A Simple Approximate Expression for Optical Crosstalk in a Linear Free Space Optical Interconnects System that Uses Micro-Lenses with Circular Apertures

Nedal Al-Ababneh

***Abstract*—**In this paper a simple expression to calculate the optical crosstalk in a lens-based linear free space optical interconnects system is proposed. This simple expression is derived using an approximate closed form expression for the light intensity distribution at the detector plane that has been evaluated in terms of zeroth order and first order first kind Bessel functions. The numerical model for the crosstalk is used as a baseline to show the validity of the proposed model under the constraint of small micro-lens aperture.

***Keywords*—** Bessel functions. Diffraction Integral, Finite aperture, Free space optical interconnects.

I. Introduction

CROSSTALK evaluation is one of the most important issues in free space optical interconnects (FSOIs) system.

In these 2D optical systems the diffraction of the light in the presence of apertures, specially micro-lenses apertures, couples the light into the neighboring channels and causes a crosstalk. In general, the crosstalk is considered as a noise which degrades the performance of the FSOIs system [1-5]. The crosstalk can be evaluated depending on the light distribution at the detector plane which in turn can be evaluated using the generalized Collins diffraction integral formula [6]. Using Collins integral the light distribution under paraxial approximation can be given as a finite integral which is difficult to solve in a closed form. Therefore, numerical methods are used to evaluate integral. To improve the calculation efficiency, methods using complex Gaussian functions and infinite series of Bessel function were proposed to solve the finite integral [7-8].

In this paper, we introduce a simple model to deal with finite integral rather than using infinite series. In this model, the light irradiance is derived for the apertured FSOIs system by solving the finite integral in terms of zeroth order and first order Bessel function under the constraint of small aperture. Using this model, the crosstalk is estimated and compared to that of the numerical model.

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II. Output Optical Field using Numerical Model

The FSOIs system is shown in Fig. 1. This system contains VCSELs array, micro-lens array, and detectors array.

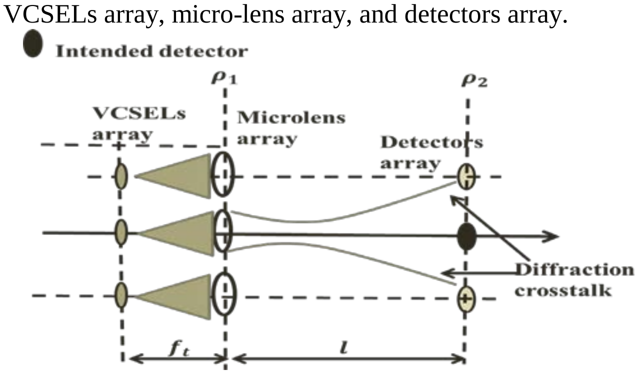

Fig. 1 Micro-lens based FSOIs

The VCSELs array is placed at the front focal length of the transmitter micro-lens. The distance between the detector array and the micro-lenses array is denoted by l . The field distribution at the detectors plane is found by using the generalized diffraction integral formula in cylindrical coordinate system assuming paraxial approximations [7, 8]

$$\mathrm{E}_2(\rho_2,\theta_2)=\frac{ik}{2\pi b}\int_0^{\infty}\int_0^{2\pi}\mathrm{E}_1(\rho_1,\theta_1)A(\rho_1)\times\exp\left[\begin{array}{l}-\frac{ika}{2b}\rho_1^2-\frac{idk}{2b}\rho_2^2\\+\frac{ik\rho_1\rho_2}{b}\cos(\theta_1-\theta_2)\end{array}\right]\rho_1 d\rho_1 d\theta_1 \quad (1)$$

ρ_1,θ_1 and ρ_2,θ_2 are the cylindrical coordinates at the input and output planes, respectively. $A(\rho_1)$ is the aperture function. $E_1(\rho_1,\theta_1)$ is the input field, $k=2\pi/\lambda$ is the wave number, and λ is the wavelength. *a*, *b*, *c*, and *d* are the transfer matrix elements of the FSOIs system and are given by:

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$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1-(l/f) & l \\ -1/f & 1 \end{bmatrix} \quad (2)$$

the input optical filed at the front surface of the micro-lens, assuming Gaussian model for the beam, can be given by

$$\mathrm{E}_1(\rho_1,\theta_1) = \exp\left[-\frac{\rho_1^2}{\omega_1^2}\right] \quad (3)$$

