# Unsteady Transverse Magneto Hydrodynamic Flow between Parallel Porous Plates with Exact Solution and Stability Analysis

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***Abstract—*** In this Paper discuss exact solution of unsteady transverse MHD flow between parallel porous plates both at rest with uniform suction. External uniform axial and transverse magnetic field and uniform suction and injection are applied perpendicularly to the plates while the fluid motion is subjected to exponential axial and transverse velocity and pressure gradient. The solution of the problem is obtained with the aid of mathematical technique. Analytical expression is given for the velocity field and the effect of the various parameters entering into the problem is discussed with the help of graph.

***Keywords—*** Fluid flow, Parallel porous Plates, MHD flow

## I. INTRODUCTION

YANG and Yu [1] studied the problem of convective magneto hydrodynamic channel flow between two parallel plates subjected simultaneously to an axial temperature gradient and a pressure gradient numerically. In their conclusion they have found that an applied transverse magnetic field may reduce the entrance length of the velocity considerably, but has little effect on the temperature development. Seth and Ghosh [2] considered the unsteady hydro magnetic flow of a viscous incompressible electrically conducting fluid in a rotating channel under the influence of a periodic pressure gradient and of uniform magnetic field, which is inclined with the axis of rotation. An analytical solution to the problem of steady and unsteady hydro magnetic flow of viscous incompressible electrically conducting fluid under the influence of constant and periodic pressure gradient in presence of inclined magnetic field has been obtained exactly by Ghosh [3] to study the effect of slowly rotating systems with low frequency of oscillation when the conductivity of the fluid is low and the applied magnetic field is weak. The MHD flow between two parallel plates is called Hartmann flow. It has many Applications in MHD power generators, MHD pumps, aerodynamic heating. Hartmann and Lazarus[4] studied the influence of a transverse uniform magnetic field on the flow of a conducting fluid between two infinite plates. Then a lot of research work concerning the Hartmann flow has been obtained under different physical effects [5-13]. The Steady of flow has been carried out by several authors. Many researchers have reported that the flow is electrically conducting fluid [14-17]. The electromagnetic force(Lorentz force) acts on the flow and this force opposes the motion of flow and there by flow is impeded, so that the external magnetic field can be used in the treatment of some kinds of diseases. MHD is the fluid mechanics of electrically conducting fluids, some of these fluids include liquid metals (such as mercury, molten iron) and ironized gases known as physicists as Plasma, one example being the solar atmosphere. The subject of MHD is largely perceived to have been initiated by sure dish electrical engineer Hannes Alfven [18] in 1942. If an electrically conducting fluid is placed in a constant magnetic field, the motion of the fluid induces current which create forces on the fluid. The production of these currents has led to the design of among other devices the MHD generates for electrically production. The governing equations that have been solved either analytically or numerically.

In this paper Unsteady flow of a conducting fluid through between parallel porous plates. The fluid is acted up and down on the time development of both velocities. The effect of suction velocity on both direction of the fluid are discussed. The result differential equation is solved by an analytical method and the solution also expressed.

## II. MODEL FORMULATION

The flow of an incompressible viscous fluid between two parallel porous plates $y = 0$ and $y = h$ is considered in the presence of a transverse magnetic field which is applied perpendicular to the walls.

Let u and v be the velocity components in the $x$ and $y$ directions respectively in the flow field at time $t$.

The equation of continuity is $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$ (1)

Equations of momentum are:

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\sigma_e B_0^2 u \quad (2)$$

$$\rho\frac{\partial v}{\partial t}=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\sigma_e B_0^2 v \quad (3)$$

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## III. Assumption

1. The Plates are porous.
2. The fluid going on up and down.
3. MHD flow is considered.
4. Flow between non conducting two parallel plates.
5. Viscosity of the fluid is considered to constant.
6. *u* and *v* are velocity components in the direction of $x$ and *y* respectively.

