The Free Vibration Analysis of Nonlocal SLGS Based on Various Plate Theories under Magnetic Field

Mehdi Mohammadimehr, Ali Ghorbanpour Arani, and Borhan Rousta Navi

Abstract— In this article, the nonlocal natural frequency of single layer nano graphene sheet using various plate theories is presented. SLGS is separately subjected to magnetic fields in x, y and z directions. Various plate theories including classic plate (CPT), first order shear deformation (FSDT), third order shear deformation (TSDT), Ambartsumain, Reissner, Touratier, Aydogdu theories are used in this article. The equations of motion for the nonlocal SLGS are derived using Hamilton's principle. Navier's solution is employed to obtain the nonlocal natural frequencies of SLGS. Compression between various plate theories showed that the results of Reissner plate theory is very closer to the obtained FSDT results and also the results of Touratier and Aydogduplate theories is coincident with TSDT results. The nonlocal natural frequenciy of single layer graphene sheet (SLGS) increases with an increase in the magnetic fields. Effect of magnetic field in z direction on the nonlocal natural frequencies of SLGS is higher than the effects of magnetic fields in x and y directions. Influence of magnetic field on the higher nonlocal natural frequencies is higher than that of for the lower nonlocal natural frequencies. Influence of magnetic fields on the nonlocal natural frequencies for Aydogdu and TSDT is higher than that of other plate theories.

Keywords— Natural vibration, magnetic fields, single layer graphene sheet (SLGS), various plate theories.

I. Introduction

TODAY, graphene sheet (a single atomic layer of carbon) attracted many researchers due to its exceptional mechanical [1] and physical [2,3] properties. Because of special electronic properties and high surface area, it has high potential for application in field-effect transistors or quantum reflective switches [7-10], optoelectronics [11] and nano-electro-mechanical systems (NEMS) [6]. Also single layer graphen sheet (SLGS) or multi layer grapheme sheet (MLGS) are employed as producing magneto-optical effects [12-13].

Mantari and Soares [14] presented generalized hybrid quasi-3D shear deformation theory for bending analysis of functionally graded plates. Adequate distribution of the transverse shear strains through the plate thickness is considered in their theory and the shear correction factor is equal to zero. Their results show that the obtained results by the new HSDTs are more accurate than the well-known trigonometric HSDT, if both of them have the same 6DOF.

Static and dynamic analysis of functionally graded plates using four-variable refined plate theory is studied by Mechab et al. [15]. They presented the new shear function the static and dynamic analysis of functionally graded plates. Their results had a good agreement with solution of Reddy. Viola et al. [16] studied general higher-order shear deformation theories for the free vibration analysis of laminated shells and panels. The shear correction factor is not used in this theory. They illustrated that use of new shear functions reduces independent parameters. A refined trigonometric shear deformation theory (RTSDT) for the thermo-elastic bending analysis of functionally graded sandwich plates is developed by Tounsi et al [17]. In this theory, number of unknown functions is equal to four while in other theory, it is equal to five. The displacement components are written as trigonometric series through the thickness of plate. They showed that their proposed theory is accurate and appropriate for analysis of thermo-elastic bending behavior of functionally graded plates. Thai and Choi [18] investigated the bending and vibration of functionally graded plates resting on elastic foundation using Zeroth order shear deformation theory. In this theory, shear forces is used instead of rotational displacements. Their results showed that bending and vibration responses of functionally graded plates are precisely predicted by this theory. Bending analysis of sandwich beams using a new nonlinear higher order theory is investigated by Dariushi and Sadighi [19]. Potential energy principle and Ritz method are used to obtain the nonlinear equations. They showed that results of nonlinear model had a good agreement with the experimental results. A trigonometric plate theory with 5-unknowns and stretching effect for bending of advanced composite plates are developed by Mantari and Soares [20]. They concluded that this theory has more accurate results for polynomial graded plates and good results as the refined quasi-3D HSDTs for exponentially graded plates.

A few researcher employed new plate theories on the nanoplate. Malekzadeh and Shojaee [21] extended nonlocal two-variable refined plate theory for the free vibration analysis of nanoplates. They used the quadratic function for the

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transverse shear strains through the nanoplate thickness. Comparing their results by other literatures shows a good agreement between them.

In this research, the natural frequencies of SLGS using various plate theories are obtained. Influence of magnetic fields on the natural frequencies of SLGS is investigated.

II. Derivation of Motion Equations

The displacements of SLGS according to the new plate theory are expressed as:

$$
\begin{aligned}
U(x,y,z) &= u(x,y,t) - z\frac{\partial w}{\partial x} + f(z)u_1 \\
V(x,y,z) &= v(x,y,z) - z\frac{\partial w}{\partial y} + g(z)v_1 \\
W(x,y,z) &= w(x,y)
\end{aligned}
\tag{1}
$$

Where $f(z)$, $g(z)$ are distribution function of transverse shear strain and it is assumed that they equal to each other. u , v and w are middle plane displacement components in x, y and z directions, respectively. u_1 and v_1 are shear displacements in x and y directions, respectively.