ω_1 the beam radius at the front surface of the transmitter micro-lens and is given by

$$\omega_1 = \omega_0\sqrt{1+\frac{\lambda^2 f^2}{\pi^2\omega_0^4}}$$

ω_0 is the waist radius of the VCSEL's beam. The aperture function of the circular micro-lens is

$$AP(\rho_1) = \begin{cases} 1 & \rho_1 \le a_1 \\ 0 & \rho_1 \succ a_1 \end{cases} \quad (5)$$

a_1 is the radius of the aperture. The integral in (1) can be evaluated by substituting (3) and (5) into (1) and using the following integral

$$J_0(x) = \frac{1}{2\pi}\int_0^{2\pi}\exp(ix\cos\theta)d\theta \quad (6)$$

to obtain

$$E_2(\rho_2) = \frac{ik}{b}\exp\left(-\frac{ikd\rho_2^2}{2b}\right)$$

$$\int_0^{a_1}\exp\left(-\rho_1^2/q^2\right)J_0\left(\frac{k\rho_2}{b}\rho_1\right)\rho_1 d\rho_1 \quad (7)$$

$$\frac{1}{q^2} = \frac{1}{\omega_1^2}+\frac{ika}{2b} \quad (8)$$

Equation (7) is a formula for the optical field at the detector array for a Gaussian beam propagating through an apertured lens based FSOIs system. This equation is difficult to express in analytical form because of the finite integral. In fact, this equation will be used to numerically calculate the crosstalk that will be used as a baseline to compare the results that can be obtained from the simple proposed model.

III. Proposed model

The integral in (7) can be solved in a closed form by noting that:

$$\exp\left(\frac{-\rho_1^2}{q^2}\right) = 1-\frac{\rho_1^2}{q^2}+\frac{1}{2!}\left(\frac{\rho_1^2}{q^2}\right)^2-\cdots \quad (9)$$

$$J_0\left(\frac{2\rho_1}{q}\right) = 1-\frac{\rho_1^2}{q^2}+\frac{1}{4}\left(\frac{\rho_1^2}{q^2}\right)^2-\cdots \quad (10)$$

Comparing (9) and (10), the exponential function be approximated as

$$\exp\left(\frac{-\rho_1^2}{q^2}\right) \approx J_0\left(\frac{2\rho_1}{q}\right) \quad (11)$$

(11)

Using (11), equation (7) can be written as

$$E_2(\rho_2) = \frac{ik}{b}\exp\left(-\frac{ikd\rho_2^2}{2b}\right)$$

$$\int_0^{a_1}J_0\left(\frac{2\rho_1}{q}\right)J_0\left(\frac{k\rho_2}{b}\rho_1\right)\rho_1 d\rho_1 \quad (12)$$

Performing the integral in (12), we get

$$E_2(\rho_2) = \frac{ik}{b}\exp\left(-\frac{ikd\rho_2^2}{2b}\right)\cdot a_1\cdot$$

$$\left[\frac{\frac{2}{q}J_1\left(\frac{2a_1}{q}\right)J_0\left(\frac{ka_1\rho_2}{b}\right)-\frac{k\rho_2}{b}J_1\left(\frac{ka_1\rho_2}{b}\right)J_0\left(\frac{2a_1}{q}\right)}{\left(\frac{2}{q}\right)^2-\left(\frac{k\rho_2}{b}\right)^2}\right] \quad (13)$$

It is important to mention that this approximated expression for the light distribution is more appropriate when the lens aperture is relatively small compared to the parameter given by equation (9) such that

$$\frac{a_1^2}{q^2} = a_1^2\left(\frac{1}{\omega_1^2}+\frac{ika}{2b}\right) \le 1 \quad (14)$$

The constraint given by the above equation relates the size of the aperture to some parameters in the optical system including the interconnect distance and the beam width. Therefore, the above constraint should be carefully examined to ensure more accurate results.

IV. Crosstalk calculation

The crosstalk as shown in Fig.1 results from the overlap between the intended detector and the diffracted light beams from other VCSELs through other transmitting micro-lenses. The overall crosstalk power received by the intended detector can be evaluated as the power received by all neighboring detectors from the light coming through intended micro-lens assuming only the intended VCSEL is on. Therefore, the diffraction induced crosstalk power can be given by

$$P_c = 4\iint_{A_1}\left|\mathrm{E}_2(x_2,y_2)\right|^2 dx_2 dy_2$$

$$+4\iint_{A_2}\left|\mathrm{E}_2(x_2,y_2)\right|^2 dx_2 dy_2 \quad (15)$$

A_1 and A_2 are the areas covered by one of the neighbor and one of the next neighbor detectors, respectively. The signal power is the power received by the intended detector from the light coming from the intended VCSEL through the intended micro-lens and is given by:

$$P_s = \iint_A \left|E_2(x_2, y_2)\right|^2 dx_2 dy_2 \qquad (16)$$

A is the area covered by intended detector. Having evaluated the signal and the crosstalk power, the signal to crosstalk ratio, SCR, can be determined as

$$SCR = \frac{P_s}{P_c} \qquad (17)$$

V. Numerical Simulations

In this section we consider an example in which λ=0.850 µm and ω_0= 10 µm for the VCSEL. The focal length is 720 µm. The distance between the detectors is 100 µm. To show the impact of using different aperture sizes on the SCR we plot in figures 2 and 3 the SCR versus the interconnect length for aperture radii of 15 µm and 20 µm, respectively. It is clear that the SCR for the FSOIs system using the smaller aperture is more close to that of the numerical model. This can be explained by the aid of using figures 4 and 5.In these figures, the constraint in equation 14 on using the proposed model is plotted versus the interconnect distance. It is clear from figure 3 that the 15 µm radius satisfies the constraint for almost all interconnect lengths. However, the 20 µm aperture doesn't satisfy the constraint for all interconnect lengths.