## IV. Notations

ρ - Density of the fluid
h - Height of the channel
μ - Coefficient of viscosity
ψ - Stream function
η - Dimensionless distance
σ - Electrical conductivity of the fluid
U - Axial component of the velocity
v - Radial Component of the velocity
$B_0$ - Electromagnetic induction
$H_0$ - Transverse magnetic field

## V. General Solution of Analytical Problem

With the help of discussions in the previous sections, Let us choose the solutions of the equations (1)-(3) respectively as

$$\left.\begin{aligned} u &= u(x,y)e^{i\omega t} \\ v &= v(x,y)e^{i\omega t} \\ p &= p(x,y)e^{i\omega t} \end{aligned}\right\} \tag{4}$$

With the boundary conditions

$$\left.\begin{aligned} u(x,0) &= 0, \qquad u(x,h) = 0 \\ v(x,0) &= v_1 \qquad v(x,h) = v_2 \end{aligned}\right\} \tag{5}$$

Let the stream functions are

$$u(x,y) = \frac{\partial \psi}{\partial y} \ \& \ v(x,y) = -\frac{\partial \psi}{\partial x} \tag{6}$$

From equations (2), (3) and (6), we have

$$\rho i\omega \frac{\partial \psi}{\partial y} = -\frac{\partial p}{\partial x} + \mu \frac{\partial}{\partial y}\left(\nabla^2 \psi\right) - \sigma_e B_0^2 \frac{\partial \psi}{\partial y} \tag{7}$$

$$-\rho i\omega \frac{\partial \psi}{\partial x} = -\frac{\partial p}{\partial y} + \mu \frac{\partial}{\partial x}\left(\nabla^2 \psi\right) - \sigma_e B_0^2 \frac{\partial \psi}{\partial x} \tag{8}$$

Differentiating equations (7) & (8) with respect to '*y*' & '*x*' Partially, we get

$$\frac{\partial^2 p}{\partial x \partial y} = \mu \frac{\partial^2}{\partial y^2}\left(\nabla^2 \psi\right) - \rho i\omega \frac{\partial^2 \psi}{\partial y^2} - \sigma_e B_0^2 \frac{\partial^2 \psi}{\partial y^2} \tag{9}$$

$$\frac{\partial^2 p}{\partial x \partial y} = -\mu \frac{\partial^2}{\partial x^2}\left(\nabla^2 \psi\right) + \rho i\omega \frac{\partial^2 \psi}{\partial x^2} - \sigma_e B_0^2 \frac{\partial^2 \psi}{\partial x^2} \tag{10}$$

From (9) and (10), we have

$$\left[\nabla^2 - \left(\frac{i\omega\rho + \sigma_e B_0^2}{\mu}\right)\right]\nabla^2 \psi = 0 \tag{11}$$

The equation of continuity can be satisfied by a stream suction of the form

$$\psi(x,\eta) = h\left(\frac{u_0}{a} - \frac{v_2 x}{h}\right) f(\eta) \tag{12}$$

Where $\eta = \frac{y}{h}$, $a = 1 - \frac{v_1}{v_2}$, $0 \le v_1 \le v_2$ and $u_0$ is the average velocity.

Substituting (12) in (11), we have

$$f^{iv}(\eta) - \alpha^2 h^2 f''(\eta) = 0, \tag{13}$$

Where $\alpha^2 = \frac{i\rho\omega + \sigma_e B_0^2}{\mu}$

## VI. Mathematical Solution of Formation of the Problem

Equation (13) reduces to the form

$$(D^4 - \alpha^2 h^2 D^2) f(\eta) = 0 \tag{14}$$

with the boundary conditions

$$\left.\begin{aligned} f(0) &= 1 - a, \quad f(1) = 1 \\ f'(0) &= 0, \qquad f'(1) = 0 \end{aligned}\right\} \tag{15}$$

Hence the solution of (14) subjecting to the boundary condition (15) is

$$f(\eta) = \frac{1}{2\alpha h \sinh \alpha h + 4(1 - \cosh \alpha h)} \times \begin{bmatrix} \alpha h \sinh \alpha h(1 - a + a\eta) + a(\cosh \alpha h(\eta - 1) - \cosh \alpha h \eta) \\ + (a - 2)\cosh \alpha h + (2 - a) \end{bmatrix}$$