For various plate theories, $f(z)$ can be written as:

CPT: $f(z) = 0$

FSDT: $f(z) = z$

TSDT: $f(z) = z(1 - 4z^2/3h^2)$

Reissner [22]: $f(z) = \frac{5z}{4}(1 - 4z^2/3h^2)$

Ambartsumain [23]: $f(z) = \frac{z}{2}(h^2/4 - z^2/3)$

Touratier [24]: $f(z) = \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)$

Aydugdo [25]: $f(z) = z\,3^{\frac{-2\left(\frac{z}{h}\right)^2}{\ln 3}}$

Using Eq. (1), the relationships between strain and displacements can be expressed as:

$$
\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x} = u_{,x} - zw_{,xx} + f(z)u_{1,x} \\
\varepsilon_y &= \frac{\partial v}{\partial y} = v_{,y} - zw_{,yy} + f(z)v_{1,y} \\
\varepsilon_z &= \frac{\partial w}{\partial z} = 0 \\
\gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = f'(z)v_1 \\
\gamma_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = f'(z)u_1 \\
\gamma_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} = u_{,y} + v_{,x} + f(z)u_{1,y} + f(z)v_{1,x} - 2zw_{,xy}
\end{aligned}
\tag{2}
$$

The constitutive equations for SLGS can be written as:

$$
\begin{aligned}
\sigma_x &= \frac{E}{(1-\nu^2)}(\varepsilon_x + \nu\varepsilon_y) \\
\sigma_y &= \frac{E}{(1-\nu^2)}(\varepsilon_y + \nu\varepsilon_x) \\
\sigma_{xy} &= \frac{E\,\varepsilon_{xy}}{2(1+\nu)} \\
\sigma_{xz} &= \frac{E\,\varepsilon_{xz}}{2(1+\nu)} \\
\sigma_{yz} &= \frac{E\,\varepsilon_{yz}}{2(1+\nu)}
\end{aligned}
\tag{3}
$$

Variation of strain energy and kinematic energy can be written as:

$$
\begin{aligned}
\delta U &= \int_V \left(\sigma_x\delta\varepsilon_x + \sigma_y\delta\varepsilon_y + \sigma_{xy}\delta\varepsilon_{xy} + \sigma_{xz}\delta\varepsilon_{xz} + \sigma_{yz}\delta\varepsilon_{yz}\right)dV \\
&= -\int_A \begin{pmatrix} N_{x,x}\delta u + K_{x,x}\delta u_1 + M_{x,xx}\delta w + N_{y,y}\delta v + K_{y,y}\delta v_1 + M_{y,yy}\delta w + N_{xy,y}\delta u + \\ K_{xy,y}\delta u_1 + 2M_{xy,xy}\delta w + N_{xy,x}\delta v + K_{xy,x}\delta v_1 - K_{xz}\delta u_1 - K_{yz}\delta v_1 \end{pmatrix} dA
\end{aligned}
\tag{4}
$$

Where N_{ij} , M_{ij} and K_{ij} are the resultant forces, the resultant moments and shear force, respectively, which is specified by the following form:

$$
\begin{aligned}
\delta T &= \rho\int_V \left(\frac{\partial U}{\partial t}\frac{\partial \delta U}{\partial t} + \frac{\partial V}{\partial t}\frac{\partial \delta V}{\partial t} + \frac{\partial W}{\partial t}\frac{\partial \delta W}{\partial t}\right)dV \\
&= -\int_A \begin{pmatrix} \rho h\frac{\partial^2 u}{\partial t^2}\delta u + I_1\frac{\partial^2 u}{\partial t^2}\delta u_1 - \frac{\rho h^3}{12}\frac{\partial^4 w}{\partial t^2\partial x^2}\delta w - I_2\frac{\partial^3 w}{\partial t^2\partial x}\delta u_1 + I_1\frac{\partial^2 u_1}{\partial t^2}\delta u + \\ I_2\frac{\partial^3 u_1}{\partial t^2\partial x}\delta w + I_3\frac{\partial^2 u_1}{\partial t^2}\delta u_1 + \rho h\frac{\partial^2 v}{\partial t^2}\delta v + I_1\frac{\partial^2 v}{\partial t^2}\delta v_1 - \frac{\rho h^3}{12}\frac{\partial^4 w}{\partial t^2\partial y^2}\delta w - \\ I_2\frac{\partial^3 w}{\partial t^2\partial v}\delta v_1 + I_1\frac{\partial^2 v_1}{\partial t^2}\delta v + I_2\frac{\partial^3 v_1}{\partial t^2\partial y}\delta w + I_3\frac{\partial^2 v_1}{\partial t^2}\delta v_1 + \rho h\frac{\partial^2 w}{\partial t^2}\delta w \end{pmatrix} dA
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} &= \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz \\
\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} &= \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} z\,dz \\
\begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} &= \int_{-h/2}^{h/2} \begin{Bmatrix} f(z)\sigma_x \\ f(z)\sigma_y \\ f(z)\tau_{xy} \end{Bmatrix} dz, \quad \begin{Bmatrix} K_{xz} \\ K_{yz} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} kf'(z)\sigma_{xz} \\ kf'(z)\sigma_{yz} \end{Bmatrix} dz
\end{aligned}
\tag{6}
$$

Where k is the shear correction factor which used only for FSDT and equal to $5/6$.

