Nonlocal Vibration Behaviour of a Pasternak Bonded Double-Piezoelectric-DWBNNT-Reinforced Microplate-System

H. Rahimipour*, A. Ghorbanpour Arani, and G. A. Sheikhzadeh

Abstract—The aim of the paper is to analyze electro-thermo nonlinear vibration of a double-piezoelectric composite microplate-system (DPCMPS) based on nonlocal piezoelasticity theory. The two microplates are assumed to be connected by an enclosing elastic medium which is simulated by Pasternak foundation. Both of piezoelectric composite microplates are made of poly-vinylidene fluoride (PVDF) reinforced by zigzag double walled boron nitride nanotubes (DWBNNTs). The micro-electromechanical model is employed to calculate mechanical, thermal and electrical properties of composite. Differential quadrature method (DQM) is employed to solve the governing differential equations. The frequency ratio of DPCMPS is investigated for three typical vibrational states, namely, out-of-phase, in-phase and the case when one microplate is fixed in the DPCMPS. A detailed parametric study is conducted to scrutinize the influences of the small scale coefficient and stiffness of the internal elastic medium.

Keywords— Nonlinear vibration; Coupled system; Nonlocal piezoelasticity; DWBNNTs.

I. Introduction

NANOCOMPOSITES hold the promise of advances that exceed those achieved in recent decades in composite materials. The nanostructure created by a nanophase in polymer matrix represents a radical alternative to the structure of conventional polymer composites. These complex hybrid materials integrate the predominant surfaces of nanoparticles and the polymeric structure into a novel nanostructure, which produces critical fabrication and interface implementations leading to extraordinary properties [1].

In recent years, small scale effect in micro- and nano-applications of beam, plate, and shell-type structures has been utilized on the basis of nonlocal elasticity theory which was initiated in the papers of Eringen [2]. Shen et al. [3] investigated nonlocal plate model for nonlinear vibration of single layer graphene sheets (SLGS) in thermal environments. Pradhan and Kumar [4] reported vibration analysis of orthotropic graphene sheets using nonlocal elasticity theory and DQM. Ghorbanpour et al. [5] studied Pasternak foundation effect on the axial and torsional waves propagation in the embedded double-walled carbon nanotubes (DWCNTs) using nonlocal elasticity cylindrical shell theory. The above studies on the nanostructures are on the basis of the nonlocal elasticity theory, which is not proper for a direct use in the piezoelectric materials. Recently, the Eringen's nonlocal elasticity theory was extended by Zhou et al. [6] for the piezoelectric materials. In the nonlocal piezoelectric materials, the stress state and the electric displacement at a given point are, respectively, as a function of the strain state and electric potential of all points in the body. Ke et al. [7] employed nonlocal piezoelasticity model to nonlinear vibration analyze of the piezoelectric nanobeams.

With respect to developmental works on mechanical behavior analysis of nano- and micro-plates, it should be noted that none of the researches mentioned above, have considered a coupled double-plate-system. Herein, Murmu and Adhikari [8] analyzed vibration of nonlocal double-nanoplate- system (NDNPS). Also, buckling behavior of the NDNPS was investigated by Murmu et al. [9]. Exact solution for nonlocal vibration of double-orthotropic nanoplates embedded in elastic medium was reported by Pouresmaeeli et al. [10]. The three papers have considered the Winkler model for simulation of elastic medium between two nanoplates. Recently, analysis of the coupled system of double-layered graphene sheets (CS-DLGSs) embedded in a visco-Pasternak foundation is carried out by Ghorbanpour et al. [11]. To the best of our knowledge, none of the works in the literature have taken in to account the nonlinear terms in the governing equations for a coupled system. This study aims to consider nonlinear terms for vibration analysis of a DPCMPS in which two microplates are connected by an enclosing Pasternak foundation. None of the aforementioned studies have considered smart coupled structures while these structures may be used in mechanical behavior control of coupled micro- and nano-structures. Recently, Buckling analysis and smart control of SLGS using elastically coupled PVDF nanoplate using the nonlocal piezoelasticity were studied by Ghorbanpour et al. [12].