Substituting the value of $f(\eta)$ in the stream function

$$\psi(x,\eta) = h\left(\frac{u_0}{a} - \frac{v_2 x}{h}\right) f(\eta)$$

Hence

$$\begin{aligned} u &= u(x,y)e^{i\omega t} \\ &= \frac{\partial \psi}{\partial y} e^{i\omega t} \\ &= \left(\frac{u_0}{a} - \frac{v_2 x}{h}\right)\left[a\alpha h e^{i\omega t}\left(\frac{2\sin \alpha h - 2\sin \alpha(y - h) - 2\cos \alpha y}{2\alpha h \sinh \alpha h + 4(1 - \cosh \alpha h)}\right)\right] \end{aligned}$$

$$\begin{aligned} v &= v(x,y)e^{i\omega t} \\ &= -\frac{\partial \psi}{\partial x} e^{i\omega t} \\ &= \frac{v_2 e^{i\omega t}}{2\alpha h \sinh \alpha h + 4(1 - \cosh \alpha h)} \times \begin{bmatrix} (1 - a)[4 + 2\alpha h \sin \alpha h - 2\cos \alpha h - 2a\cos \alpha h + 2a] \\ + a[2\alpha y \sin \alpha h + 2\cos \alpha(y - h) + 2\cos \alpha y \end{bmatrix} \end{aligned}$$

The Pressure drop is given up

$$p(x,\eta)-p(0,0)=K\left(\frac{u_0}{a}-\frac{v_2 x}{2h}\right)x+\frac{\mu v_2}{h^2}f'(\eta)-i\rho\omega v_2\int_0^{\eta} f(\eta)d\eta$$

Where $K=\dfrac{\mu}{h^2}f'''(0)$

## VII. GRAPHICAL REPRESENTATION OF ANALYTICAL SOLUTION

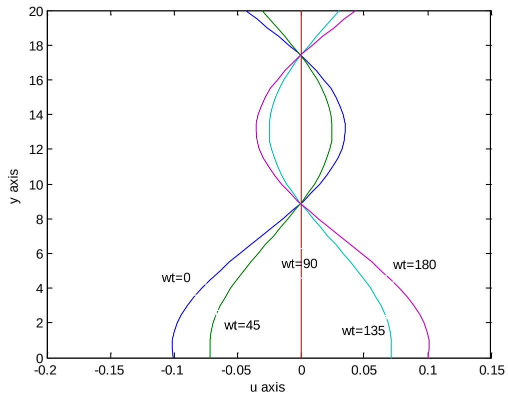


Fig. 1 u0=1.5;v2=-1;x=1;y=0:0.5:20;h=20;a=1; α=0.25.

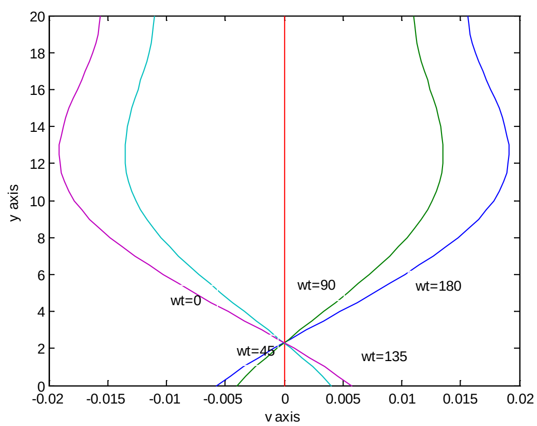


Fig. 2 u0=1.5;v2=-1;x=1;y=0:0.5:20;h=20;a=1; α=0.25

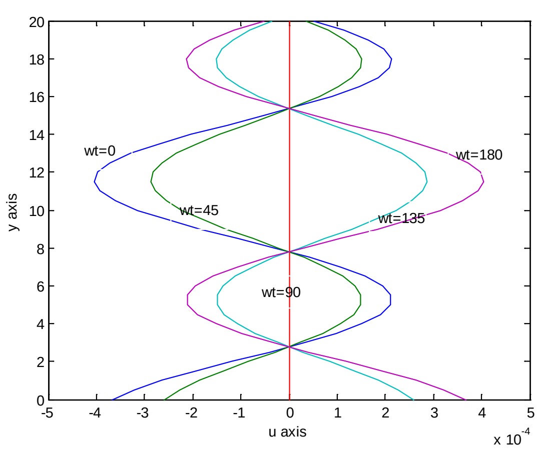


Fig. 3 u0=1.5;v2=-1;x=1;y=0:0.5:20;h=20;a=1; α=0.5.