Also coefficients of I_1, I_2 and I_3 are determined as:

$$I_1=\int_{-h/2}^{h/2}\rho f\left(z\right)dz\,,\; I_2=\int_{-h/2}^{h/2}\rho z f\left(z\right)dz \qquad\qquad (7)$$
$$I_3=\int_{-h/2}^{h/2}\rho f^{\,2}\left(z\right)dz$$

Variation of work done due to the magnetic field using Maxwell equations [26] can be obtained as:

$$\begin{aligned}
\vec{h} &= \nabla\times\left(\vec{U}\times\vec{H}\right)\\
\vec{J} &= \nabla\times\vec{h}\\
\vec{f_l} &= \eta\left(\vec{J}\times H\right)\\
\delta W_{ext}^{f_l} &= \int_V\left(f_{xl}\delta U + f_{yl}\delta V + f_{zl}\delta W\right)dV
\end{aligned} \qquad (8)$$

where $\vec{U}$, $\vec{H}$, $\vec{h}$, $\vec{J}$, $\vec{f_l}$ and η are the displacement vector, magnetic vector, perturbation of magnetic field vector, electric current density vector, Lorentz force and magnetic permeability, respectively.

If magnetic field is only applied in z direction $\left(0,0,H_z\right)$, variation of work done by Lorentz force can be written as:

$$\delta W_{ext}^{fl}=\int_A\eta H_z^2\begin{pmatrix}\left(hu_{,xy}+h_1u_{1,xy}+h_1v_{1,yy}+h_2v_1+hv_{,yy}\right)\delta v+\\ \left(h_1u_{,xy}-h_3w_{,xxy}+h_4u_{1,xy}+h_4v_{1,yy}+h_5v_1+h_1v_{,yy}+hv_{,yy}-h_3w_{,yyy}\right)\delta v_1+\\ \left(\frac{-\rho h^3}{12}w_{,xxyy}+h_3u_{1,xyy}+h_3v_{1,yyy}+h_6v_{1,y}-\frac{\rho h^3}{12}w_{,yyyy}\right)\delta w\end{pmatrix}dA \qquad (9)$$

In the same way, if magnetic field vector is only applied in y direction, then the variation of work done by Lorentz force can be obtained as:

$$\delta W_{ext}^{fl}=\int_A\eta H_z^2\begin{pmatrix}\left(hu_{,xx}+h_1u_{1,xx}+h_1u_{1,yy}+hu_{,yy}\right)\delta u+\\ \left(h_1u_{,xx}-h_3w_{,xxx}+h_4u_{1,xx}+h_1u_{,yy}+h_4u_{1,yy}-h_3w_{,xyy}\right)\delta u_1+\\ \left(\frac{-\rho h^3}{12}w_{,xxyy}+h_3u_{1,xxx}+h_3u_{1,yyx}+w_{,yy}-w_{,xx}+h_6u_{1,x}-\frac{\rho h^3}{12}w_{,xxxx}\right)\delta w\end{pmatrix}dA \qquad (10)$$

Also for magnetic field in x direction, variation of work done by Lorentz force can be expressed as:

$$\delta W_{ext}^{fl}=\int_A\eta H_z^2\begin{pmatrix}\left(hv_{,xx}+h_1v_{1,xx}+h_1v_{1,yy}+hv_{,yy}\right)\delta v+\\ \left(h_1v_{,xx}-h_3w_{,yxx}+h_4v_{1,xx}+h_1v_{1,yy}+h_4v_{1,yy}-h_3w_{,yyy}\right)\delta v_1+\\ \left(\frac{-\rho h^3}{12}w_{,xxyy}+h_3v_{1,xxy}+h_3v_{1,yyy}-w_{,yy}+w_{,xx}+h_6v_{1,y}-\frac{\rho h^3}{12}w_{,yyyy}\right)\delta w\end{pmatrix}dA \qquad (11)$$

Hamilton's principle yields the following equation:

$$\int\left(\delta T+\delta W_{ext}-\delta U-\delta U^{s}\right)dt=0 \qquad (12)$$