However, to date, no report has been found in the literature on the vibration of an elastically coupled DPCMPS. Motivated

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by these considerations, in order to improve optimum design of smart microstructure, we aim to study the electro-thermo nonlinear nonlocal vibration of an elastically coupled DPCMPS. Herein, the two PVDF microplates reinforced by DWBNNTs are coupled by an enclosing Pasternak foundation. Considering the nonlinear strain-displacement relations and charge equation, the nonlinear governing equations are derived using energy method and Hamilton's principle. Hence, the DQM is presented to solve the nonlinear governing equations and estimate the frequency ratio of clamped supported DPCMPS. In present study, the influences of nonlocal parameter and elastic medium constants have been taken into account.

II. Formulation

A. Classical plate theory

Based on the classical plate theory (CPT) which satisfies Kirchhoff assumption, the strain equations in terms of the mid-plane displacements are derived as follows [13]

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}+\dfrac{1}{2}(\dfrac{\partial w_0}{\partial x})^2\\ \dfrac{\partial v_0}{\partial y}+\dfrac{1}{2}(\dfrac{\partial w_0}{\partial y})^2\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial y}+\dfrac{\partial w_0}{\partial x}\dfrac{\partial w_0}{\partial y}\end{Bmatrix}+z\begin{Bmatrix}-\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix}, \quad (1)$$

B. Modeling of the problem

An elastically coupled DPCMPS having the length l, the width b and the thickness h, assuming that $h << l,b$, is shown in Fig. 1. The origin of the Cartesian coordinate system is considered at one corner of the middle surface of the microplate. The x, y and z axes are taken along the length, width, and thickness of the microplates, respectively. The two microplates are made from PVDF and reinforced by DWBNNTs in x-direction so that both microplates are identical. The DPCMPS is subjected to uniform temperature change and polarized in x-direction. The two microplates are coupled by an elastic medium which is simulated by the Pasternak foundation. As is well known this foundation model is characterized by two parameters: the Winkler modulus k_w and shear modulus k_g.

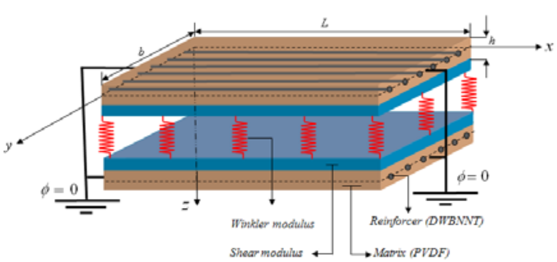


Fig. 1 Schematic of double-smart composite microplate-system

C. Constitutive equations for piezoelectric materials

In a piezoelectric material, application of an electric field to it will cause a strain proportional to the mechanical field strength, and vice versa. According to a piezoelectric microplate under electro-thermal loads, constitutive equations can be represented as [12]:

$$(1-\mu\nabla^2)\begin{bmatrix}\sigma_{xx}^{nl}\\ \sigma_{yy}^{nl}\\ \sigma_{xy}^{nl}\end{bmatrix}=\begin{bmatrix}\overline{C}_{11} & \overline{C}_{12} & 0\\ \overline{C}_{21} & \overline{C}_{22} & 0\\ 0 & 0 & \overline{C}_{66}\end{bmatrix}\left(\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}-\begin{pmatrix}\alpha_x\\ \alpha_y\\ 0\end{pmatrix}\Delta T\right)-\begin{bmatrix}e_{11} & 0 & 0\\ e_{12} & 0 & 0\\ 0 & e_{26} & 0\end{bmatrix}\begin{Bmatrix}E_{xx}\\ E_{yy}\\ E_{zz}\end{Bmatrix}, \quad (2)$$

