An Exact Solution of the Problem of Unsteady MHD Flow Through Parallel Porous Plates

C.K.Kirubhashankar, Dr.S.Ganesh, and A.Mohamed Ismail

***Abstract*—** This Paper discusses the exact solution of unsteady MHD flow through parallel porous plates with uniform suction at the lower plate. The fluid is driven by a constant pressure gradient and an external uniform field is applied parallel to the plates. The effect of the variation in the viscosity and electric conductivity of the fluid and the uniform magnetic field on the velocity is discussed. Analytical expression is given for the velocity field and the effects of the various parameters of the governing parameters entering into the problem are discussed with the help of graph.

***Keywords*—** Fluid flow, Parallel porous Plates, MHD flow

I. Introduction

MHD fluid flow in a parallel plate channel is an interesting area in the study of fluid mechanics because of its relevance to various engineering applications. The MHD flow in the planar channels leads to a startup process implying thereby a viscous layer at the boundary is suddenly set into motion and becomes important in the application of various branches of geophysics, astrophysics and fluid engineering. Magneto hydrodynamic flow has many applications in aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, accelerators, fluid droplets, MHD pumps, power generators and purification of crude oil. Flow through porous medium have numerous engineering and geophysical applications. Hartmann and Lazarus [1] studied the influence of a transverse uniform magnetic field on the flow of a conducting fluid between two infinite parallel, stationary and insulated plates. The equations which describe the MHD flow are a combination of continuity equation and Navier-Stokes equations of fluid dynamics. The governing equations are differential equations that have to be solved either analytically or numerically. Berman A.S [2] studied the laminar flow in channel with porous walls. Hartmann J. [3] considered the magnetic field in the laminar flow of an electrically conducting liquid. Hassaninen I. A. and Mansour M.A. [4] has investigated the magnetic flow through the porous medium between two infinite plates. Hamza E.A [5]has studied the suction and injection effects of flow between parallel plates. Soundalgekar V. and Uplekar [6] studied the studied the effect of heat transfer considering constant temperature. Sing & Ram [7] considered laminar flow of an electrically conducting fluid through a channel in the presence of transverse magnetic field under the influence of periodic pressure gradient and solved the resulting differential equation by the method of Laplace transform. Cox.S.M [8] considered the two dimensional flow of a viscous fluid in a channel with porous walls. Craik and Criminale [9,10] described a procedure for finding classes of exact solutions of the Navier-Stokes equations. A somewhat similar formulation was given by Lagnado *et al*. [11], but was restricted to two-dimensional basic flows. Ganesh [12] studied unsteady MHD stokes flow of a viscous fluid between two parallel porous plates. They considered the fluid being withdrawn through both walls of the channel at the same rate.

In this paper, the unsteady flow of an incompressible viscous fluid between two parallel porous plates when there is a periodic at the lower plate is

A.Mohamed Ismail, Research Scholar, Sathyabama University, Chennai email: ismailbtl@yahoo.co.in, 09445123960

Dr.S.Ganesh, Professor, Sathyabama University, Chennai, email: sganesh19 @yahoo.com, 09444168002.

C.K.Kirubhashankar, Research Scholar, Sathyabama University, Chennai email: kirubha.c.k@gmail.com, 09445274183

II. Model Formulation

The flow of an incompressible viscous fluid between two parallel porous plates $y = 0$ and $y = h$ is considered in the presence of a transverse magnetic field which is applied perpendicular to the walls.

Let u and v be the velocity components in the x and y directions respectively in the flow field at time t.

The equation of continuity is $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ (1)

Equations of momentum are:

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\rho \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \sigma_e B_0^2 v \tag{3}$$

III. Assumption

1. The Plates are porous.
2. MHD flow is considered.
3. Flow between non conducting two parallel plates.
4. u and v are velocity components in the direction of x and y respectively.