Substituting Eqs. (4), (5) and (9) into Eq. (12) and separating of variables, the governing equations of motion for SLGS can be written as:

$$\begin{aligned}
&\frac{Eh}{\left(1-\nu^2\right)}u_{,xx}+a_1u_{1,xx}+\frac{Eh\nu}{\left(1-\nu^2\right)}v_{,xy}+a_1v_{1,xy}+\frac{Eh}{2\left(1+\nu\right)}\left(u_{,yy}+v_{,xy}\right)+\\
&\quad a_3\left(u_{1,yy}+v_{1,xy}\right)=\left(1-\left(e_0a\right)^2\nabla^2\right)\left(\rho h\frac{\partial^2u}{\partial t^2}+I_1\frac{\partial^2u_1}{\partial t^2}\right)\\
&a_1u_{,xx}-a_4w_{,xxx}+a_5u_{1,xx}+a_6v_{,xy}-a_7w_{,yyx}+a_4v_{1,xy}+a_3u_{,yy}+a_3v_{,xy}+\\
&\quad +a_9u_{1,yy}+a_9v_{1,xy}-2a_{10}w_{,xyy}+a_9\left(u_{1,yy}+v_{1,xy}\right)-a_{11}u_1=\\
&\quad \left(1-\left(e_0a\right)^2\nabla^2\right)\left(I_1\frac{\partial^2u}{\partial t^2}-I_2\frac{\partial^3w}{\partial t^2\partial x}+I_3\frac{\partial^2u_1}{\partial t^2}\right)\\
&\frac{Eh\nu}{\left(1-\nu^2\right)}u_{,xy}+a_6u_{1,xy}+\frac{Eh}{\left(1-\nu^2\right)}v_{,yy}+a_2v_{1,yy}+\frac{Eh}{2\left(1+\nu\right)}\left(u_{,xy}+v_{,xx}\right)+\\
&\quad a_3\left(u_{1,xy}+v_{1,xx}\right)+\left(1-\left(e_0a\right)^2\nabla^2\right)\eta H_z^2\left(hu_{,xy}+h_1u_{1,xy}+h_1v_{1,yy}+h_2v_1+hv_{,yy}\right)=\\
&\quad \left(1-\left(e_0a\right)^2\nabla^2\right)\left(\rho h\frac{\partial^2v}{\partial t^2}+I_1\frac{\partial^2v_1}{\partial t^2}\right)\\
&a_6u_{,xy}-a_7w_{,xxy}+a_8u_{1,xy}+a_2v_{,yy}-a_{13}w_{,yyy}+a_{12}v_{1,yy}+a_3u_{,xy}+a_3v_{,xx}+\\
&\quad +a_9u_{1,yy}+a_9v_{1,xy}-2a_{10}w_{,xxy}+a_9\left(u_{1,xy}+v_{1,xx}\right)-a_{14}v_1+\\
&\quad \eta H_z^2\left(h_1u_{,xy}-h_3w_{,xxy}+h_4u_{1,xy}+h_4v_{1,yy}+h_5v_1+h_1v_{,yy}+hv_{,yy}-h_3w_{,yyy}\right)=\\
&\quad \left(1-\left(e_0a\right)^2\nabla^2\right)\left(I_1\frac{\partial^2v}{\partial t^2}-I_2\frac{\partial^3w}{\partial t^2\partial y}+I_3\frac{\partial^2v_1}{\partial t^2}\right)\\
&-\frac{Eh^3}{12\left(1-\nu^2\right)}w_{,xxxx}-\left(\frac{Eh^3\nu}{6\left(1-\nu^2\right)}+\frac{Eh^3}{6\left(1+\nu\right)}\right)w_{,xxyy}-\frac{Eh^3}{12\left(1-\nu^2\right)}w_{,yyyy}+a_4u_{1,xxx}+\\
&\quad a_7v_{1,yxx}+a_7u_{1,yyx}+a_{13}v_{1,yyy}+2a_{10}v_{1,yxx}+2a_{10}u_{1,yyx}+\\
&\quad \eta H_z^2\left(1-\left(e_0a\right)^2\nabla^2\right)\left(-\frac{h^3}{12}w_{,xxyy}+h_3u_{1,xyy}+h_3v_{1,yyy}+h_6v_{1,y}-\frac{h^3}{12}w_{,yyyy}\right)=\\
&\quad \left(1-\left(e_0a\right)^2\nabla^2\right)\left(-\frac{\rho h^3}{12}\frac{\partial^4w}{\partial t^2\partial x^2}-\frac{\rho h^3}{12}\frac{\partial^4w}{\partial t^2\partial y^2}+I_2\frac{\partial^3u_1}{\partial t^2\partial x}+I_2\frac{\partial^3v_1}{\partial t^2\partial y}\right)
\end{aligned} \qquad (14)$$