$$(1-\mu\nabla^2)\begin{bmatrix}D_{xx}^{nl}\\ D_{yy}^{nl}\\ D_{xy}^{nl}\end{bmatrix}=\begin{bmatrix}e_{11} & e_{12} & 0\\ 0 & 0 & e_{26}\\ 0 & 0 & 0\end{bmatrix}\left(\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}-\begin{pmatrix}\alpha_x\\ \alpha_y\\ 0\end{pmatrix}\Delta T\right)-\begin{bmatrix}\in_{11} & 0 & 0\\ 0 & \in_{22} & 0\\ 0 & 0 & \in_{22}\end{bmatrix}\begin{Bmatrix}E_{xx}\\ E_{yy}\\ E_{zz}\end{Bmatrix}, \quad (3)$$

where $e_{ij}, \in_{ij}\ (i,j=1,\ldots,6), \alpha_k\ (k=x,y)$ and ΔT are piezoelectric constants, dielectric constants, thermal expansion coefficients and temperature gradient, respectively. Electric field tensor E can be written in term of electric potential ϕ as [12]:

$$E=-\nabla\phi. \quad (4)$$

Using approach adopted by Tan and Tong [14] in which they use representative volume element (RVE) based on micro-electro-mechanical models, the mechanical, thermal and electrical properties of the DPCMPS can be obtained from Ref. [14].

D. Equations of motion

The governing differential equations of motion are derived using the Hamilton's principle which is given as [12]:

$$\int_0^T(\delta U+\delta V-\delta K)dt=0 \quad (5)$$

Where δU is the virtual strain energy, δV is the virtual work done by external applied forces and δK is the virtual kinetic energy. With defining dimensionless parameters as follows

$$\begin{gathered}\beta_x=\frac{h}{l},\quad \beta_y=\frac{h}{b},\quad Q_1=\frac{\overline{C}_{12}}{\overline{C}_{11}},\quad Q_2=\frac{\overline{C}_{22}}{\overline{C}_{11}},\\ Q_3=\frac{\overline{C}_{66}}{\overline{C}_{11}},\quad T_x=\alpha_{xx}T, T_y=\alpha_{yy}T,\quad K_G^*=\frac{k_g}{\overline{C}_{11}l},\\ K_W^*=\frac{k_w l}{\overline{C}_{11}},\quad \overline{m}_0=\frac{m_0}{\rho_0 l},\quad \overline{m}_2=\frac{m_2}{\rho_0 l^3},\\ \Omega=\omega l\sqrt{\frac{\rho_0}{\overline{C}_{11}}},\quad \phi_0=l\sqrt{\frac{\overline{C}_{11}}{\in_{11}}},\quad \overline{e}_1=\frac{e_{11}\phi_0}{\overline{C}_{11}l}=\frac{e_{11}}{\sqrt{\overline{C}_{11}\in_{11}}},\\ \overline{e}_2=\frac{e_{12}\phi_0}{\overline{C}_{11}l}=\frac{e_{12}}{\sqrt{\overline{C}_{11}\in_{11}}},\quad \overline{e}_3=\frac{e_{26}\phi_0}{\overline{C}_{11}l}=\frac{e_{26}}{\sqrt{\overline{C}_{11}\in_{11}}},\\ \mu^*=\frac{\mu}{l^2}, \zeta=\frac{x}{l},\quad \eta=\frac{y}{b},\quad W^*=\frac{W}{h},\quad \Phi=\frac{\phi}{\phi_0}.\end{gathered} \quad (6)$$