IV. Notations

ρ - Density of the fluid

h - Height of the channel

μ - Coefficient of viscosity

ψ - Stream function
η - Dimensionless distance
σ - Electrical conductivity of the fluid
U - Axial component of the velocity
v - Radial Component of the velocity
B_0 - Electromagnetic induction

V. General Solution of Analytical Problem

With the help of discussions in the previous sections, Let us choose the solutions of the equations (1)-(3) respectively as

$$\left.\begin{aligned} u &= u(x,y)e^{i\omega t} \\ v &= v(x,y)e^{i\omega t} \\ p &= p(x,y)e^{i\omega t} \end{aligned}\right\} \tag{4}$$

With the boundary conditions

$$\left.\begin{aligned} u(x,0) &= 0, \qquad & u(x,h) &= U \\ v(x,0) &= -v_0 \qquad & v(x,h) &= 0 \end{aligned}\right\} \tag{5}$$

The stream function $\psi(x,y)$ is defined as

$$\psi(x,\eta) = \left(hu(0) - v_0 x\right) f(\eta) \tag{6}$$

$$u(x,y) = \frac{\partial \psi}{\partial y} \;\&\; v(x,y) = -\frac{\partial \psi}{\partial x} \tag{7}$$

From equations (2), (3) and (6), we have

$$\rho i\omega \frac{\partial \psi}{\partial y} = -\frac{\partial p}{\partial x} + \mu \frac{\partial}{\partial y}\left(\nabla^2 \psi\right) \tag{8}$$

$$-\rho i\omega \frac{\partial \psi}{\partial x} = -\frac{\partial p}{\partial y} + \mu \frac{\partial}{\partial x}\left(\nabla^2 \psi\right) - \sigma_e B_0^2 \frac{\partial \psi}{\partial x} \tag{9}$$

Using equations (6) & (7) in (8) and (9), we get

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = \left(u(0) - \frac{v_0}{h}x\right)\left[i\omega f' - \frac{v}{h^2} f'''(\eta)\right] \tag{10}$$

$$-\frac{1}{\rho h}\frac{\partial p}{\partial \eta} = i\omega v \; f(\eta) - \frac{v v_0}{h^2} f''(\eta) - \sigma B_0^2 v_0 f(\eta) \tag{11}$$

Partially differentiation of equations (10) & (11) with respect to 'η' & 'x' respectively, we have

$$\frac{\partial^2 p}{\partial \eta \partial x} = \left(h(0) - \frac{v_0 x}{h}\right)\frac{\partial}{\partial \eta}\left(i\omega f' - \frac{v}{h^2} f'''(\eta)\right) \tag{12}$$

$$\frac{\partial^2 p}{\partial x \partial \eta} = 0 \tag{13}$$

$$\left[\nabla^2 - \left(\frac{i\omega\rho}{\mu}\right)h^2\right]\nabla^2 \psi = 0 \tag{14}$$

Substituting (6) in (14), we have

$$f^{iv}(\eta) - \alpha^2 h^2 f''(\eta) = 0, \tag{15}$$

where $\alpha^2 = \dfrac{i\rho\omega}{\mu}$

VI. Mathematical Solution of Formation of the Problem

Equation (15) reduces to the form

$$(D^4 - \alpha^2 h^2 D^2) f(\eta) = 0 \tag{16}$$

With the boundary conditions

$$\left.\begin{aligned} f(0) &= -1, \qquad & f(1) &= 0 \\ f'(0) &= 0, \qquad & f'(1) &= 0 \end{aligned}\right\} \tag{17}$$

Hence the solution of (16) subjecting to the boundary condition (17) is

$$f(\eta) = c_1 + c_2\eta + c_3 e^{\alpha h \eta} + c_4 e^{-\alpha h \eta}$$

Substituting the value of $f(\eta)$ in the stream function

$$\psi(x,\eta) = \left(hu(0) - v_0 x\right) f(\eta)$$

Hence

$$\begin{aligned} u &= u(x,y)e^{i\omega t} \\ &= \frac{\partial \psi}{\partial y} e^{i\omega t} \\ &= \left(u(0) - \frac{v_0 x}{h}\right)\alpha h e^{i\omega t} \\ &\quad \times \left[\frac{\sinh \alpha h - \sinh \alpha y - \sinh \alpha (y-h)}{2 + \alpha h \sinh \alpha h - \cosh \alpha h}\right] \end{aligned}$$

$$\begin{aligned} v &= v(x,y)e^{i\omega t} \\ &= -\frac{\partial \psi}{\partial x} e^{i\omega t} \\ &= v_0 e^{i\omega t}\left[\frac{3 + \alpha(y+h)\sinh \alpha h - 2\cosh \alpha h - 2\cosh \alpha y + 2\cosh \alpha(y-h)}{2 + \alpha h \sinh \alpha h - \cosh \alpha h}\right] \end{aligned}$$