$$\begin{aligned}
&N_{x,x}+N_{xy,y}=\rho h\frac{\partial^2u}{\partial t^2}+I_1\frac{\partial^2u_1}{\partial t^2}\\
&K_{x,x}+K_{xy,y}-K_{xz}=I_1\frac{\partial^2u}{\partial t^2}-I_2\frac{\partial^3w}{\partial t^2\partial x}+I_3\frac{\partial^2u_1}{\partial t^2}\\
&N_{y,y}+N_{xy,x}+\\
&\qquad \eta H_z^2\left(hu_{,xy}+h_1u_{1,xy}+h_1v_{1,yy}+h_2v_1+hv_{,yy}\right)=\rho h\frac{\partial^2v}{\partial t^2}+I_1\frac{\partial^2v_1}{\partial t^2}\\
&K_{y,y}+K_{xy,x}-K_{yz}+\\
&\quad \eta H_z^2\left(h_1u_{,xy}-h_3w_{,xxy}+h_4u_{1,xy}+h_4v_{1,yy}+h_5v_1+h_1v_{,yy}+hv_{,yy}-h_3w_{,yyy}\right)=\\
&\quad I_1\frac{\partial^2v}{\partial t^2}-I_2\frac{\partial^3w}{\partial t^2\partial y}+I_3\frac{\partial^2v_1}{\partial t^2}\\
&M_{x,xx}+M_{y,yy}+2M_{xy,xy}+\\
&\quad \eta H_z^2\left(-\frac{\rho h^3}{12}w_{,xxyy}+h_3u_{1,xyy}+h_3v_{1,yyy}+h_6v_{1,y}-\frac{\rho h^3}{12}w_{,yyyy}\right)=\\
&\quad -\frac{\rho h^3}{12}\frac{\partial^4w}{\partial t^2\partial x^2}-\frac{\rho h^3}{12}\frac{\partial^4w}{\partial t^2\partial y^2}+I_2\frac{\partial^3u_1}{\partial t^2\partial x}+I_2\frac{\partial^3v_1}{\partial t^2\partial y}
\end{aligned} \qquad (13)$$

Using the nonlocal Eringen theory [27] and substituting Eq. (6) into Eq. (13) yields:

$$\begin{aligned}
&\frac{Eh}{\left(1-\nu^2\right)}u_{,xx}+a_1u_{1,xx}+\frac{Eh\nu}{\left(1-\nu^2\right)}v_{,xy}+a_1v_{1,xy}+\frac{Eh}{2\left(1+\nu\right)}\left(u_{,yy}+v_{,xy}\right)+\\
&\quad a_3\left(u_{1,yy}+v_{1,xy}\right)=\left(1-\left(e_0a\right)^2\nabla^2\right)\left(\rho h\frac{\partial^2u}{\partial t^2}+I_1\frac{\partial^2u_1}{\partial t^2}\right)\\
&a_1u_{,xx}-a_4w_{,xxx}+a_5u_{1,xx}+a_6v_{,xy}-a_7w_{,yyx}+a_4v_{1,xy}+a_3u_{,yy}+a_3v_{,xy}+\\
&\quad +a_9u_{1,yy}+a_9v_{1,xy}-2a_{10}w_{,xyy}+a_9\left(u_{1,yy}+v_{1,xy}\right)-a_{11}u_1=\\
&\quad \left(1-\left(e_0a\right)^2\nabla^2\right)\left(I_1\frac{\partial^2u}{\partial t^2}-I_2\frac{\partial^3w}{\partial t^2\partial x}+I_3\frac{\partial^2u_1}{\partial t^2}\right)\\
&\frac{Eh\nu}{\left(1-\nu^2\right)}u_{,xy}+a_6u_{1,xy}+\frac{Eh}{\left(1-\nu^2\right)}v_{,yy}+a_2v_{1,yy}+\frac{Eh}{2\left(1+\nu\right)}\left(u_{,xy}+v_{,xx}\right)+\\
&\quad a_3\left(u_{1,xy}+v_{1,xx}\right)+\left(1-\left(e_0a\right)^2\nabla^2\right)\eta H_z^2\left(hu_{,xy}+h_1u_{1,xy}+h_1v_{1,yy}+h_2v_1+hv_{,yy}\right)=\\
&\quad \left(1-\left(e_0a\right)^2\nabla^2\right)\left(\rho h\frac{\partial^2v}{\partial t^2}+I_1\frac{\partial^2v_1}{\partial t^2}\right)\\
&a_6u_{,xy}-a_7w_{,xxy}+a_8u_{1,xy}+a_2v_{,yy}-a_{13}w_{,yyy}+a_{12}v_{1,yy}+a_3u_{,xy}+a_3v_{,xx}+\\
&\quad +a_9u_{1,yy}+a_9v_{1,xy}-2a_{10}w_{,xxy}+a_9\left(u_{1,xy}+v_{1,xx}\right)-a_{14}v_1+\\
&\quad \eta H_z^2\left(h_1u_{,xy}-h_3w_{,xxy}+h_4u_{1,xy}+h_4v_{1,yy}+h_5v_1+h_1v_{,yy}+hv_{,yy}-h_3w_{,yyy}\right)=\\
&\quad \left(1-\left(e_0a\right)^2\nabla^2\right)\left(I_1\frac{\partial^2v}{\partial t^2}-I_2\frac{\partial^3w}{\partial t^2\partial y}+I_3\frac{\partial^2v_1}{\partial t^2}\right)\\
&-\frac{Eh^3}{12\left(1-\nu^2\right)}w_{,xxxx}-\left(\frac{Eh^3\nu}{6\left(1-\nu^2\right)}+\frac{Eh^3}{6\left(1+\nu\right)}\right)w_{,xxyy}-\frac{Eh^3}{12\left(1-\nu^2\right)}w_{,yyyy}+a_4u_{1,xxx}+\\
&\quad a_7v_{1,yxx}+a_7u_{1,yyx}+a_{13}v_{1,yyy}+2a_{10}v_{1,yxx}+2a_{10}u_{1,yyx}+\\
&\quad \eta H_z^2\left(1-\left(e_0a\right)^2\nabla^2\right)\left(-\frac{h^3}{12}w_{,xxyy}+h_3u_{1,xyy}+h_3v_{1,yyy}+h_6v_{1,y}-\frac{h^3}{12}w_{,yyyy}\right)=\\
&\quad \left(1-\left(e_0a\right)^2\nabla^2\right)\left(-\frac{\rho h^3}{12}\frac{\partial^4w}{\partial t^2\partial x^2}-\frac{\rho h^3}{12}\frac{\partial^4w}{\partial t^2\partial y^2}+I_2\frac{\partial^3u_1}{\partial t^2\partial x}+I_2\frac{\partial^3v_1}{\partial t^2\partial y}\right)
\end{aligned} \qquad (14)$$