The dimensionless nonlinear nonlocal motion equations of DPCMPS can be written as

$$
\begin{aligned}
&\frac{-1}{12}\beta_x^4\frac{\partial^4 W_{(m)}^*}{\partial\zeta^4}-\frac{1}{6}\beta_x^2\beta_y^2(Q_1+2Q_3)\frac{\partial^4 W_{(m)}^*}{\partial\eta^2\partial\zeta^2}\\
&-\frac{1}{12}Q_2\beta_y^4\frac{\partial^4 W_{(m)}^*}{\partial\eta^4}-(T_x+Q_1T_y)\beta_x^2\frac{\partial^2 W_{(m)}^*}{\partial\zeta^2}\\
&+\beta_x^2\overline{e}_1\frac{\partial^2 W_{(m)}^*}{\partial\zeta^2}\frac{d\Phi_{(m)}}{d\zeta}+\beta_y^2(Q_1T_x+Q_2T_y)\frac{\partial^2 W_{(m)}^*}{\partial\eta^2}+\\
&\beta_y^2\overline{e}_2\frac{\partial^2 W_{(m)}^*}{\partial\eta^2}\frac{d\Phi_{(m)}}{d\eta}+\Omega^2\left[\begin{array}{l}\overline{m}_0\beta_xW_{(m)}^*-(\mu^*\overline{m}_0+\overline{m}_2)\\(\beta_x\frac{\partial^2 W_{(m)}^*}{\partial\zeta^2}+\frac{\beta_y^2}{\beta_x}\frac{\partial^2 W_{(m)}^*}{\partial\eta^2})\end{array}\right.\\
&+\mu^*\overline{m}_2(\beta_x\frac{\partial^4 W_{(m)}^*}{\partial\zeta^4}+2\frac{\beta_y^2}{\beta_x}\frac{\partial^4 W_{(m)}^*}{\partial\eta^2\partial\zeta^2}\\
&\left.+\frac{\beta_y^4}{\beta_x^3}\frac{\partial^4 W_{(m)}^*}{\partial\eta^4})\right]+F_{(m)}=0,\quad (m)=1,2
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
&\beta_x^4\overline{e}_1(\frac{\partial W_{(m)}^*}{\partial\zeta}\frac{\partial^2 W_{(m)}^*}{\partial\zeta^2})+\overline{e}_2\beta_x^2\beta_y^2(\frac{\partial W_{(m)}^*}{\partial\eta}\frac{\partial^2 W_{(m)}^*}{\partial\eta\partial\zeta})\\
&+\overline{e}_3\beta_x^2\beta_y^2(\frac{\partial W_{(m)}^*}{\partial\eta}\frac{\partial^2 W_{(m)}^*}{\partial\eta\partial\zeta}+\frac{\partial W_{(m)}^*}{\partial\zeta}\frac{\partial^2 W_{(m)}^*}{\partial\eta^2})\\
&-\beta_x^2\frac{d^2\Phi_{(m)}}{d\zeta^2}=0,\quad (m)=1,2
\end{aligned}
\tag{8}
$$