VII. Results and Discussion

The numerical values of *u* and *v* velocity profiles have been calculated for different values of *x* and *y*. It is assumed that profiles at different cross sections of the channel and different suction. Form the figures it is seen that the magnitude of the axial velocity increases as x increases from 1 to 6 for different values of ωt. From Fig 2 and Fig 4 when v_0 is increased to 3 from 1, it is seen that magnitude of the axial velocity increases. It is also seen that in fig 8, the radial velocity vanishes for ωt = π/2 and radial velocity profiles are non linear for the other values of ωt. The above is true for all values of v_0.The radial velocity profile for different values of v_0 when ωt = 0.

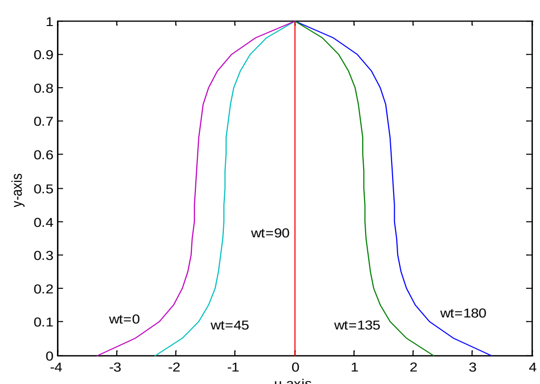

Fig. 1 Axial velocity profile for u0=0.5; v0=-1;x=1;y=0:0.5:1;h=1; α=10.

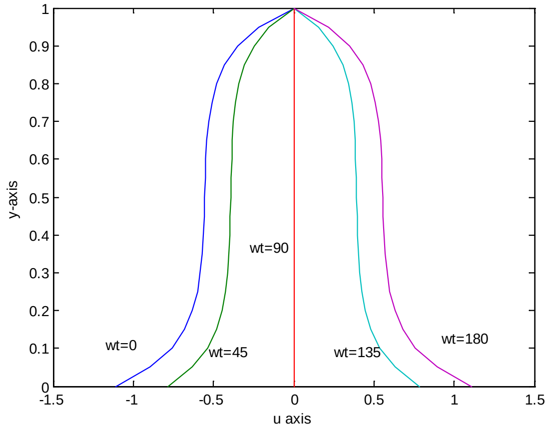

Fig. 2 Axial velocity profile for u0=0.5; v0=1;x=1;y=0:0.5:1;h=1; α=10.

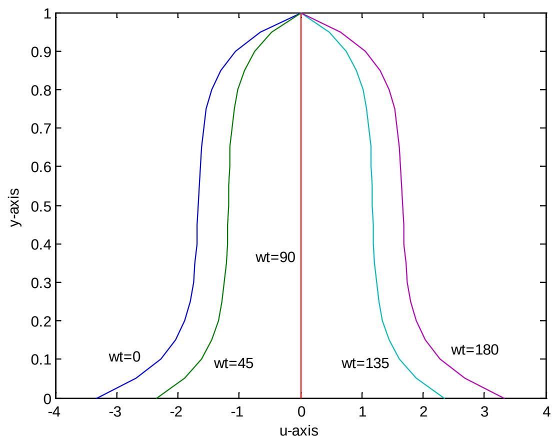

Fig. 3 Axial velocity profile for u0=0.5; v0=2;x=1;y=0:0.5:1;h=1; α=10.

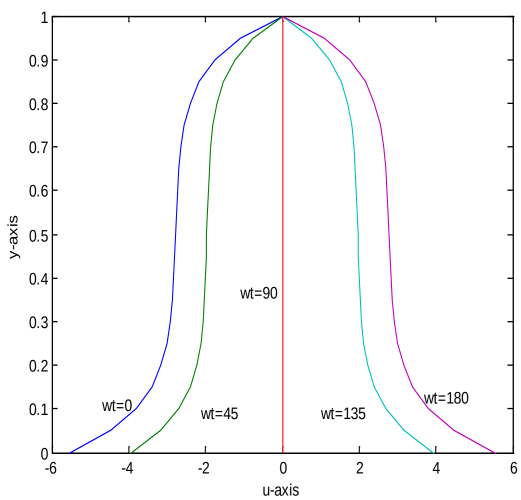

Fig. 4 Axial velocity profile for u0=0.5; v0=3;x=1;y=0:0.5:1;h=1; α=10.