Where e_0a and ∇^2 are the nonlocal parameter and laplacian operator, respectively.

To solve governing equations of the motion, Navier's type solution is considered as:

$$u(x,y,t)=\sum_{i=1}^{m}\sum_{j=1}^{n}u_{mn}e^{i\omega t}\cos(m\pi x/a)\sin(n\pi y/b)$$
$$v(x,y,t)=\sum_{i=1}^{m}\sum_{j=1}^{n}v_{mn}e^{i\omega t}\sin(m\pi x/a)\cos(n\pi y/b)$$
$$u_1(x,y,t)=\sum_{i=1}^{m}\sum_{j=1}^{n}u_{1mn}\cos(m\pi x/a)\sin(n\pi y/b) \quad (15)$$
$$v_1(x,y,t)=\sum_{i=1}^{m}\sum_{j=1}^{n}v_{1mn}\sin(m\pi x/a)\cos(n\pi y/b)$$
$$w(x,y)=\sum_{i=1}^{m}\sum_{j=1}^{n}w_{mn}\sin(m\pi x/a)\sin(n\pi y/b)$$

Substituting Eq. (15) into Eq. (14), the following equation can be obtained as:

$$\left(K-\omega^2 M\right)\begin{Bmatrix} w \\ \phi \end{Bmatrix}=0 \quad (16)$$

Where ω is the natural frequency of SLGS.

III. Results and discussion

Material properties of SLGS and other parameters which used in this research are listed in Table I.

TABLE I
MATERIAL PROPERTIES OF SLGS AND OTHER PARAMETERS

Elastic modulus	Poisson's ratio	Magnetic permeability	Density	Length	Width
1000GPa	0.3	$4\pi\times10^{-7}$ $Kg.m/A^2.\sec^2$	7000 Kg/m^3	10nm	10nm

Table II illustrated the non-dimensional natural frequencies of SLGS for various nonlocal parameter and different plate theories. It can be observed that the results of Reissner plate theory is very closer to FSDT results and also results of Touratier and Aydogdu plate theories is coincident with TSDT results. Furthermore the non-dimensional natural frequencies of SLGS decrease with increasing of the nonlocal parameter. Deference between results of various theories is considerable in higher natural frequencies.

In Fig. s 1, 2 and 3 depict first third natural frequencies against the nonlocal parameter, respectively. It can be seen that the natural frequencies decreases with increasing of the nonlocal parameter. Nonlocal natural frequencies of SLGS based on Reissner plate theory is approximately equal to that of based on FSDT. Results of other plate theory are close together.