Where

$$
\begin{aligned}
F_1=&+\mu^*K_W^*\left(\beta_x(\frac{\partial^2 W_{(1)}^*}{\partial\zeta^2}-\frac{\partial^2 W_{(2)}^*}{\partial\zeta^2})+\frac{\beta_y^2}{\beta_x}(\frac{\partial^2 W_{(1)}^*}{\partial\eta^2}-\frac{\partial^2 W_{(2)}^*}{\partial\eta^2})\right)-\\
&\mu^*K_G^*\left(\beta_x(\frac{\partial^4 W_{(1)}^*}{\partial\zeta^4}-\frac{\partial^4 W_{(2)}^*}{\partial\zeta^4})+2\frac{\beta_y^2}{\beta_x}(\frac{\partial^4 W_{(1)}^*}{\partial\eta^2\partial\zeta^2}-\frac{\partial^4 W_{(2)}^*}{\partial\eta^2\partial\zeta^2})\right.\\
&\left.+\frac{\beta_y^4}{\beta_x^3}(\frac{\partial^4 W_{(1)}^*}{\partial\zeta^4}-\frac{\partial^4 W_{(2)}^*}{\partial\zeta^4})\right)-K_W^*\beta_x(W_{(1)}^*-W_{(2)}^*)\\
&+K_G^*\left(\beta_x(\frac{\partial^2 W_{(1)}^*}{\partial\zeta^2}-\frac{\partial^2 W_{(2)}^*}{\partial\zeta^2})+\frac{\beta_y^2}{\beta_x}(\frac{\partial^2 W_{(1)}^*}{\partial\eta^2}-\frac{\partial^2 W_{(2)}^*}{\partial\eta^2})\right),\\
&\beta_x^4\overline{e}_1(\frac{\partial W_{(m)}^*}{\partial\zeta}\frac{\partial^2 W_{(m)}^*}{\partial\zeta^2})+\overline{e}_2\beta_x^2\beta_y^2(\frac{\partial W_{(m)}^*}{\partial\eta}\frac{\partial^2 W_{(m)}^*}{\partial\eta\partial\zeta})\\
&+\overline{e}_3\beta_x^2\beta_y^2(\frac{\partial W_{(m)}^*}{\partial\eta}\frac{\partial^2 W_{(m)}^*}{\partial\eta\partial\zeta}+\frac{\partial W_{(m)}^*}{\partial\zeta}\frac{\partial^2 W_{(m)}^*}{\partial\eta^2})\\
&-\beta_x^2\frac{d^2\Phi_{(m)}}{d\zeta^2}=0,\quad (m)=1,2
\end{aligned}
\tag{9}
$$

Where subscript (1) and (2) denote the upper and the lower microplates.

E. Vibrational states of DPCMPS

In this paper, vibrational states of DPCMPS are including out-of-phase sequence, in-phase sequence, and one-microplate being stationary.

F. Out-of-phase vibration

In this case, both microplates vibrate asynchronously, and $W_{(1)}^*-W_{(2)}^*\neq 0$.

G. In-phase vibration

In this case, both microplates vibrate synchronously, hence the relative displacement between them disappear $(W_{(1)}^*=W_{(2)}^*)$. Thus, any one of the two microplates could represent the vibration of the coupled system.

H. One microplate being stationary

In this case, one microplate is fixed in the DPCMPS (i.e. $W_{(2)}^*=0$). In mentioned case, the DPCMPS behaves as if upper microplate is embedded in Pasternak foundation.

In order to carry out the eigenvalue analysis, the domain and boundary points are separated and in vector forms they are denoted as {d} and {b}, respectively. Hence, the discretized form of the motion equations together with the boundary conditions can be expressed in matrix form using DQM as

$$
\left(\underbrace{\left[K_L+K_{NL}\right]}_{[K]}-\Omega^2\left[M\right]\right)\begin{bmatrix}\{d\}\\\{b\}\end{bmatrix}=0, \tag{10}
$$

In which $[M]$, $[K_L]$ and $[K_{NL}]$ are the mass matrix, linear stiffness matrix and nonlinear stiffness matrix.

III. Numerical results and discussion

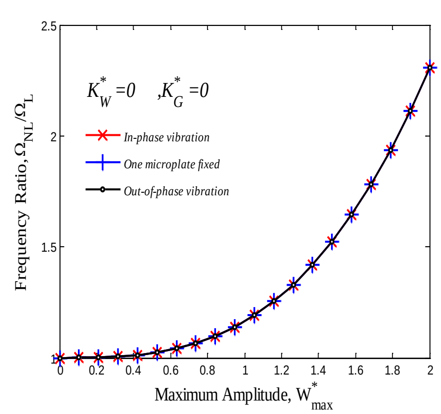
The final converged solution using the numerical procedure using DQM is illustrated as the influences of the elastic medium and nonlocal parameter on the frequency ratio of the DPCMPS. The frequency ratio is defined as

$$
\text{Frequency Ratio}=\frac{\Omega_{NL}}{\Omega_L},
$$

Where Ω_{NL} and Ω_L are the nonlinear and linear frequencies of the DPCMPS, respectively.