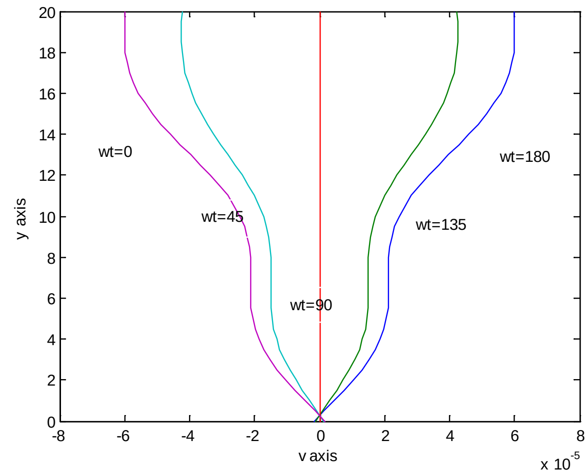


Fig. 4 u0=1.5;v2=-1;x=1;y=0:0.5:20;h=20;a=1; α=0.5.

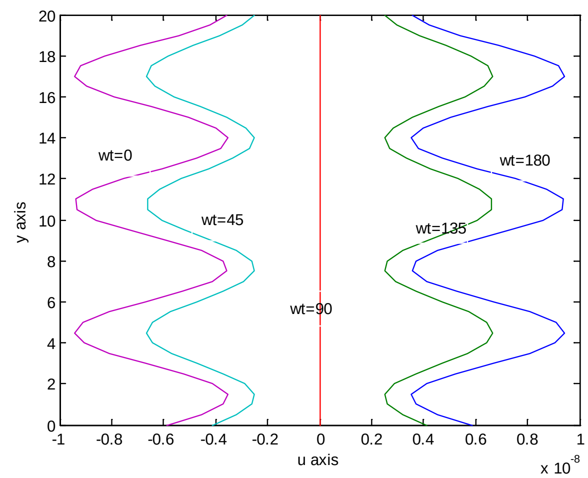


Fig. 5 u0=1.5;v2=-1;x=1;y=0:0.5:20;h=20;a=1; α=1

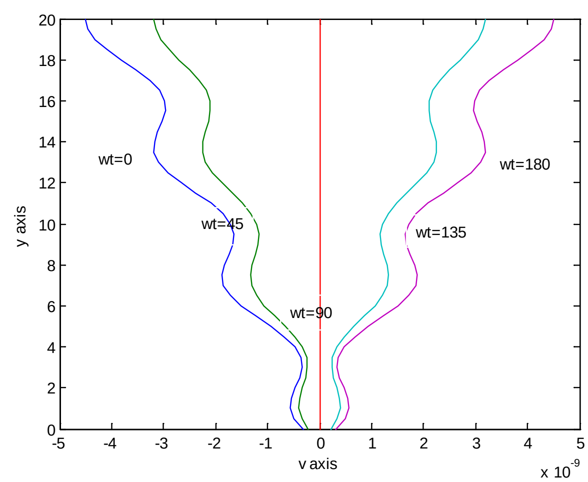


Fig. 6 u0=1.5;v2=-1;x=1;y=0:0.5:20;h=20;a=1; α=1

## VIII. CONCLUSION

In this Paper discuss the transient Hartman flow of a conducting fluid under the influence of magnetic field has been studied, considering the two parallel plate effects in the presence of uniform suction and injection. An analytical solution for the equations of motion has been developed while the differential equation has been solved analytically. The effect of magnetic field, suction and injection on the velocity and pressure distribution has been investigated. It is found that the effect of the suction and injection velocity u and v depends upon the magnetic field. For large value of magnetic field increasing MHD flow increases u. For small values of magnetic field increasing the MHD flow slightly decreases u. An exact solution of unsteady transverse MHD flow between parallel porous plates both at rest with uniform suction. External uniform axial and transverse magnetic field and uniform suction and injection are applied perpendicularly to the plates while the fluid motion is subjected to exponential axial and transverse velocity and pressure gradient. The solution of the problem is obtained with the aid of mathematical technique. Analytical expression is given for the velocity field and the effect of the various parameters entering into the problem is discussed with the help of graph.

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