TABLE II
THE NON-DIMENSIONAL NATURAL FREQUENCIES OF SLGS FOR VARIOUS PLATE THEORIES ($\bar{\omega}=\omega_{mn}h\sqrt{2\rho(1+\upsilon)/E}$)

$\bar{\omega}$	e_0a	CPT [28]	FSDT [28]	TSDT [28]	TVRPT [28]	Touratier
$\bar{\omega}_{11}$	0	0.0963	0.093	0.0935	0.0930	0.0935
	1	0.08	0.085	0.0854	0.0850	0.0854
	$\sqrt{2}$	0.0816	0.0788	0.0791	0.0788	0.0791
	$\sqrt{3}$	0.0763	0.0737	0.0741	0.0737	0.07401
$\bar{\omega}_{22}$	0	0.385	0.3414	0.345	0.341	0.347
	1	0.288	0.255	0.258	0.254	0.259
	$\sqrt{2}$	0.239	0.212	0.215	0.212	0.216
	$\sqrt{3}$	0.209	0.167	0.167	0.167	0.189
$\bar{\omega}_{33}$	0	0.867	0.689	0.702	0.644	0.709
	1	0.520	0.413	0.421	0.410	0.426
	$\sqrt{2}$	0.406	0.323	0.329	0.321	0.333
	$\sqrt{3}$	0.345	0.273	0.279	0.271	0.282

TABLE III
THE NON-DIMENSIONAL NATURAL FREQUENCIES OF SLGS FOR VARIOUS PLATE THEORIES ($\bar{\omega}=\omega_{mn}h\sqrt{2\rho(1+\upsilon)/E}$)

$\bar{\omega}$	e_0a	Reissner	Ambartsumain	Aydogdu
$\bar{\omega}_{11}$	0	0.0930	0.0943	0.0935
	1	0.0849	0.0861	0.0855
	$\sqrt{2}$	0.0787	0.0798	0.0792
	$\sqrt{3}$	0.0737	0.0747	0.0741
$\bar{\omega}_{22}$	0	0.340	0.356	0.347
	1	0.254	0.266	0.259
	$\sqrt{2}$	0.212	0.221	0.216
	$\sqrt{3}$	0.185	0.1948	0.189
$\bar{\omega}_{33}$	0	0.682	0.743	0.712
	1	0.409	0.446	0.428
	$\sqrt{2}$	0.319	0.348	0.334
	$\sqrt{3}$	0.271	0.295	0.283

The influence of magnetic fields on the first, second and third nonlocal natural frequencies of SLGS for various plate theories with respect to the nonlocal parameter is displayed in Fig. s 4, 5 and 6, respectively. It can be observed that results of various plate theories are very closer together. The nonlocal natural frequencies increase with the presence of magnetic fields for various plate theories. Furthermore, this increase rate foe magnetic field in z direction is higher than the magnetic fields in other direction.

IV. Conclusion

In this research, the free vibration of SLGS based on various plate theories such as CPT, FSDT, TSDT, Ambartsumain, Reissner, Touratier, Aydogdu theories subjected to the magnetic fields is investigated. The governing equations of motion for SLGS are obtained by Hamilton's principle. Navier's type solution is employed to solve these equations. Present results are compared by the results of other researchers. It is found that the natural frequencies increase with increasing of the magnetic field and decreases with an increase in the nonlocal parameter. Also the obtained results of Reissner are closer to FSDT results and also the results of Touratier and Aydogdu are coincident with TSDT results.

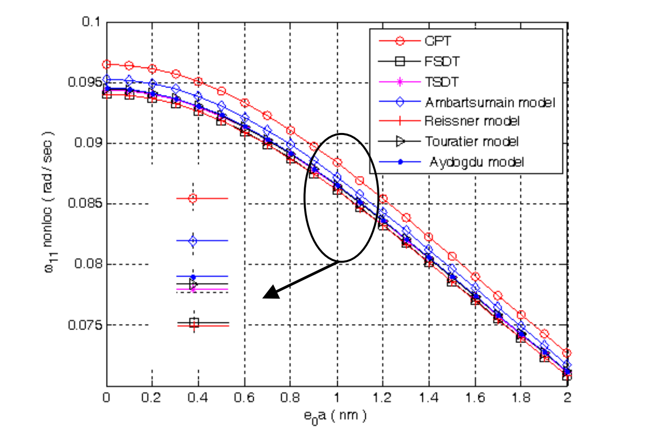

Fig. 1 The first natural frequency against the nonlocal parameter for various plate theories

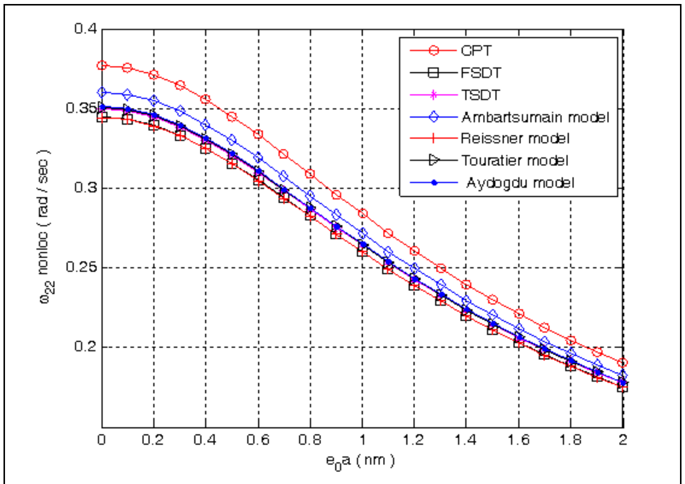

Fig. 2 The second natural frequency against the nonlocal parameter for various plate theories