In order to show the effect of dimensionless coupling elastic medium between two piezoelectric composite microplates, the frequency ratio (Ω_{NL}/Ω_L) versus the dimensionless maximum amplitude $(W_{\max}^*)$ is demonstrated in Fig. 3a-d for three different cases of vibration characteristic. In general, the frequency ratio decreases with increasing elastic medium constants. This is because increasing Winkler and Pasternak coefficients increases the system stiffness. The frequency ratio of cases 1 and 2 decreases with increasing elastic medium constants.

a



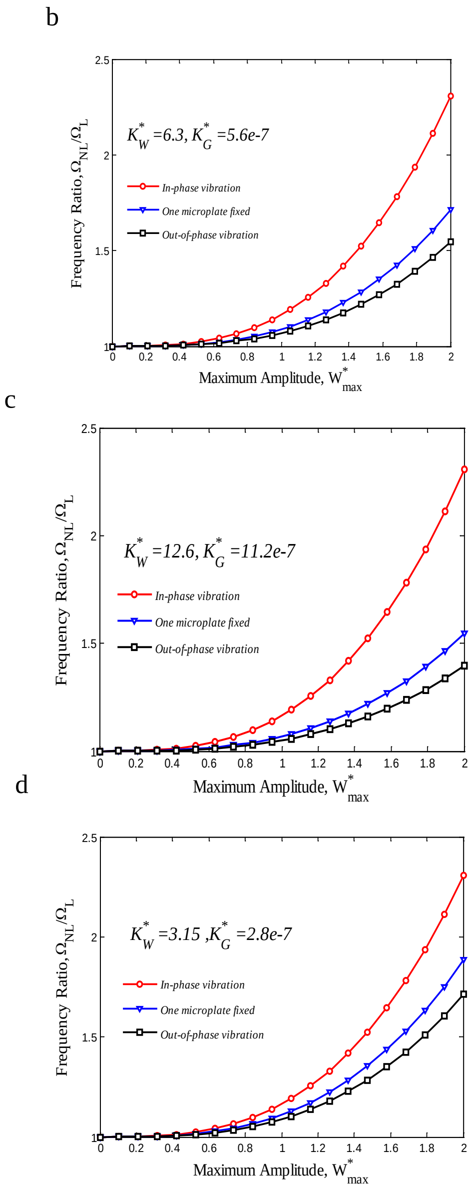


Fig. 3 The effect of dimensionless coupling elastic medium on the frequency ratio versus the dimensionless maximum amplitude.

Furthermore, Ω_{NL}/Ω_L for case 2 is higher than case 1 because in case 2, the DPCMPS treats as a single piezoelectric composite microplate with the elastic foundation effect. Therefore, the stiffness of the system in case 2 is lower than case 1 and consequently, its frequency ratio is higher with respect to case 1. It is also worth mentioning that the frequency ratio of case 3 is independent of elastic medium stiffness.

Fig. 4a-d demonstrates the effects of dimensionless nonlocal parameter (μ^*) on the frequency ratio versus the dimensionless maximum amplitude for three cases of out-of-phase vibration, vibration with one microplate fixed and in-phase vibration. It should be noted that the nonlocal parameter $\mu^*=0$ corresponds to the classical microplate without the nonlocal effect. As can be seen, the frequency ratio for the case of in-phase vibration is higher than cases of out-of-phase vibration and one microplate fixed. The higher frequency ratio for in-phase vibration is due to the absence of coupling effect of the spring's foundation between the two piezoelectric composite microplates. It is also concluded that the small scale effect in the case of in-phase vibration is higher than that in the out-of-phase vibration and one microplate fixed cases. Obviously, increasing the μ^* increases the frequency ratio.