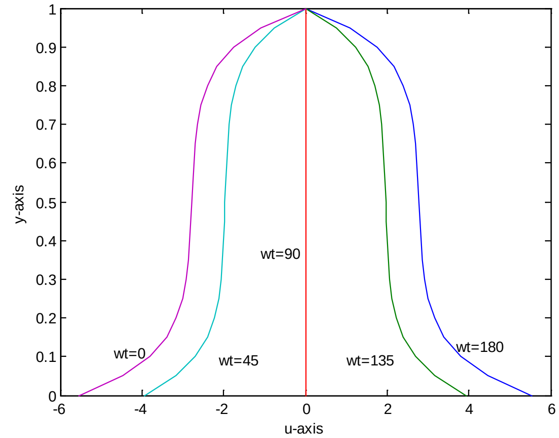

Fig. 5 Axial velocity profile for u0=0.5; v0=3;x=2;y=0:0.5:1;h=1; α=10.

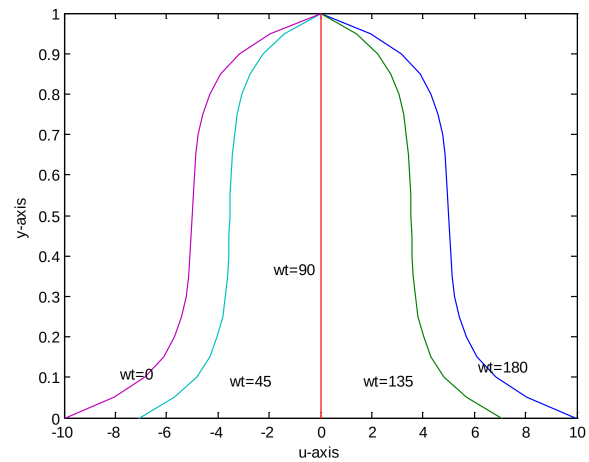

Fig. 6 Axial velocity profile for u0=0.5; v0=3;x=4;y=0:0.5:1;h=1; α=10.

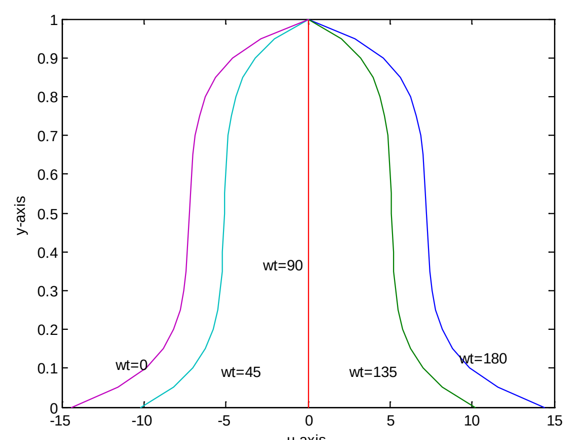

Fig. 7 Axial velocity profile for u0=0.5; v0=3;x=6;y=0:0.5:1;h=1; α=10

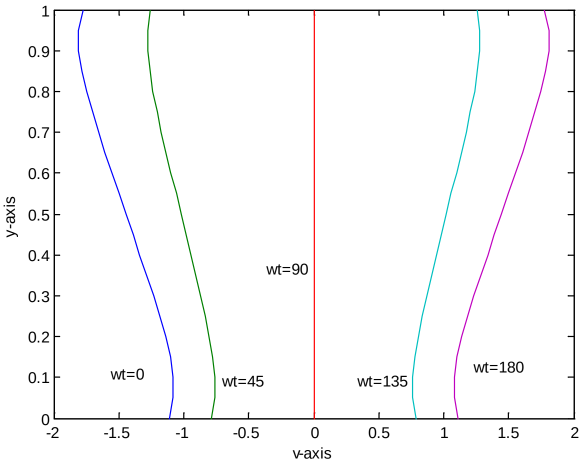

Fig. 8 Radial velocity profile for u0=0.5; v0=-1;x=1;y=0:0.5:1;h=1; α=10.

VIII. CONCLUSION

In the above analysis a class of solutions of the unsteady stokes flow of viscous fluid between two parallel porous plates is presented, when there exists variable suction at the lower plate.

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