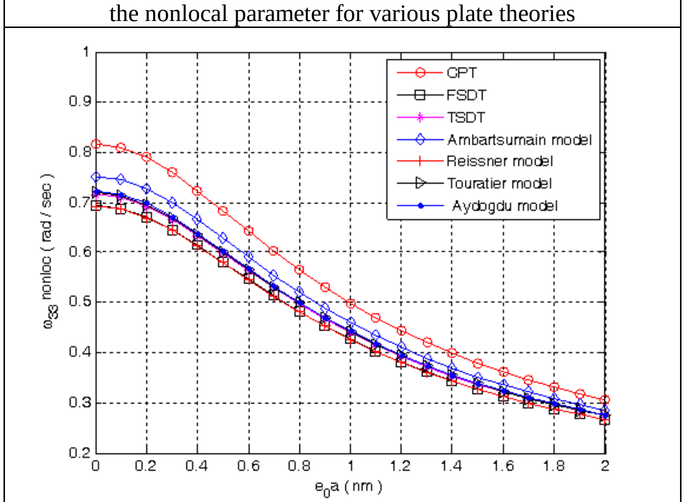

Fig. 3 The third natural frequency against the nonlocal parameter for various plate theories

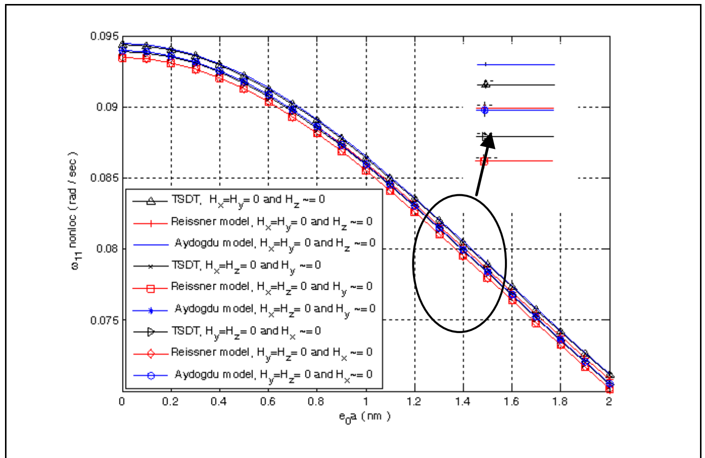

Fig. 4 The effect of magnetic fields on the first natural frequency for various plate theories

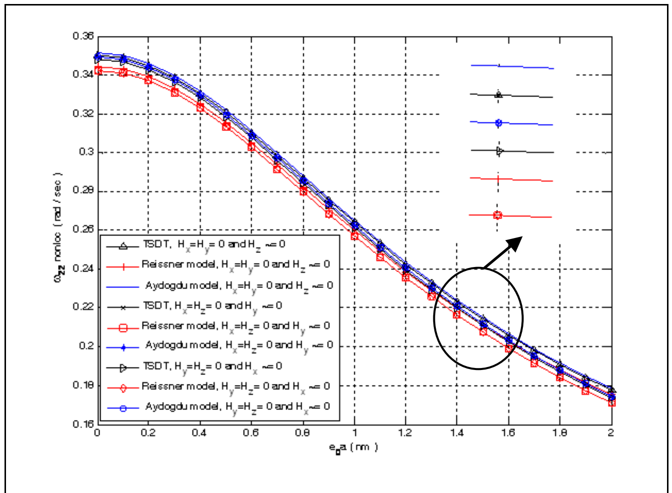

Fig. 5 The influence of magnetic fields on the second natural frequency for various plate theories

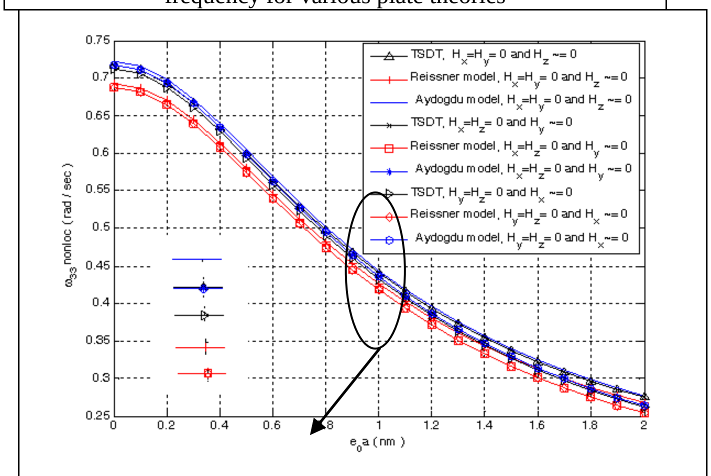

Fig. 6: The effect of magnetic fields on the third natural frequency for various plate theories

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