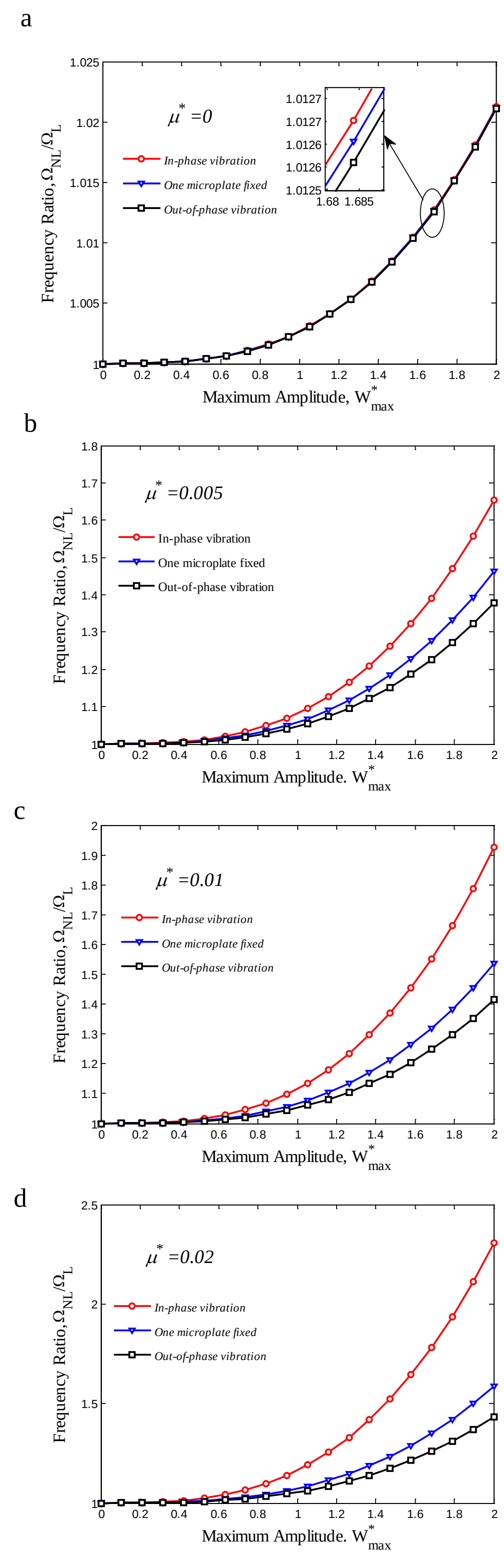


Fig. 4 The effect of dimensionless nonlocal parameter on the frequency ratio versus the dimensionless maximum amplitude

This is due to the fact that the increase of nonlocal parameter decreases the interaction force between microplate atoms, and that leads to a softer structure.

IV. Conclusion

Vibration response of piezoelectric nano/micro composites has applications in designing many NEMS/MEMS devices such as hydraulic sensors and actuators. In the present study, electro-thermo nonlinear vibration of a double-piezoelectric composite microplate made of PVDF reinforced by DWBNNTs is investigated for three typical vibrational states, namely, out-of-phase, in-phase and the case when one microplate fixed. The internal elastic medium between two microplates is simulated as Pasternak foundation. Considering charge equation, the nonlinear motion equations are derived based on nonlocal piezoelasticity theory. The DQM is applied to obtain to the nonlinear frequency ratio of the DPCMPS so that the effects of the small scale coefficient and stiffness of the internal elastic medium are discussed. The results indicate that the effects of small scale parameter in the case of in-phase vibration is higher than that in the out-of-phase vibration and one microplate fixed cases. It is also worth mentioning that the frequency ratio of the in-phase vibration is independent of elastic medium stiffness.

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References

[1] R.Kotsilkova, *Thermoset Nanocomposites for Engineering Applications*. UK: Smithers Rapra Technology, 2007.

