Lorenzen and Vance [21] cost model. They extended these investigations to the intangible external cost along with the in and out of control ARL.

Another economic design for variable parameters was describe by Costa and Rahim [23]. They develop a cost model, to show the cost of false alarms, of finding and eliminating the assignable cause, the associated with production in an out of control state, and the cost of sampling and testing. They assume an exponential distribution. These authors tried to examine the economic advantages of implementing variable parameters X charts.

B. *Types of charts*

It is reasonable to compare VP X chart with CUSUM and EWMA charts [24], in detecting small process mean shifts, when varying the action limits between two values. After his advance, investigators in this theme, (including all of the literature found for this paper that involve variable parameters), used his contributions. According to Fu, et al. [6], many cases uses more than one control chart is used simultaneously to monitor processes. As Lin and Chou [25] mentioned in his article *Non-normality and the variable parameters X control charts*, the usual assumption when a control chart is design, is that the data is normally distributed. For them, the performance of the X chart is evaluated under non-normality.

In most cases, when an X chart is being used to control the mean of a process, some other type of chart, such as an R chart, should be used to monitor the process variance [26]. According to their advances, an underlying principle can be used to decide the size, instant to take the next sample and the width coefficient (k) used to know the control limits variation. Stoumbos and Reynolds Jr. [27] used a combined model using X charts and EWMA. The application of this new scheme could be used to reduce the sampling effort and cost necessary to insure a required detection capability.

Another type of chart that was used to implement variable parameters is the T^2. The article wrote by Faraz and Parsian used double warning lines in this chart using VSI and VSS. They compared the T^2 chart with the MEWMA and said that the second one give better results in small mean shifts, but in larger ones their proposal provide lower costs associated with nonconforming products. It is very important to highlight that this authors showed that using double warning lines improves the performance of the chart using VSI and VSS.

In the year 2006 an economic-statistical design was done by Serel and Moskowitz using EWMA charts. Instead, Yang and Rahim [13] made a proposal based on X and S^2 . In this paper the authors showed the application of the X and S^2 control charts in order to control the over-adjustment processes. Their objective was to find a model that provides economic adjustments and the optimal parameters so that the average long-terms cost of the process is minimized and the proposed control charts have the required statistical properties.

A CUSUM based chart was develop by Patel and Divecha [28] named the ACUSUM which represent an autocorrelated cumulative sum control chart. This chart detect shifts of all kinds with first order autocorrelated process data.

The main disadvantage of an R chart is its slowness to signal small increases on the variability [29]. In the article *Adaptive R charts with variable parameters* the author, Pei-His Lee Lee [29], proposed a method to improve the efficiency in signaling increases on the variances. He conclude that VP R chart is faster signalin small process variance increases compared to other adaptive R charts, pointing that adaptive R charts are very similar but R charts required fewer items to be inspected. He propose the VP R chart to be used, when signal small standard deviation, if not, then use R charts to control its standard deviation.

Another author who wrote about different charts was Nenes [30]. He evaluate the cost of using X, EWMA and CUSUM charts and conclude that EWMA and CUSUM charts are superior when varying the parameters than X charts.

Non-central chi-square charts are more effective than the joint X and R charts in detecting small mean shifts or variance changes of a performance variable [31]. A disadvantage highlighted from these authors in the implementation of Chi-square charts is its cost. The article propose a method that simplifies the process monitoring when the user only matters if the process is in or out of control. In the same year Epprecht, et al. [17] used EWMA charts to monitor correlated variables. They found the values for the parameters of the charts that minimize the average run length (ARL) for a given process shift.

C. Other aplications

Investigations have being made on the performance of the continuous sampling plan 1 (CSP-1), when production run lengths are short or moderate or when the input control is not independent identically distributed. These authors proposed average outgoing quality limits (AOQL) to supplement the tables MIL–STD–1235B for use in the CSP-1 plans. The objective of their article was to examine the effectiveness of the CSP-1 plan having short runs, moderated length or when the input processes do not preserve.

McShane and Turnbull [4] affirm that you should be very careful in the way AOQ or AOQL are interpreted, since they are the measures of the effectiveness of CSP-1 plans. They also state that even though processes are statistically in control, the long run average measures can be deceiving for finite production run, because the AOQ and AOQL may differ from their finite run counterparts, and they do not contemplate variability. Barbosa Correa, et al. [32] focuses their study in CSP sampling plans for acceptance using Markovian and Bayesian revisions. They applied their proposal to production in series and by lots that support the quality activities and reduction of costs by inspection. In their paper, they conclude that business or a person who considered a 100% correct the prediction done by inspectors, inspecting tools and machines, can have economic disadvantages.

V. CONCLUSIÓN

After analyzing the literature, the authors observed that there is plenty information about the implementation of the Markov chain in X charts, but there is not much in others such as T2, S2 and R. Some articles where develop with a design of EWMA and CUSUM charts, but for the authors' appreciation the implementation of these charts is beneficial in some circumstances since they yield to better economic and statistic results.

In addition to the validation of each article, and other important aspect found in the search is that all of these authors used the X charts to implement their methods. Most of them agree in the implementation of Variable Parameters and validate different methods where the use of this type of sampling is better than fix parameters. It was not easy to find literature about the implementation of the Markov chain in other type of monitoring quality processes, but the authors analyzed double sampling as an implementation of Markov in the evaluation of the quality of lots. An interesting investigation can include both processes at the time: Double sampling and variable parameters.

Another important aspect that authors conclude from this review is that there is not much information about dependent processes, and they exists lots of processes that depend from a past step and the expected results can be affected by it.

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