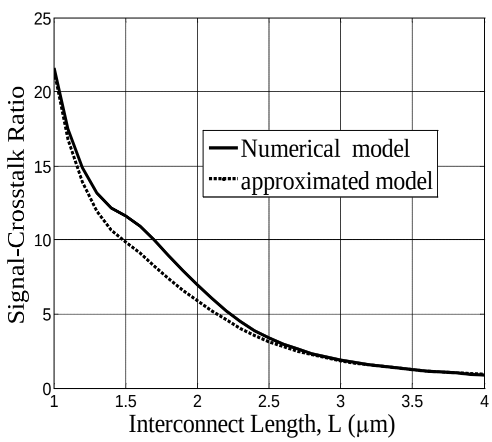

Fig. 2 SCR versus interconnect length with h_1=15 µm.

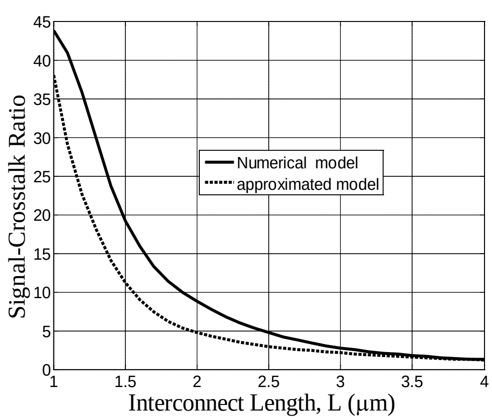

Fig. 3 SCR versus interconnect length with h_1=20 µm.

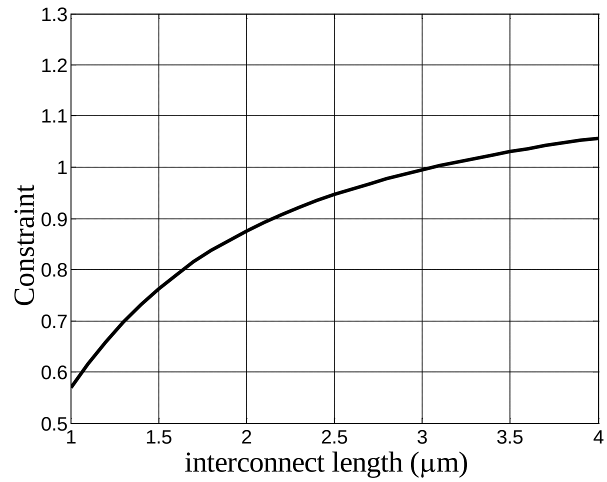

Fig. 4 Constraint versus interconnect length with h_1=15 µm.

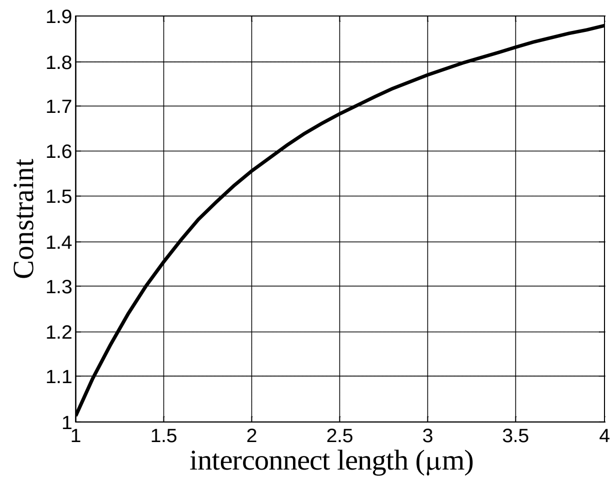

Fig. 5 SCR versus interconnect length with h_1=20 µm.

VI. Conclusion

We have introduced a simple expression for the optical crosstalk in first order FSOIs systems. The expression was derived by evaluating the finite integral in terms of zeroth order and first orders Bessel functions. An analytical formula for optical field at the detector plane was derived for the FSOIs system using this model. We have shown that the use of this simple model produced results which are closed to that of the numerical model under the constraint of small aperture size.

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