[2] Eringen AC. “Nonlocal polar elastic continua,” *Int. J. Eng. Sci.* vol. 10, pp.1–16, 1972. http://dx.doi.org/10.1016/0020-7225(72)90070-5

[3] L. Shen, H.S. Shen and C.L. Zhang, “Nonlocal plate model for nonlinear vibration of single layer graphene sheets in thermal environments,” *Comput. Mater. Sci.* vol.48, pp.680–685, 2010. http://dx.doi.org/10.1016/j.commatsci.2010.03.006

[4] S.C. Pradhan and A. Kumar, “Vibration analysis of orthotropic graphene sheets embedded in Pasternak elastic medium using nonlocal elasticity theory and differential quadrature method,” Comput. Mater. Sci. vol.50, pp.239–245, 2010. http://dx.doi.org/10.1016/j.commatsci.2010.08.009

[5] A. Ghorbanpour Arani, A.A. Mosallaie Barzoki, R. Kolahchi and A. Loghman, “Pasternak foundation effect on the axial and torsional waves propagation in embedded DWCNTs using nonlocal elasticity cylindrical shell theory,” *J. Mech. Sci. Tech.* vol.25, pp.2385-2391, 2011. http://dx.doi.org/10.1007/s12206-011-0712-5

[6] Z.G. Zhou and B. Wang, “The scattering of harmonic elastic anti-plane shear waves by a Griffith crack in a piezoelectric material plane by using the non-local theory,” *Int. J. Eng. Sci.* vol.40, pp.303–17, 2002. http://dx.doi.org/10.1016/S0020-7225(01)00069-6

[7] L.L. Ke, Y.S. Wang and Z.D Wang, “Nonlinear vibration of the piezoelectric nanobeams based on the nonlocal theory,” *Compos. Struct.* Vol.94, pp.2038-2047, 2012. http://dx.doi.org/10.1016/j.compstruct.2012.01.023

[8] T. Murmu and S. Adhikari, “Nonlocal vibration of bonded double-nanoplate-systems,” *Compos: Part B: Eng.* vol.42, pp.1901–1911, 2011. http://dx.doi.org/10.1016/j.compositesb.2011.06.009

[9] T. Murmu, J. Sienz, J. Adhikari and C. Arnold, “Nonlocal buckling behavior of bonded double-nanoplate-systems,” *J. Appl. Phys.* vol.110, pp.084316, 2011 http://dx.doi.org/10.1063/1.3644908

[10] S. Pouresmaeeli, S.A Fazelzadeh and E. Ghavanloo, “Exact solution for nonlocal vibration of double-orthotropic nanoplates embedded in elastic medium,” *Compos: Part B: Eng.* vol.43, pp.3384–3390, 2012. http://dx.doi.org/10.1016/j.compositesb.2012.01.046

[11] A. Ghorbanpour Arani, A. Shiravand, M. Rahi and R. Kolahchi, “Nonlocal vibration of coupled DLGS systems embedded on Visco-Pasternak foundation,” *Physica B* vol.407, pp.4123–4131, 2012. http://dx.doi.org/10.1016/j.physb.2012.06.035

[12] A. Ghorbanpour Arani, R. Kolahchi and, H. Vossough, “Buckling analysis and smart control of SLGS using elastically coupled PVDF nanoplate based on the nonlocal Mindlin plate theory,” *Physica B* vol.407, pp.4458-4465, 2012. http://dx.doi.org/10.1016/j.physb.2012.07.046

[13] A. Ghorbanpour Arani, R. Kolahchi, A.A. Mosallaie Barzoki, M.R. Mozdianfard and S.M. Noudeh Farahani, “Elastic foundation effect on nonlinear thermo-vibration of embedded double-layered orthotropic graphene sheets using differential quadrature method,” *Proc. IMech. Part C: J. Mech. Eng. Sci.* vol.227, pp.1-8, 2012.

[14] P. Tan and L. Tong “Micro-electromechanics models for piezoelectric-fiber-reinforced composite materials,” *Compos. Sci. Tech.* vol.61, pp.759–769, 2001. http://dx.doi.org/10.1016/S0266-3538(01)00014-8